

Spin-selective Peierls transition in interacting one-dimensional conductors with spin-orbit interaction

Bernd Braunecker,¹ George I. Japaridze,^{2,3} Jelena Klinovaja,¹ and Daniel Loss¹
¹*Department of Physics, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland*

²*Andronikashvili Institute of Physics, Tamarashvili 6, 0177 Tbilisi, Georgia*

³*Ilia State University, Cholokashvili Ave. 3-5, 0162 Tbilisi, Georgia*

(Received 7 July 2010; published 29 July 2010)

Interacting one-dimensional conductors with Rashba spin-orbit coupling are shown to exhibit a spin-selective Peierls-type transition into a mixed spin-charge-density-wave state. The transition leads to a gap for one-half of the conducting modes, which is strongly enhanced by electron-electron interactions. The other half of the modes remains in a strongly renormalized gapless state and conducts opposite spins in opposite directions, thus providing a perfect spin filter. The transition is driven by magnetic field and by spin-orbit interactions. As an example we show for semiconducting quantum wires and carbon nanotubes that the gap induced by weak magnetic fields or intrinsic spin-orbit interactions can get renormalized by 1 order of magnitude up to 10–30 K.

DOI: [10.1103/PhysRevB.82.045127](https://doi.org/10.1103/PhysRevB.82.045127)

PACS number(s): 71.30.+h, 71.10.Pm, 71.70.Ej, 72.25.-b

I. INTRODUCTION

Electron-electron (e-e) interactions play a central role in determining the physical properties of low-dimensional electron conductors. They lead to interesting correlated-electron physics but also provide a handle to design systems with specific properties that cannot be reached with noninteracting particles. In this paper we focus on one-dimensional (1D) electron conductors with strong spin-orbit interaction (SOI). Such systems have spin-split bands with generally a crossing of two bands with opposite spin projection. Through an external magnetic field or intrinsic SOI the degeneracy at the crossing point can be lifted and a gap Δ opens. We show in this paper that in this situation e-e interactions play a crucial role, which has not been investigated so far. The interactions lead to a substantial enhancement of the gap and strongly modify the nature of the remaining conducting modes. Underlying the strong response to interactions is the instability of any 1D conductor to the formation of charge- and spin-density waves. A spatially modulated potential can make this instability dominant and drive the system into an ordered density-wave phase, known as the electronic Peierls transition. We show below that the renormalization of Δ can be identified with a Peierls-type transition depending on spin and chirality, affecting only one-half of the conducting modes but both spin components. For short, we refer to it as *spin-selective Peierls transition*.

A sketch of the conductor is shown in Fig. 1. The main effect of SOI is illustrated in Fig. 2 and consists in a spin-dependent shift of the electron dispersion by a momentum σk_{so} , where $\sigma = \uparrow, \downarrow = +, -$ is the spin polarization along an axis $\boldsymbol{\eta}$ determined by the SOI. A uniform magnetic field perpendicular to $\boldsymbol{\eta}$ opens the gap Δ at the band crossing point at $k=0$. It was shown in previous work^{1–6} that this leads to a reduced conductance and remaining spin-filtered conducting states. This effect was very recently observed in high-mobility GaAs/AlGaAs hole quantum wires.⁷

We show here that e-e interactions substantially modify the physics within this gap. Indeed, tuning the chemical po-

tential μ to the middle of Δ leads to the commensurability condition $k_F = k_{so}$ (with k_F the Fermi momentum) at which the e-e interactions have remarkable consequences. They enhance Δ and for strong e-e interactions an enhancement by more than an order of magnitude is possible. As an immediate consequence, the spin filter effect is stabilized, removing the need of fine-tuning external parameters such as chemical potential and magnetic field. We show below that this renormalization can be mapped onto a Peierls-type mechanism. The interactions also modify the right- (*R*) and left- (*L*) moving modes with momenta close to $\pm 2k_F$, which remain in a spin-filtered conducting state, but form a strongly renormalized electron liquid. Furthermore, we show that the same transition can be achieved without SOI through a spiral magnetic field as obtained, for instance, by placing nanomagnets near the conductor, or by the Overhauser field generated by nuclear spins through a self-ordering feedback mechanism due to their interaction with electrons.^{8,9} For systems with SOI, a spiral magnetic field with wave number $2(k_F - k_{so})$ can also be used to obtain the renormalized gap at higher electron densities $k_F > k_{so}$.

