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Some Experimental Signaturesof the Standing Wave Braneworld

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We introduce the new 5D braneworld with the real scalar field in the bulk. The model represents the brane which bounds collective oscillations of standing gravitational and scalar fields waves in the bulk. These waves are out of phase, i.e. the energy of oscillations passes back and forth between the scalar and gravitational waves. We investigate two limiting cases depending on the values of the width of the brane and of the size of the horizon in extra space. When the amplitude of the standing waves is small these sizes coincide. In the case of large oscillations it is shown that the mass of the lightest KK mode is determined by the smaller scale, which corresponds to the horizon size. So KK modes can be born in accelerators at relatively low energies, what gives a chance to check the present model.

Braneworld models involving large extra dimensions have been very useful in addressing several open questions in high energy physics. Most of the braneworlds are realized as time-independent field configurations. However, in [1] there were proposed a non-stationary model, where the braneworld was generated by standing gravitational waves coupled to a phantom-like bulk scalar field, rapid oscillations of these waves provide universal gravitational trapping of zero modes of all kinds of matter fields on the brane [2].

Here we consider the new non-stationary 5D braneworld generated by standing waves of the gravitational and real scalar fields [3], instead of the phantom-like scalar fields of [1].

Let us consider 5D space-time without bulk cosmological constant containing a brane and a nonself interacting real scalar field coupled to gravity:

$$(1)\quad S=\int d^5x\sqrt{g}\left(\frac{M^3}{2}R+\frac{1}{2}g^{MN}\partial_M\varphi\partial_N\varphi+L_B\right),$$

L_B is the brane Lagranjian and M is the 5D fundamental scale. We assume that the 5D scalar field is a function of t and absolute value of the extra coordinate, $|r|$, and use the metric ansatz:

$$(2)\quad ds^2=\frac{e^S}{(1-a|r|)^{2/3}}(dt^2-dr^2)-(1-a|r|)^{2/3}(e^u dx^2+e^u dy^2+e^{-2u}dz^2),$$

Where a is a positive constant and $S=S(|r|)$, $u=u(t,|r|)$ are some functions.

For our setup the solution to the system of Einstein and scalar field equations is [3]:

$$(3)\quad u(t,|r|)=A\sin(\omega t)J_0\left(\frac{\omega}{a}-\omega|r|\right),$$

$$\varphi(t,|r|)=\sqrt{\frac{3M^3}{2}}A\cos(\omega t)J_0\left(\frac{\omega}{a}-\omega|r|\right),$$

$$S(|r|)=\frac{3\omega^2(1-a|r|)^2}{2a^2}A^2\left[J_0^2\left(\frac{\omega}{a}-\omega|r|\right)+J_1^2\left(\frac{\omega}{a}-\omega|r|\right)-\right.$$

$$\left.-\frac{a}{\omega(1-a|r|)}J_0\left(\frac{\omega}{a}-\omega|r|\right)J_1\left(\frac{\omega}{a}-\omega|r|\right)\right]-B,$$

Where A and B are some dimensionless constants and J_0 and J_1 are Bessel functions of the first kind, and the brane energy-momentum tensor has the form:

$$(4)\quad \sigma_A^B=M^3\delta(r)diag[\tau_t,\tau_x,\tau_y,\tau_z,0],$$

Where the brane tensions are:

$$(5)\quad \tau_t=2a$$

$$\tau_x = \tau_y = \frac{2}{3}a + aB + A\omega\sin(\omega t)J_1\left(\frac{\omega}{a} - \omega|r|\right),$$

$$\tau_z = \frac{2}{3}a + aB - 2A\omega\sin(\omega t)J_1\left(\frac{\omega}{a} - \omega|r|\right),$$

To interpret (3) as describing the brane at , which bounds the scalar and gravitational bulk standing waves, we assume that,

(6) $$u|_{r=0} = 0, \quad \varphi|_{r=0} = 0, \quad S|_{r=0} = 0,$$

These conditions can be achieved assuming the relations:

(7) $$\frac{\omega}{a} = Z_n^{(J_0)}, \qquad B = \frac{3\omega^2 A^2}{2a^2} J_1^2\left(\frac{\omega}{a}\right),$$

where $Z_n^{J_0}$ is n-th zero of the function J_0.

