Spin Filtering by Field-Dependent Resonant Tunneling

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We consider theoretically transport in a spinful one-channel interacting quantum wire placed in an external magnetic field. For the case of two pointlike impurities embedded in the wire, under a small voltage bias the spin-polarized current occurs at special points in the parameter space, tunable by a single parameter. At sufficiently low temperatures complete spin polarization may be achieved, provided repulsive interaction between electrons is not too strong.

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Introduction.—Control and manipulation of spin degrees of freedom in nanoscale electronic devices is an active new field of research [1,2]. In quantum wires spin-selective transmission of electrons was considered in the past in a number of publications [3–7]. In [3] a strong asymmetry of the spin dependent conductances in a Luttinger liquid (LL) with a magnetic impurity was observed, which is related to the Zeeman energy splitting Δ of the impurity states. In [4] the authors consider the spin dependent backscattering of repulsive electrons from a single weak impurity in the presence of a strong magnetic field $\Delta > \Delta_C \approx 0.2E_F$ where E_F denotes the Fermi energy. Contrary to weak fields, the backscattering of electrons having spin parallel to the field may be suppressed making the impurity transparent, whereas electrons antiparallel to the field are still reflected.

In the present Letter we report on spin-selective transmission of electrons in a quantum wire through a quantum dot formed by two impurities. The mechanism consists in lifting the degeneracy of the condition for resonant tunneling of up and down electrons through the quantum dot [8,9] by an external magnetic field H. Whereas the transmission for the spin direction which fulfills the resonance condition is finite for repulsive interaction, it vanishes for the other spin direction due to the Coulomb blockade in the quantum dot. The mechanism requires sufficiently low temperatures such that the Zeeman splitting, $\Delta = g\mu_B H$, and the Coulomb energy of the quantum dot, $\sim E_F/n$, are large compared to T. Here n denotes the number of electrons in the quantum dot.

For weak impurities we find a resonance in the region of repulsive electron interaction where the transmission for one spin direction is perfect, provided the impurity is weaker than a critical value, whereas the other spin direction is completely blocked. For strong impurities transmission is found to change smoothly from perfect to zero when the interaction strength is increased. As a difference to the case $H = 0$ considered in [8], we find that the resonance condition for $H \neq 0$ is not same in the two limiting cases of strong and weak impurities and leads to two scenarios shown in Fig. 2.

A similar setup, but under very different conditions, has been considered recently in [6]. There, the Coulomb blockade effect was ignored and the magnetic field was assumed to be unrealistically strong, $\Delta = \mathcal{O}(E_F)$.

Model.—We consider electrons in a one dimensional wire along the x axis exposed to an external magnetic field. Since electrons are confined in the directions transverse to x, orbital effects are suppressed and the only field effect of the magnetic field is to polarize the electrons. In the noninteracting case the Zeeman energy splits the Fermi momentum k_{Fs} ($s = \uparrow, \downarrow$) of the up and down spin electrons by $|k_{F\uparrow} - k_{F\downarrow}|/(k_{F\uparrow} + k_{F\downarrow}) \approx \Delta/E_F \ll 1$. The Hamiltonian for electrons in the external impurity potential $V(x)$ can be described by the Tomonaga-Luttinger model [10]

$$H = \sum_s \int dx\{-i\hbar v_F[\psi_{Rs}^\dagger \partial_x \psi_{Rs} - \psi_{Ls}^\dagger \partial_x \psi_{Ls}] + V(x)\rho_s(x)\} + \frac{1}{2}\sum_{s,s'}\int dx dx' W(x-x')\rho_s(x)\rho_{s'}(x'), \tag{1}$$

where $\psi_{Rs}(x)$, $\psi_{Ls}(x)$ are the annihilation operators for right- and left-moving spin-s electrons, $\psi_s = \psi_{Rs} + \psi_{Ls}$ is the annihilation operator for spin-s electrons, $\rho_s = \psi_s^\dagger \psi_s$ is the spin-s electron density, and $W(x - x')$ is the screened Coulomb interaction between electrons [11].