II. MAIN PHYSICS

To show the crucial importance of e-e interactions, let us consider a generic model for the 1D conductor, described by the Hamiltonian $H = H_{el} + H_{so} + H_{\Delta}$, where

$$H_{el} = \sum_{\sigma} \int dr \psi_{\sigma}^{\dagger}(r) \left(\frac{-i\hbar\partial_r}{2m} \right)^2 \psi_{\sigma}(r) + U \quad (1)$$

is the electron Hamiltonian

$$H_{so} = \sum_{\sigma\sigma'} \int dr \psi_{\sigma}^{\dagger}(r) (\boldsymbol{\eta} \cdot \boldsymbol{\sigma})_{\sigma\sigma'} (-i\hbar\partial_r) \psi_{\sigma'}(r) \quad (2)$$

is the SOI, and

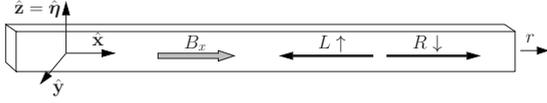


FIG. 1. Sketch of the quantum wire. The spin z axis is chosen along the SOI axis $\boldsymbol{\eta}$. A magnetic field B_x (if required) is applied perpendicularly to $\boldsymbol{\eta}$, for instance, along the spin x axis. The position coordinate along the wire is denoted by r . The labels $L\uparrow$ and $R\downarrow$ indicate the spin-filtered conducting modes.

$$H_{\Delta} = -\Delta \sum_{\sigma, \sigma'} \int dr \psi_{\sigma}^{\dagger}(r) (\boldsymbol{\sigma}_x)_{\sigma\sigma'} \psi_{\sigma'}(r) \quad (3)$$

describes the gap. In these Hamiltonians $\psi_{\sigma}(r)$ is the electron operator for spin σ at position r , m is the band mass, U is a general e-e interaction term, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector formed by the Pauli matrices. The vector $\boldsymbol{\eta}$ determines the amplitude and direction of the SOI and depends on the material and confinement of the 1D system (for instance, for Rashba SOI it is usually perpendicular to the wire axis). We choose the spin z direction such that $\boldsymbol{\eta} \cdot \boldsymbol{\sigma} = \eta \sigma_z$. The gap Δ is typically created by a uniform magnetic field B_x applied perpendicularly to $\boldsymbol{\eta}$ along the spin x direction but it can appear also through intrinsic SOI as in carbon nanotubes^{10,11} (see discussion in Sec. IV). In the case of magnetic field, $\Delta = B_x g \mu_B / 2$, with g the Landé g -factor and μ_B the Bohr magneton. Further SOI and orbital coupling to the magnetic field may be included without changing the described physics.

In the absence of e-e interactions ($U=0$) the electron dispersion can easily be calculated and is shown in Fig. 2 without (dashed lines) and with (solid lines) the gap. The main effect of the SOI is a spin-dependent shift of the single-particle dispersions by momentum $k_{so} = m \eta / \hbar$. The spin-flip scattering of Eq. (3) opens the gap Δ at $k=0$.

The opening of this gap is equivalent to a Peierls transition. The latter refers to the formation of an electron density

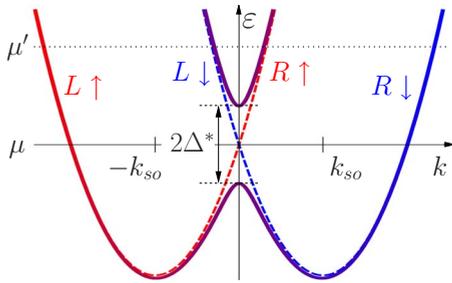


FIG. 2. (Color online) Electron dispersion ε_k for the 1D conductor. With the choice of spin axes as in Fig. 1, the $\sigma = \uparrow, \downarrow$ bands are shifted to the left and right by the wave vectors $\mp k_{so}$, respectively. Higher subbands are assumed to have no influence. Close to the chemical potential μ the modes are classified into L and R movers. The $L\downarrow$ and $R\uparrow$ branches cross at $k=0$, and μ passes through the crossing point by tuning the density to $k_F = k_{so}$. A gap 2Δ opens at $k=0$ through a magnetic field B_x or through intrinsic SOI, and is strongly enhanced by e-e interactions to $2\Delta^*$. The dashed lines correspond to $\Delta=0$. Through the application of a spiral magnetic field a gap can also open at any higher chemical potential μ' .

