*Physics*

# Magnetization Plateau in the Spin S=1/2 Two-Leg Ladder with Trimer Modulation of the Rung Exchange

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**ABSTRACT. The ground-state magnetic phase diagram of a two-leg spin ladder with trimerized modulated rung exchange is studied using the continuum-limit bosonization approach. In the limit where the rung exchange is dominant, the model is mapped onto the effective quantum sine-Gordon model with topological term. Six quantum phase transitions at different critical magnetic fields are identified. We have shown that the magnetization curve of the system exhibits two plateaus at magnetization equal to the 1/3 and 2/3 of the saturation value. The width of the plateaus is proportional to the excitation gap at given magnetization and scales as $\delta^{\nu}$, where $\delta$ is the amplitude of rung-exchange modulation and the critical exponent is obtained as $\nu = 1.13$ in the case of a ladder with antiferromagnetic legs and $\nu = 1.50$ in the case of ladder with ferromagnetic legs.** *© 2012 Bull. Georg. Natl. Acad. Sci.*

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**Introduction.** Low-dimensional quantum magnetism has been the subject of intense research for decades. Perpetual interest in the study of these systems is determined by their rather unconventional low-energy properties (see for a review [1]). An increased current activity in this field is connected with the large number of qualitatively new and phenomena dominated by the quantum effects recently discovered in these systems [2,3] as well as with the opened wide prospects for use of low-dimensional magnetic materials in modern nanoscale technologies.

The spin S=1/2 two-leg ladders represent one, particular subclass of low-dimensional quantum magnets which also has attracted much interest for a number of reasons. On the one hand, there was remarkable progress in recent years in the fabrication of such ladder compounds [4]. On the other hand, spin-ladder models pose interesting theoretical problems, since the excitation spectrum of a two-leg antiferromagnetic ladder is gapped and therefore, in the presence of a magnetic field these systems reveal an extremely complex behavior, dominated by quantum effects. The magnetic field driven quantum phase transitions in ladder

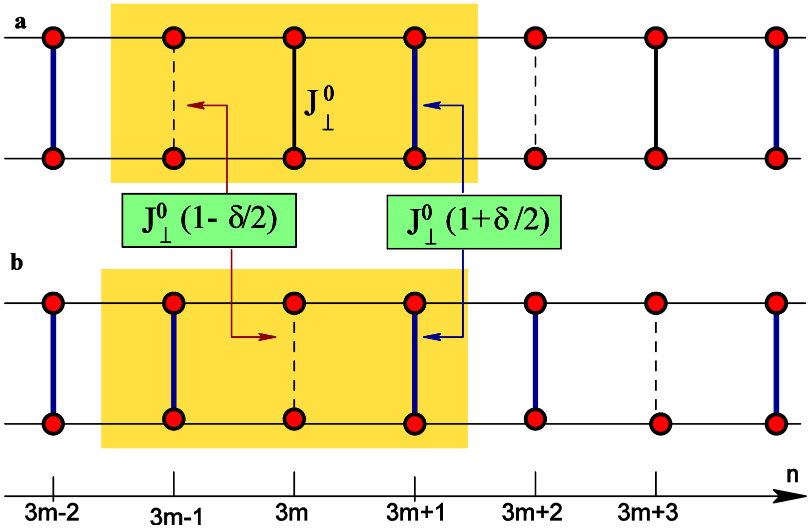


**Fig. 1.** Schematic representation of spin ladder with period three modulation of the rung exchange. Different width and color of vertical links corresponds to different values of the rung exchange, indicated in the figure by arrows. Yellow blocks mark unit cells of the ladder in the case of the "saw tooth" (a) and "domain" type (b) modulation of rung exchange. The horizontal axis marks rung numbers.