If the amplitude of the bulk standing waves is small, A<<1, from (3) it is clear that the functions u, S and φ does not play significant role. So from the very beginning one can put

$$S(|r|) = u(t,|r|) = \varphi(t,|r|) = 0$$

(8)

And consider the metric ansatz without oscillatory metric functions:
(9)

$$ds^2 = \frac{1}{(1-a|r|)^{2/3}}(dt^2 - dr^2) - (1-a|r|)^{2/3}(dx^2 + dy^2 + dz^2),$$

This metric has the horizon at $|r| = 1/a$, where R_{tt} and R_{rr} components of the Ricci tensor get infinite values, while all gravitational invariants are finite. It resembles the situation with the Schwarzild Black Hole. However, in contrast to the Black Hole case the determinant of (9) becomes at $|r| = 1/a$. As the result nothing can cross the horizon of (9), and matter fields are confined inside of the 3-brane of the width $\sim 1/a$ in the extra space. To provide experimentally acceptable localization of matter fields the actual size of the extra dimension must be sufficiently small, $\leq 1/M_H$, where M_H denotes the Higgs scale. So the curvature scale a must be large $M_H \leq a \leq M$, where M is 5D fundamental scale.

In the case of large oscillations, $A^2 \sim B >> 1$, and assuming that curvature scale a is relatively small, the width of the brane is determined by the metric function $S \sim B \sim A^2$ in (2). So trapping of matter fields is caused by the pressure of the bulk oscillations and not by the existence of the horizon in the extra space.

To illustrate this trapping mechanism let us consider localization of a real massless scalar field $\Phi(x^A)$. We look for the solution in the form:

(10) $\Phi(t,x,y,z,r) = \phi(x^\mu)\rho(r),$

Where the 4D factor of the scalar field wavefunction,

(11) $\phi(x^\mu) = e^{-i(Et-p_x x-p_y y-p_z z)},$

Obeys the dispersion relation:

(12) $\eta^{\nu\beta}\partial_\nu\partial_\beta\phi(x^\mu) = -m^2\phi(x^\mu).$

For the extra dimension factor, $\rho(|r|)$, we require the boundary conditions:

(13) $\rho'\big|_{|r|=0} = 0, \quad \rho\big|_{|r|\to 1/a} = 0,$

Where prime denotes the derivative with the respect to $|r|$. Then Klein-Gordon equation for $\Phi(x^A)$ reduces to:

(14) $\rho'' - \frac{a\,\mathrm{sgn}(r)}{1-a|r|}\rho' - E^2\left[\frac{e^S}{(1-a|r|)^{4/3}} - 1\right]\rho == -m^2\frac{e^S}{(1-a|r|)^{4/3}}\rho$

Close to the brane, $|r| \to 0$, the solution to (14) for the zero mode (m=0) wavefunction $\rho_0(r)$, which obeys the boundary conditions (13), has the following series expansion:

(15) $\rho_0(r)\big|_{|r|\to 0} = C_1\left(1-\frac{1}{6}BEa|r|^3\right)+O(a^4|r|^4),$

where C_1 is the integration constant expressed by Airy functions [3].

In the second limiting region $|r|\to 1/a$, the series expansion of the zero mode (m=0) solution of (14) is done by:

(16) $\rho_0(r)\big|_{|r|\to 1/a} = C_2\left[1+\frac{9c^2}{4a^2}(1-a|r|)^{2/3}\right]+O((1-a|r|)^{4/3}),$

where $c^2 = E^2 e^{-B}$ and C_2 are some constants [3].

For the zero mode, $\rho_0(r)$, with the asympthotes (15) and (16), the 5D scalar field action,

(17)

$$S_0 = \frac{1}{2}\int dx^5 \{Q_1(r)\partial_t\phi^2 - Q_2(r)(\partial_x\phi^2+\partial_y\phi^2+\partial_z\phi^2)-Q_3(r)\phi^2\},$$

where

(18) $Q_1(r)=(1-a|r|)\rho_0^2, \qquad Q_2(r)=(1-a|r|)^{-1/3}e^S\rho_0^2,$

$Q_3(r)=(1-a|r|)\rho_0'^2,$

is integrable over the extra coordinate r. This means that the scalar field zero mode wavefunction is localized on the brane.

To estimate the masses of KK excitations on the brane we consider the scalar particles with zero 3-momentum, $p^2=0$. Then the solution of the equation (14) for the massive modes, after imposing the boundary conditions (13), gives the KK mass spectrum on the brane:

(19) $m_n = aZ_n^{(J_1)},$

Where $Z_n^{(J_1)}$, is the n-th zero of the Bessel function J_1. The mass gap between the zero and the first massive modes will be:

(20) $\Delta_m = m_1 = aZ_1^{(J_1)} \approx 3.8a$

So when the curvature scale a, which determines the size of extra dimension, is smaller than the scale associated with of the brane, the KK modes of the mass $\approx 3.8a$, can be created in accelerators at relatively low energies, what gives a chance to check this model.

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