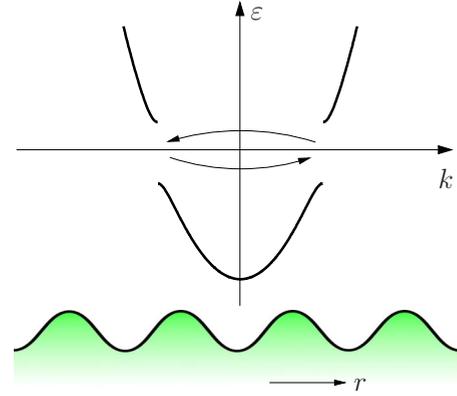


FIG. 3. (Color online) Illustration of the electronic Peierls transition in a one-dimensional conductor. The scattering of electrons on an external periodic potential (represented in real space below the band structure) causes hybridization between the states near $\pm k_F$ if the potential has the wave number $2k_F$. As a consequence a gap opens at $\pm k_F$ and the system becomes insulating.

wave in a 1D electron system in the presence of an external potential with a spatial periodicity commensurate with $2k_F$. Scattering on this potential with momentum transfer $\pm 2k_F$ leads to mixing between the modes with momenta close to $\pm k_F$ and turns the system into an insulator (see Fig. 3). An equivalent situation is obtained in the present system by considering the spin-dependent gauge transformation $\psi_{\sigma}(r) \rightarrow e^{i\sigma k_{so} r} \psi_{\sigma}(r)$, which eliminates the σk_{so} shifts such that $H_0 + H_{so} \rightarrow H_0$ (see Fig. 4). The uniform Δ , however, turns into the spiral field, $\Delta(r) = \Delta [\cos(2k_{so} r) \hat{x} - \sin(2k_{so} r) \hat{y}]$, fully equivalent to a spiral magnetic field in a system without SOI, where \hat{x}, \hat{y} are the unit vectors in the spin x, y directions. At commensurability $k_F = k_{so}$ this spiral plays the same role as the periodic potential for the regular Peierls instability. Yet it is spin selective because the helicity breaks the symmetry between the spin directions and so the gap Δ opens only between the modes $R\uparrow$ and $L\downarrow$.

This identification with a Peierls-type mechanism tells us that e-e interactions substantially enhance Δ . We first visualize this with a simple mean-field picture, equivalent to the Stoner argument, and give quantitative results afterward. Let us assume that Δ is caused by a uniform field B_x . This field causes a paramagnetic partial spin polarization $\langle S_x \rangle$ by the relation $\langle S_x \rangle = -\chi B_x$ with $\chi > 0$ a susceptibility. We assume for illustration that the e-e interactions can be represented by a simple interaction density of the type $U n_{\sigma} n_{\sigma'}$, where n_{σ} is the density of σ electrons. Important is the mean-field exchange coupling $-4U \langle S_x \rangle S_x$ (using $\langle S_{y,z} \rangle = 0$) with $S_{x,y,z}$ the components of the electron spin density. This exchange term acts as an effective magnetic field and leads to $\langle S_x \rangle = -\chi [B_x - 4U \langle S_x \rangle] = -\chi B_x^*$ with the renormalized effective magnetic field $B_x^* = B_x / (1 - 4\chi U)$ and so to the renormalized gap $\Delta^* = \Delta / (1 - 4\chi U)$.