systems were intensively investigated both theoretically [5-11] and experimentally [12-17]. Usually, these most exciting properties of low-dimensional quantum spin systems exhibit strongly correlated effects driving them toward regimes with no classical analog. Properties of the low-dimentional systems in these regimes or "quantum phases" depend in turn on the properties of their ground state and low-lying energy excitations. Therefore search for the gapped phases emerging from different sources and study of ordered phases and quantum phase transitions associated with the dynamical generation of new gaps is an important direction in theoretical studies of quantum spin systems. Particular realization of such scenario appears in the case where the spin-exchange coupling constants are spatially modulated. The spin-Peierls effect in spin chains represents a prototype example of such behavior [18]. The first studies of spin chains with trimerized exchange modulation were performed by Hida [19] and Okamoto [20] in an attempt to describe some organic compounds with periodic couplings. Later, Oshikawa and collaborators [21] undertook the first systematic study of this problem and they provided a necessary condition for the appearance of magnetization plateau in 1D spin systems. Magnetic properties of the spin S=1/2 ladders with alternating rung exchange has been first studied in [22, 23].

**The model**. In this paper we study the spin S=1/2 two-leg ladder with modulated rung exchange given by the Hamiltonian

$$\hat{H} = J_{\parallel}\sum_{n,\alpha}\vec{S}_{n,\alpha}\vec{S}_{n+1,\alpha} - H\sum_{n,\alpha}S_n^z + \sum_{n,\alpha}J_{\perp}(n)\vec{S}_{n,1}\cdot\vec{S}_{n,2}, \tag{1}$$

where $\vec{S}_{n,\alpha}$ is the spin S=1/2 operator on rung $n$ ($n$=1,...,$N$) and leg $\alpha$ $(\alpha = 1,2)$. The interleg coupling is antiferromagnetic, $J_{\perp}(n) > 0$ and fulfils the condition $J_{\perp}(n+3) = J_{\perp}(n)$. We restrict our consideration by the following two types of rung exchange modulation: the "saw tooth" (or the "A") type:

$$J_{\perp}^{A}(n) = J_{\perp}^{0}\left[1 + \frac{\delta}{\sqrt{3}}\sin\left(\frac{2\pi}{3}n\right)\right], \tag{2a}$$

which corresponds to the case $J_\perp(3m) = J_\perp^0,\ J_\perp(3m\pm1) = J_\perp^0(1\pm\delta/2)$ (see Fig. 2.1a) and the "domain" (or the "B") type

$$J_\perp^B(n) = J_\perp^0\left[1+\frac{\delta}{2}\left(\frac{1}{3}-\frac{4}{3}\cos\left(\frac{2\pi}{3}n\right)\right)\right], \tag{2b}$$

which corresponds to the case $J_\perp(3m) = J_\perp^0(1-\delta/2),\ J_\perp(3m\pm1) = J_\perp^0(1+\delta/2)$ (see Fig. 2.1b).

**Derivation of the effective model**. We restrict our consideration to the limit of strong rung exchange and magnetic field $H, J_\perp^\pm >> J_\|, \delta J_\perp^0 \equiv \Delta$ and follow the route already used to study the standard ladder models in the same limit [7,8]. We start from the case $J_\| = 0$, where an eigenstate of $\hat{H}$ can be written as a product of on-rung states. At each rung two spins form either a singlet state $|s_n^0\rangle$ or one of the triplet states: $|t_n^0\rangle$, $|t_n^+\rangle$ and $|t_n^-\rangle$ with energies $E_S = -3J_\perp^n/4$, $E_t^0 = J_\perp^n/4$, and $E_t^\pm = J_\perp^n/4 \pm H$, respectively. When the magnetic field, $H$, is small, the ground state consists of a product of rung singlet states, while at $H \approx J_\perp^n$ the $|t_n^-\rangle$ becomes almost degenerate with $|s_n^0\rangle$, while other states have much higher energy. Integrating out the high energy states and introducing the effective pseudo-spin $\tau = 1/2$ operators, $\vec{\tau}_n$ which acts on these states as