We first consider the system without impurities. Then the model (1) describes an interacting quantum wire with four Fermi points [13]. In that situation it is useful to split terms arising from the interaction into intersubband and intrasubband terms [14]. For repulsive and spin independent interaction electrons stay in their bands during scattering processes and the only allowed intersubband process is the forward scattering [15]. While mutually noninteracting subsystems consisting of spin up and spin down electrons are described in the bosonized representation by the standard LL Euclidean action [10] in terms of bosonic fields $\varphi_\uparrow$, $\varphi_\downarrow$ with the Luttinger parameter (LP) $K = (1 + \frac{\tilde{W}(0)-\tilde{W}(2k_F)}{\pi\hbar v_F})^{-1/2}$, the intersubband interaction is diagonalized in symmetric $\varphi_\rho = (\varphi_\uparrow + \varphi_\downarrow)/\sqrt{2}$ and antisymmetric $\varphi_\sigma = (\varphi_\uparrow - \varphi_\downarrow)/\sqrt{2}$ combinations. φ_ρ describe charge and φ_σ spin degrees of freedom. The action of the system in the absence of impurities is then given by (for details see [16,17])

$$\frac{S_0}{\hbar} = \sum_{\ell=\rho,\sigma} \frac{1}{2\pi K_\ell} \int dx d\tau \left[\frac{1}{v_\ell}(\partial_\tau \varphi_\ell)^2 + v_\ell(\partial_x \varphi_\ell)^2\right], \quad (2)$$

where $K_\ell = K(1 \pm \frac{K^2 \tilde{W}(0)}{\pi\hbar v_F})^{-1/2}$ with the convention that the upper (lower) sign corresponds to $\ell = \rho(\sigma)$. The velocities of excitations are $v_\ell = v_F/K_\ell$, where v_F is the Fermi velocity.

Nontrivial effects come from impurities. We consider two pointlike impurities, modeled as δ functions of the strength V and placed at $\pm a/2$. Introducing the displacement fields at the impurity positions $\phi_{1s}(\tau) = \varphi_s(-a/2, \tau)$ and $\phi_{2s}(\tau) = \varphi_s(a/2, \tau)$, the bosonized form of electron-impurity interaction reads [8,16] $S_1 = \sum_s (V k_{Fs}/\pi) \int d\tau[\cos(2\phi_s + k_{Fs}a) + \cos(2\phi_s - k_{Fs}a)]$.

To analyze the full action $S_0 + S_1$ it is useful to integrate out degrees of freedom outside the impurities. In that way one gets an action in terms of four fluctuating fields in imaginary time. For low frequencies, $|\omega| \ll v_\ell/a$, the effective action reads

$$S_{\text{eff}} = \sum_{\ell=\rho,\sigma} \sum_{k=\pm} \int \frac{d\omega}{16\pi^2} \frac{\hbar|\omega|}{K_\ell} |\Phi_{\ell k}(\omega)|^2 + \int d\tau V_{\text{eff}}, \quad (3)$$

where effective potential energy V_{eff} reads

$$\begin{aligned} V_{\text{eff}}(\phi_{1\uparrow}, \phi_{2\uparrow}, \phi_{1\downarrow}, \phi_{2\downarrow}) = &\sum_\ell \frac{1}{2} U_\ell \Phi_{\ell^-}(\tau)^2 \\ &+ \sum_s V_s[\cos(2\phi_{1s} + k_{Fs}a) \\ &+ \cos(2\phi_{2s} - k_{Fs}a)]. \quad (4) \end{aligned}$$

Here we have introduced $U_\ell = \frac{\hbar v_\ell}{2\pi a K_\ell}$, $V_s = V k_{Fs}/\pi$, and the fields $\Phi_{\ell k} = \phi_{2\uparrow} + k\phi_{1\uparrow} \pm (\phi_{2\downarrow} + k\phi_{1\downarrow})$, where $k = \pm$ and our sign convention for $\ell = \rho, \sigma$ applies. Φ_{ρ^-} and Φ_{σ^-} determine the total charge and spin, respectively, between the impurities.

The effective potential energy (4) consists of two types of terms: the charging energy $E_C = \sum_\ell U_\ell \Phi_{\ell^-}^2/2$ suppresses the accumulation of charge and spin on the island between impurities, while the V_s term tends to pin the displacement fields at the impurity positions. The part $|\omega||\Phi_{\ell^-}|^2$ of the action (3) is a fluctuation correction to E_C and is important at resonance points for strong impurities, when Φ_{ℓ^-} are undetermined; see below.

In the following we will examine the system described by (3) in two limiting cases, for strong and weak impurity strengths. In the realistic case of repulsive interaction, we have $K_\rho < 1$, $K_\sigma > 1$ and $U_\sigma < U_\rho$. We study the model at zero temperature, while influence of temperature is briefly considered at the end. Our strategy is to first determine the ground state from V_{eff} without fluctuations, see Eq. (3), and then to include fluctuations in order to check the stability of that ground state.