While this mean-field argument qualitatively shows that e-e interactions can strongly enhance Δ , it is quantitatively inaccurate for 1D systems where the physics is dominated by fluctuations. To overcome this limitation we use the Luttinger liquid (LL) theory.¹² Much attention was given recently to the connection between LL and SOI physics,¹³⁻²⁶

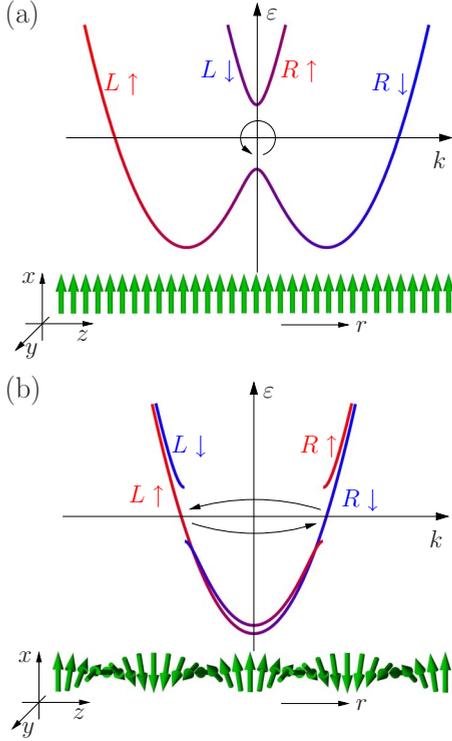


FIG. 4. (Color online) Illustration of the connection to the Peierls mechanism. (a) Band structure of the SOI shifted bands as in Fig. 2. The arrows below the band structure represent the uniform magnetic field B_x in real space. Spin-flip scattering on B_x (indicated by the circular arrow) hybridizes the modes at $k=0$ and opens the gap $\Delta \propto B_x$. (b) The same band structure in the gauge transformed basis $\psi_\sigma(r) \rightarrow e^{i\sigma k_{so} r} \psi_\sigma(r)$. The spin-split bands overlap and the uniform magnetic field turns into a spiral field in the spin x, y plane with wave number $2k_{so}$. The scattering on the magnetic field turns into a spin-flip scattering on the periodic external potential with momentum transfer $\pm 2k_{so}$, as indicated by the arrows, and leads to the hybridization between the modes L_\downarrow and R_\uparrow at $\pm k_{so}$. At $k_F = k_{so}$ this is equivalent to the Peierls mechanism shown in Fig. 3, yet affects only one half of the electron modes. Electron-electron interactions strongly renormalize Δ and modify the properties of the remaining gapless states. For clarity of the presentation the spin axes are chosen here differently than in Fig. 1.

yet the addressed regimes are far from the regime considered in this work, and the resulting physics is different. The LL theory is exact for a linear electron dispersion, and band curvature can lead to modified physics.^{27–29} However, since the instability to density wave order is inherent in any 1D conductor, we expect that the Peierls-type renormalization persists for general situations, and we use the LL theory for quantitative predictions.

The physics of a LL in a spiral magnetic field was studied in detail in Refs. 8 and 9, where it was shown that a Hamiltonian of the type of Eq. (3) can be written as

$$H_\Delta = - \int dr \frac{\Delta}{\pi a} \{ \cos[\phi_{R\uparrow} + \phi_{L\downarrow} + 2(k_{so} - k_F)r] + \cos[\phi_{R\downarrow} + \phi_{L\uparrow} + 2(k_{so} + k_F)r] \}, \quad (4)$$

where a is the lattice constant and $\phi_{\ell\sigma}$, for $\ell=L, R$, are boson

fields such that $\partial_r \phi_{\ell\sigma}$ describe electron density fluctuations about the four Fermi points of the branches labeled in Fig. 2 (dashed lines). In terms of the standard boson fields¹² for charge (ϕ_c, θ_c) and spin fluctuations (ϕ_s, θ_s) we have $\phi_{R\uparrow} + \phi_{L\downarrow} = \sqrt{2}(\phi_c - \theta_s)$ and $\phi_{R\downarrow} + \phi_{L\uparrow} = \sqrt{2}(\phi_c + \theta_s)$. The first cosine term in Eq. (4) describes the hybridization between the two crossing branches. At $k_F = k_{so}$ it becomes nonoscillating and relevant in the renormalization-group (RG) sense. The second cosine, expressing scattering between the R_\downarrow and L_\uparrow branches at $\pm 2k_F$, then strongly oscillates (irrelevant for the RG) and can be neglected for systems of length $L \gg \pi/4k_F$. The e-e interactions strongly enhance the amplitude of the relevant term and it was shown in Refs. 8 and 9 that the resulting renormalized gap is given by

$$\Delta^* = \Delta(\xi/a)^{1-\kappa} \quad (5)$$

with $\kappa = (K_c + K_s^{-1})/2$ and ξ the correlation length of the gapped modes. Here $K_{c,s}$ are the LL parameters for charge and spin fluctuations, which fully encode the interaction U .¹² For a noninteracting system $K_c = K_s = 1$ and $\Delta^* = \Delta$. Repulsive e-e interactions lead to $0 < K_c < 1$. In the absence of spin SU(2) breaking interactions other than Eq. (2) we have $K_s = 1$. Otherwise $K_s \neq 1$, and the interplay with the SOI can lead to interesting spin-density-wave phases.^{20,22,23} At commensurability $k_{so} = k_F$, however, the nonoscillating term in Eq. (4) is much more relevant and we can neglect such processes. For an infinite system at zero temperature ξ is given by⁹ $\xi = \xi_\infty = (\pi \hbar v_F / \Delta a)^{1/(2-\kappa)}$ with v_F the slope of the dispersion at $k_F = k_{so}$. In a realistic system ξ is furthermore limited by the system length L and the thermal length $\lambda_T = \hbar v_F / k_B T$ (with k_B the Boltzmann constant) such that, with an order 1 uncertainty, $\xi = \min\{L, \lambda_T, \xi_\infty\}$. For the experimentally available systems considered in Sec. IV we have $\xi = \xi_\infty$ except at very low fields below some millitesla. The gap is stable upon a slight detuning $\Delta k = |k_F - k_{so}|$ from commensurability as long as $\xi \ll \pi / \Delta k$.¹²

III. CONSEQUENCES

For temperatures $k_B T < \Delta^*$ the combination $\phi_{R\uparrow} + \phi_{L\downarrow}$ is pinned to a minimum of the relevant cosine term in Eq. (4). This leads to a spin polarization in x direction, which we estimate as⁹ $\langle S_x \rangle \equiv \langle \psi_{L\downarrow}^\dagger \psi_{R\uparrow} \rangle = \langle \psi_{R\uparrow}^\dagger \psi_{L\downarrow} \rangle \sim \Delta^* / 2E_F$ with $E_F = \hbar v_F k_F / 2$. This is essentially the mean-field result from above (with $\chi \sim 1/E_F$) but with the corrected enhancement factor $(\xi/a)^{1-\kappa}$. Since ξ depends on Δ (except for very small Δ where $\xi \neq \xi_\infty$) the response is generally nonlinear. Let $M_x = (\mu_B g / 2) \langle S_x \rangle$ be the magnetization of the system. For $\Delta \propto B_x$, we find with $\langle S_x \rangle \propto \Delta^* \propto g$ that $M_x \propto g^2 (\xi/a)^{1-\kappa}$. Hence we can interpret the enhancement also as a B_x -dependent g -factor renormalization, $g \rightarrow g(\xi/a)^{(1-\kappa)/2}$, yet only for the gapped modes.

The ungapped modes L_\uparrow and R_\downarrow remain in a LL state describing the low-energy physics with strongly modified parameters. This can be captured by writing the corresponding electron operators as^{9,12} $\psi_{L\uparrow} \propto e^{-i(\phi+\theta)}$ and $\psi_{R\downarrow} \propto e^{i(\phi-\theta)}$, where we have used $\phi_c = \theta_s + \text{const}$. The LL theory is then described by the Hamiltonian⁹ $H = \hbar v \int (dr/2\pi) [K^{-1}(\partial_r \phi)^2 + K(\partial_r \theta)^2]$ with $K = 2K_c / (1 + K_s K_c)$ and $v = v_F (K_c^{-1} + K_c) / (1 + K_s K_c)$. The

gapless modes preserve the $L\uparrow$ and $R\downarrow$ character, and the conductor acts as a perfect spin filter for injected electrons. Indeed, local charge fluctuations ρ_c and spin current j_s are identical, $\rho_c(r)=j_s(r)=-\partial_r\phi(r)/\pi$, showing that any charge fluctuation transports spin through the system. This also implies that backscattering without a spin flip is strongly suppressed, and that the system is stable against nonmagnetic backscatterers such as impurities or disorder.