Strong impurities.—In the limit of very strong barriers, $V_\uparrow$, $V_\downarrow \gg U_\rho$, U_σ, E_F, the ground state of the system is defined by subsequent minimization of the pinning and the charging energy, see Eq. (4). The pinning energy terms are minimal for $2\phi_{ps} = (-1)^p k_{Fs}a + \pi(1 - 2n_{ps})$, $p = 1, 2$ where n_{ps} are integers. The high degeneracy of the pinning energy is broken by the charging energy. Plugging ϕ_{ps} into E_C and defining $n_s = n_{2s} - n_{1s}$ one gets

$$\begin{aligned} E_C(n_\uparrow, n_\downarrow) = &\frac{U_\rho}{2}(k_{F\uparrow}a + k_{F\downarrow}a - \pi(n_\uparrow + n_\downarrow))^2 \\ &+ \frac{U_\sigma}{2}(k_{F\uparrow}a - k_{F\downarrow}a - \pi(n_\uparrow - n_\downarrow))^2. \quad (5) \end{aligned}$$

To characterize different nonequivalent minima of (5) it is useful to restrict the Fermi momenta to satisfy $n < k_{F\uparrow}a/\pi$, $k_{F\downarrow}a/\pi \le n + 1$, where $n \ge 0$ is an integer. This implies $n \le n_\uparrow, n_\downarrow \le n + 1$. The particle number on the island is $n_\uparrow + n_\downarrow$. The ground states resulting from the minimization of the charging energy (5) are shown in Fig. 1. For generic values of $k_{Fs}a$, the ground state is uniquely determined. However, at special lines different ground states meet. These lines define the *resonance conditions*: while the number of particles on the island with one spin direction is fixed at the same value on both sides of the boundary, the number of electrons with the opposite spins changes by ± 1. $E_C(n_\uparrow, n_\downarrow) = E_C(n_\uparrow \pm 1, n_\downarrow)$ and $E_C(n_\uparrow, n_\downarrow) = E_C(n_\uparrow, n_\downarrow \pm 1)$ are the resonance conditions for the up and down spin electrons, respectively. As a result a particle having the degenerate spin can tunnel through the quantum dot in a sequential tunneling process without changing its energy. Hence we have a spin-selective barrier transparency.

We further solve the model along the boundary line where $E_C(n + 1, n) = E_C(n + 1, n + 1)$. Similar results hold for other cases. The fields $\phi_{p\uparrow}$, $p = 1, 2$ are locked by the strong impurity pinning and have fixed values of $n_\uparrow$. Approximating the nonlinear cosine term by a quadratic term for the $\phi_{p\uparrow}$ one can integrate out them from the action (3) [19]. The resulting effective action then reads

$$\begin{aligned} S'_{\text{eff}} = &\int \frac{d\omega}{4\pi^2} \frac{\hbar|\omega|}{K_{\text{eff}}} (|\phi_{1\downarrow}(\omega)|^2 + |\phi_{2\downarrow}(\omega)|^2) \\ &+ \int d\tau V_{\text{eff}}\left(-\frac{k_{F\uparrow}a + \pi}{2}, \frac{k_{F\uparrow}a + \pi}{2}, \phi_{1\downarrow}, \phi_{2\downarrow}\right), \quad (6) \end{aligned}$$

with $K_{\text{eff}} = \frac{2K_\rho K_\sigma}{K_\rho + K_\sigma}$. It describes the resonant tunneling of spin down electrons and is analogous to the case of spinless electrons [8]. The partition function is dominated by tun-

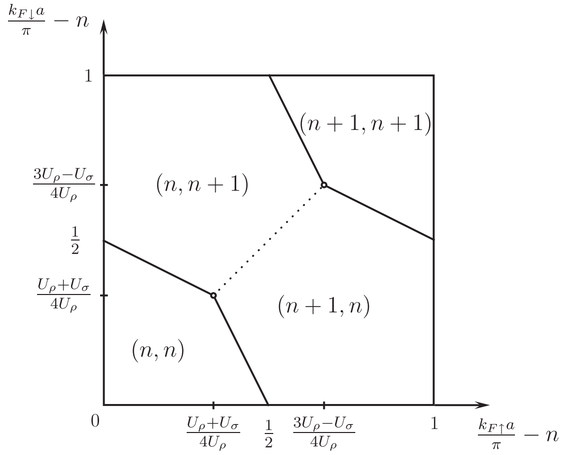

FIG. 1. Ground state energy configurations of the charging $E_C(n_\uparrow, n_\downarrow)$ for repulsive interaction $U_\sigma < U_\rho$. Points at boundaries between different ground state configurations correspond to the resonance points, special points where the ground state degeneracy is present. Boundaries with solid lines describe resonances for either up or down spins, while the dotted line is the Kondo resonance for spin exchange process at the island which exists without magnetic field, when $k_{F\uparrow} = k_{F\downarrow}$.