Replacing the uniform magnetic field B_x by the spiral magnetic field $B_{xy}(r)=B_{xy}[\cos(4k_{so}r)\hat{x}+\sin(4k_{so}r)\hat{y}]$ (if feasible) exchanges the relevance of the cosine terms in Eq. (4) by switching $(k_{so}+k_F)\leftrightarrow(k_{so}-k_F)$. Consequently a gap opens for the modes at $k\approx\pm 2k_F$ while the modes at $k\approx 0$ remain unaffected. The wave packets for the $k\approx 0$ states extend over the whole system and we expect them to be only weakly affected by local scatterers such as impurities. More important are e-e interactions such as $k=0$ umklapp scattering, which can lead to the opening of an additional gap, clearly distinguishable from Δ^* by its dependence on system properties.

Different spiral magnetic fields allow to obtain the gap Δ^* already at higher electron densities (position of μ' in Fig. 2). If $k_F=k_{so}+\Delta k$, the application of $B_{xy}(r)=B_{xy}[\cos(2\Delta kr)\hat{x}-\sin(2\Delta kr)\hat{y}]$ eliminates the oscillation of the first cosine term in Eq. (4) and opens the gap. Since $k_{so}\neq 0$ the transition is obtained with a spiral field of longer wavelength than in the absence of SOI.

IV. EXPERIMENTAL REALIZATIONS

Candidates for this physics are InAs nanowires^{30,31} and AlGaAs quantum wires,⁷ where Δ is induced by B_x ; or single-wall carbon nanotubes, where Δ appears intrinsically from SOI.^{10,11} The latter are interesting candidates as they can be produced at high purity and have strong e-e interactions.³²⁻³⁴ Importantly, the nanotube curvature induces intrinsic SOI,^{10,11} which leads to a gap at the band crossing of metallic nanotubes. However, due to the hollow nature of the nanotubes, the spin-dependent shift k_{so} is absent. The latter can be induced by Rashba SOI with an electric field across the nanotube. Following Ref. 35, we estimate that a splitting such that a band structure as in Fig. 2 can be resolved requires an electric field on the order of 0.1–1 V/nm.³⁶ The effect of a transverse electric field on the band structure was investigated with perturbation theory in Ref. 37. At such strong fields, however, the perturbative treatment must be replaced by a full incorporation into the Bloch theory. The final Dirac theory remains very similar and SOI has the same effect. In particular, there remains the intrinsic band gap on the order of $\Delta\sim 40\ \mu\text{eV}$ (tunable by tube radius).^{10,11} Using^{8,9} $a=2.46\ \text{\AA}$, $K_c\approx 0.2$, and $K_s\approx 1$ (but SOI may further modify K_s , see also Ref. 26), we obtain $\Delta^*\sim 1\ \text{meV}$, enhanced by about a factor 30. (Note that here in the same way as in Refs. 8 and 9 the spin-selective Peierls

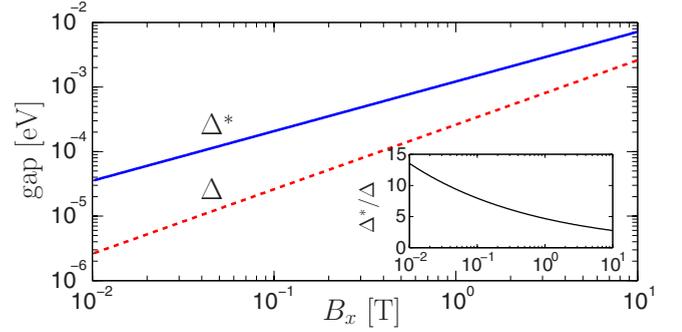


FIG. 5. (Color online) Gaps Δ and Δ^* as a function of magnetic field B_x for typical values of InAs quantum wires. The inset shows the enhancement Δ^*/Δ for the same values of B_x .

transition leads to an effective decoupling of the two valleys at the K and K' points, and the important LL parameter remains $K_c\approx 0.2$.)

In contrast, InAs nanowires have the spin shift k_{so} but require a magnetic field to open the gap. For such wires it has been shown^{30,31} that a SOI length of $\lambda_{so}\approx 130\ \text{nm}$ can be obtained, corresponding to $k_{so}=2\pi/\lambda_{so}=5\times 10^7\ \text{m}^{-1}$. Similar numbers are found in InAs quantum wells, which have SOI strengths of³⁸⁻⁴⁰ $\hbar\eta=(0.6-4)\times 10^{-11}\ \text{eV m}$ corresponding to $k_{so}=1.2\times 10^7\ \text{m}^{-1}$ (using⁴¹ $m=0.040m_e$, with m_e the electron mass, $g=-9$, and a maximal η). Comparable values have also been reported for AlGaAs quantum wires,⁷ yet in the hole doped regime, which is not explicitly addressed with this theory. For such low densities it is reasonable to assume $K_c\sim 0.4$ and $K_s=1$. With $a=6.06\ \text{\AA}$ we obtain the values shown in Fig. 5. Notably an enhancement by more than a factor 10 is achieved at low fields.

V. CONCLUSIONS

We have shown that in 1D conductors with SOI the gap Δ opened by a magnetic field at the crossing point of the spin-split bands is substantially enhanced by e-e interactions. In the local rest frame of the electron spin Δ becomes a spiral field and assumes the role of a spin-selective periodic potential that drives the system through a Peierls-type transition. The interactions also renormalize the remaining gapless modes and strongly stabilize their spin-filter effect. Remarkably, it is possible to obtain the same effects even without a time-reversal symmetry breaking magnetic field in carbon nanotubes, where a gap Δ exists intrinsically from SOI alone.

ACKNOWLEDGMENTS

We thank M. J. Schmidt for helpful discussions. B.B., J.K., and D.L. acknowledge the support by the Swiss NSF and NCCR Nanoscience (Basel), and G.I.J. the support by the Georgian NSF under Grant No. ST/09-447.