neling events connecting degenerate minima of the strong impurity potential. Using the Coulomb gas representation [8,20,21] one can produce the renormalization group equations for the tunneling transparency $t_\downarrow$ of barriers for spin-$\downarrow$ electrons. For strong impurity potential $V_\downarrow$ it reads $d_l t_\downarrow = t_\downarrow(1 - 1/(2K_{\text{eff}}))$, from which we get that for $K_{\text{eff}} > \frac{1}{2}$ the transparency $t_\downarrow$ increases, or equivalently, the strength of $V_\downarrow$ flows to smaller values at low energies. Outside the resonance lines, t_s flows to zero for any repulsion, similar to the single impurity case.

Weak impurities.—In the limit of weak impurities, $V_\uparrow$, $V_\downarrow \ll U_\rho$, U_σ, the action (3) is minimized for $\Phi_{\ell^-} = 0$. This corresponds to fixed charge and spin on the island. Integrating out the Φ_{ℓ^-} fluctuations from (3), new scattering processes of the form $\sum_s 2V_s \cos(k_{Fs}a) \times \cos(\phi_{1s} + \phi_{2s}) + V^{(2)} \sin(k_{F\uparrow}a) \sin(k_{F\downarrow}a) \cos\Phi_{\rho^+}$ are generated, where $V^{(2)} = V_\uparrow V_\downarrow \frac{U_\sigma - U_\rho}{2U_\rho U_\sigma}$. Other generated higher order processes are irrelevant for repulsive interaction. The resonance condition for the spin-s particles is now given by $\cos(k_{Fs}a) = 0$.

For the generic situation $\cos(k_{Fs}a) \neq 0$, the single electron backscattering processes are the most important ones. To leading order in the impurity potential, the renormalization group (RG) flow equations is $d_l V_s = V_s[1 - (K_\rho + K_\sigma)/2]$, from which we conclude that backward scattering terms V_s are relevant for $K_\rho + K_\sigma < 2$. Since the point impurity is a local quantity it cannot renormalize bulk quantities such as K_ρ, K_σ, and the flow of V_s is vertical [8,22]. The flow diagram for $V_\downarrow$ is shown in Fig. 2(a). Since the two limiting cases have opposite flow, it is plausible to expect a line of attractive fixed points somewhere in between, corresponding to a new phase, where spins of one direction (here down spins) have nonzero transmission at zero temperature, while the other spin direction is blocked [23].

In the resonance case $\cos(k_{F\downarrow}a) = 0$ and $|\cos(k_{F\uparrow}a)| \notin \{0, 1\}$, the two-particle scattering processes should be taken into account (only for spin-$\downarrow$ electrons since spin-$\uparrow$ already have backscattering in the lowest nonvanishing order). From the RG flow equation $d_l V^{(2)} = V^{(2)}(1 - 2K_\rho)$, we conclude that spin-$\downarrow$ electrons are effectively free at low energies for $K_\rho > 1/2$. In Fig. 2(b) we show the flow of $V_\downarrow$. Again the flows of two limiting cases are opposite, resulting in a separatrix in between the two resulting phases: perfectly conducting for spin down for small enough $V_\downarrow$ and insulating for larger $V_\downarrow$. Spin up electrons are always in the insulating phase in that case. Outside the middle region, the flow of $V_\downarrow$ is as in the single impurity case: toward zero for attractive interaction and toward infinity for very repulsive interaction.

In order to check the correctness of the assumption of massive fluctuations for the $\phi_{p\uparrow}$ fields made when we derived (6), we will examine (3) in cases when the $\phi_{p\downarrow}$ fields are either freely fluctuating or completely frozen, limits that are appropriate close to the noninteracting point and in the strongly repulsive region, respectively, see Fig. 2(a). Integrating out $\phi_{p\downarrow}$ one gets an action that matches the action of a single impurity in LL. In the former case with the LP $(K_\rho + K_\sigma)/2$, and in the latter with K_{eff}. In both cases any repulsion ultimately renormalizes $V_\uparrow$ to infinity, which justifies massive fluctuations of $\phi_{p\uparrow}$.