- ¹P. Štředa and P. Šeba, *Phys. Rev. Lett.* **90**, 256601 (2003).
- ²Y. V. Pershin, J. A. Nesteroff, and V. Privman, *Phys. Rev. B* **69**, 121306(R) (2004).
- ³P. Devillard, A. Crépieux, K. I. Imura, and T. Martin, *Phys. Rev. B* **72**, 041309(R) (2005).
- ⁴S. Zhang, R. Liang, E. Zhang, L. Zhang, and Y. Liu, *Phys. Rev. B* **73**, 155316 (2006).
- ⁵D. Sánchez, L. Serra, and M.-S. Choi, *Phys. Rev. B* **77**, 035315 (2008).
- ⁶J. E. Birkholz and V. Meden, *Phys. Rev. B* **79**, 085420 (2009).
- ⁷C. H. L. Quay, T. L. Hughes, J. A. Sulpizio, L. N. Pfeiffer, K. W. Baldwin, K. W. West, D. Goldhaber-Gordon, and R. de Picciotto, *Nat. Phys.* **6**, 336 (2010).
- ⁸B. Braunecker, P. Simon, and D. Loss, *Phys. Rev. Lett.* **102**, 116403 (2009).
- ⁹B. Braunecker, P. Simon, and D. Loss, *Phys. Rev. B* **80**, 165119 (2009).
- ¹⁰W. Izumida, K. Sato, and R. Saito, *J. Phys. Soc. Jpn.* **78**, 074707 (2009).
- ¹¹J.-S. Jeong and H.-W. Lee, *Phys. Rev. B* **80**, 075409 (2009).
- ¹²A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, 1998); T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).
- ¹³A. V. Moroz, K. V. Samokhin, and C. H. W. Barnes, *Phys. Rev. Lett.* **84**, 4164 (2000).
- ¹⁴A. V. Moroz, K. V. Samokhin, and C. H. W. Barnes, *Phys. Rev. B* **62**, 16900 (2000).
- ¹⁵M. Governale and U. Zülicke, *Phys. Rev. B* **66**, 073311 (2002).
- ¹⁶A. Iucci, *Phys. Rev. B* **68**, 075107 (2003).
- ¹⁷Y. Yu, Y. Wen, J. Li, Z. Su, and S. T. Chui, *Phys. Rev. B* **69**, 153307 (2004).
- ¹⁸R. G. Pereira and E. Miranda, *Phys. Rev. B* **71**, 085318 (2005).
- ¹⁹Y.-X. Li, Y. Guo, and B.-Z. Li, *Phys. Rev. B* **72**, 075321 (2005).
- ²⁰V. Gritsev, G. I. Japaridze, M. Pletyukhov, and D. Baeriswyl, *Phys. Rev. Lett.* **94**, 137207 (2005).
- ²¹F. Cheng and G. Zhou, *J. Phys.: Condens. Matter* **19**, 136215 (2007).
- ²²J. Sun, S. Gangadharaiah, and O. A. Starykh, *Phys. Rev. Lett.* **98**, 126408 (2007).
- ²³S. Gangadharaiah, J. Sun, and O. A. Starykh, *Phys. Rev. B* **78**, 054436 (2008).
- ²⁴A. Schulz, A. De Martino, P. Ingenhoven, and R. Egger, *Phys. Rev. B* **79**, 205432 (2009).
- ²⁵G. I. Japaridze, H. Johannesson, and A. Ferraz, *Phys. Rev. B* **80**, 041308(R) (2009).
- ²⁶A. Schulz, A. De Martino, and R. Egger, *Phys. Rev. B* **82**, 033407 (2010).
- ²⁷M. Khodas, M. Pustilnik, A. Kamenev, and L. I. Glazman, *Phys. Rev. B* **76**, 155402 (2007).
- ²⁸A. Imambekov and L. I. Glazman, *Science* **323**, 228 (2009).
- ²⁹A. Imambekov and L. I. Glazman, *Phys. Rev. Lett.* **102**, 126405 (2009).
- ³⁰C. Fath, A. Fuhrer, L. Samuelson, V. N. Golovach, and D. Loss, *Phys. Rev. Lett.* **98**, 266801 (2007).
- ³¹S. Nadj-Perge, S. M. Frolov, J. W. W. van Tilburg, J. Danon, Y. V. Nazarov, R. Algra, E. P. A. M. Bakkers, and L. P. Kouwenhoven, *Phys. Rev. B* **81**, 201305(R) (2010).
- ³²C. Schönenberger, *Semicond. Sci. Technol.* **21**, S1 (2006).
- ³³A. K. Hüttel, G. A. Steele, B. Witkamp, M. Poot, L. P. Kouwenhoven, and H. S. J. van der Zant, *Nano Lett.* **9**, 2547 (2009).
- ³⁴H. O. H. Churchill, A. J. Bestwick, J. W. Harlow, F. Kuemmeth, D. Marcos, C. H. Stwertka, S. K. Watson, and C. M. Marcus, *Nat. Phys.* **5**, 321 (2009).
- ³⁵H. Min, J. E. Hill, N. A. Sinitsyn, B. R. Sahu, L. Kleinman, and A. H. MacDonald, *Phys. Rev. B* **74**, 165310 (2006).
- ³⁶J. Klinovaja, M. J. Schmidt, B. Braunecker, and D. Loss (unpublished).
- ³⁷D. S. Novikov and L. S. Levitov, *Phys. Rev. Lett.* **96**, 036402 (2006); arXiv:cond-mat/0204499 (unpublished).
- ³⁸I. Žutić, J. Fabian, and S. Das Sarma, *Rev. Mod. Phys.* **76**, 323 (2004).
- ³⁹R. H. Silsbee, *J. Phys.: Condens. Matter* **16**, R179 (2004).
- ⁴⁰P. J. Simmonds, S. N. Holmes, H. E. Beere, and D. A. Ritchie, *J. Appl. Phys.* **103**, 124506 (2008).
- ⁴¹P. J. Simmonds, F. Sfigakis, H. E. Beere, D. A. Ritchie, M. Pepper, D. Anderson, and G. A. C. Jones, *Appl. Phys. Lett.* **92**, 152108 (2008).