Transport.—Now we will consider the conductance of our system using the anticipated flow diagram, see Fig. 2. Assuming the applied voltage across the dot is V_G and at the ends of the wire is V_L, an additional term should be included in the action (3), which reads $-eV_G \int d\tau \Phi_{\rho^-}/\pi - eV_L \int d\tau \Phi_{\rho^+}/(2\pi)$. The voltage V_L pushes the electrons to advance in one direction along the wire, while the gate voltage V_G serves as a single tuning parameter. Because of nonzero V_G, the shifted Fermi mo-

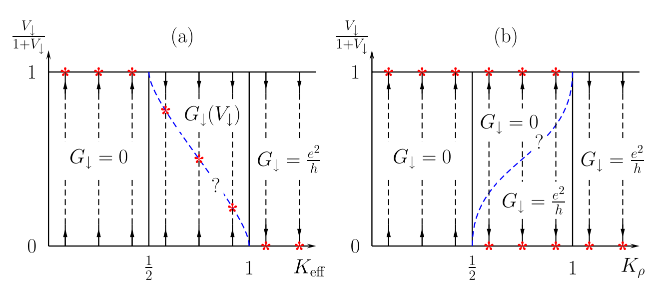

FIG. 2 (color online). The renormalization group flow diagram for $V_\downarrow$ as a function of interaction for parameters when the resonance is achieved for strong but not for weak impurities (a) and for weak but not for strong impurities (b). The middle region contains a line of fixed points in case (a), and a phase transition line in case (b), precise form of which is unknown. The noninteracting point is $K_\rho = K_\sigma = K_{\text{eff}} = 1$.

menta $k'_{Fs} = k_{Fs} - \frac{eV_G K_\rho^2}{\hbar v_F}$ should be taken in the above results, e.g., for the resonance conditions. This means the latter can be achieved by adjusting V_G for fixed both magnetic field and distance between impurities.

Without impurities or for attractive interaction in the low energy limit the system is described by Eq. (2) and has the perfect non-spin-polarized conductance $G_\uparrow = G_\downarrow = e^2/h$ [27–29]. The situation drastically changes when impurities are present. In the nonresonant case, our model translates into the single impurity problem with the LP $K_{\rm eff}$. Therefore, the conductance is suppressed at low V_L for repulsive interaction for both spin directions as $\sim V_L^{2/K_{\rm eff}-2}$.

On the resonance that corresponds to Fig. 2(a), i.e., for strong impurities when the charge state for spin-$\downarrow$ electrons is degenerate on the island, one gets spin-polarized conductance. Inside the region where the new line of fixed points appears, different scattering is experienced by two spin orientations. While $G_\uparrow$ is suppressed at low voltages as $\sim V_L^{2/K_{\rm eff}-2}$ near the point $K_{\rm eff} = 1/2$, and as $\sim V_L^{4/(K_\rho+K_\sigma)-2}$ for $K_\rho + K_\sigma \to 2^-$, $G_\downarrow$ is not suppressed even at very low voltages. It is controlled by the fixed point value $V_\downarrow^*(K_{\rm eff})$ which determines the effective strength of impurity scattering for a given $K_{\rm eff}$. We can estimate the conductance as $G_\downarrow(K_{\rm eff}) \approx \frac{e^2}{h}\frac{1}{1+[\pi V_\downarrow^*(K_{\rm eff})/E_F]^2}$. Within our approach we are not able to determine $V_\downarrow^*(K_{\rm eff})$. We expect that the fermionic method used in Ref. [30], which is beyond the scope of the present Letter, could give more results.

On the resonance that corresponds to weak impurities, Fig. 2(b), the system again has spin-polarized conductance which is controlled by the fixed points. In the lowest nontrivial order we have $G_\downarrow = e^2/h$ for $K_\rho > 1/2$ and $G_\uparrow \sim V_L^{4/(K_\rho+K_\sigma)-2}$, for not too big initial values of impurity strengths. Otherwise the spin polarization is destroyed and the conductance behaves as in the nonresonant case.

So far we considered zero temperatures. At finite temperature the picture will be qualitatively unchanged until the electron thermal energy is much smaller than the charging and Zeeman energy. In the opposite case, which is the high frequency limit $|\omega| \gg v_\ell/a$, or $T \gg K_\ell U_\ell$, for the starting action one would get Eqs. (3) and (4) with the replacements $K_\ell \to K_\ell/2$, $U_\ell \to 0$. Then the coherent effects of impurities are missing and our system effectively has the single impurity behavior [8,21].

Conclusions.—We have shown that a quantum wire with two impurities in an external magnetic field may have spin-filter properties for repulsive interaction. Our study is based on the resonance tunneling phenomenon which may be tuned by a single parameter for only one spin polarization.

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