$$\tau_n^z|s_n^0\rangle = -\tfrac{1}{2}|s_n^0\rangle,\ \ \tau_n^+|s_n^0\rangle = |t_n^+\rangle,\ \ \tau_n^-|s_n^0\rangle = 0;$$
$$\tau_n^z|t_n^+\rangle = +\tfrac{1}{2}|t_n^+\rangle,\ \ \tau_n^-|t_n^+\rangle = |s_n^0\rangle,\ \ \tau_n^+|t_n^+\rangle = 0,$$

we obtain the following effective Hamiltonian of the anisotropic Heisenberg chain with anisotropy parameter 1/2 in the uniform and spatially modulated longitudinal magnetic fields

$$H_{eff}^i = \sum_n \{J_\|(\tau_n^x\cdot\tau_{n+1}^x + \tau_n^y\cdot\tau_{n+1}^y + \tfrac{1}{2}\tau_n^z\cdot\tau_{n+1}^z) - [h_0^i + h_1^i(n)]\cdot\tau_n^z\} \quad i = A, B, \tag{3}$$

where

$$h_0^A = H - J_\perp^0 - J_\|/2 - \delta J_\perp^0/6, \qquad h_0^B = H - J_\perp^0 - J_\|/2, \tag{4a}$$

and

$$h_1^A(n) = \frac{1}{\sqrt{3}}\cdot\delta J_\perp^0\sin\left(\frac{2\pi}{3}n\right), \qquad h_1^B(n) = \frac{2}{3}\cdot\delta J_\perp^0\cos\left(\frac{2\pi}{3}n\right). \tag{4b}$$

Thus, the effective Hamiltonian is nothing but the anisotropic XXZ Heisenberg chain in uniform and spatially trimer modulated magnetic fields. The exchange anisotropy parameter of the effective model $\lambda = 1/2$, ($\lambda = -1/2$) for ladder with antiferromagnetic (ferromagnetic) legs $J_\| > 0$, ($J_\| < 0$). It is worth noting that a similar problem has been studied intensively in past years [24-26]. At $\delta = 0$, the effective Hamiltonian reduces to the XXZ Heisenberg chain in a uniform longitudinal magnetic field. The magnetization curve of this model has only saturation plateau corresponding to the fully polarized chain. At $H$=0 and $J_\perp^0 >> J_\|$, spins coupled by strong rung exchange form singlet pairs and the nonmagnetic and gapped ground state of the ladder system is well described by superposition of on-rung singlets. In terms of effective $\tau$ - spin model, the ground state corresponds to the ferromagnetic order with magnetization per site $m = -1/2$. In the opposite limit of very strong magnetic field $H >> J_\perp^0$, fully polarized state of the ladder with magnetization per rung $M$=1, is represented in terms of effective $\tau$ -spin chain as the fully polarized state with magnetization per site $m=1/2$. This gives the following relation between the magnetization per rung of the ladder system and the magnetization per site of the effective chain $m=M-1/2$.

The performed mapping allows to estimate the critical field $H_{on}$ corresponding to the transition from a gapped rung-singlet phase to a gapless paramagnetic phase, the saturation field $H_{sat}$ corresponding to the transition onto the fully polarized phase as well as the critical fields $H_{c1}^{\pm}$ and $H_{c2}^{\pm}$ which mark borders of the magnetization plateaus at $M=M_{sat}/3$ and $M=2M_{sat}/3$. The direct way to express $H_{on}$ and $H_{sat}$ in terms of ladder parameters is to perform the Jordan-Wigner transformation which maps the problem onto a system of interacting spinless fermions [27]:

$$\hat{H}_{sf}^{i}=\sum_{n}[t(a_{n}^{+}a_{n+1}+h.c.)+V\cdot a_{n}^{+}a_{n}a_{n+1}^{+}a_{n+1}-(\mu_{0}^{i}+\mu_{1}^{i}(n))\cdot a_{n}^{+}a_{n}],\qquad i=A,B \tag{5}$$

where $t=V=J_{\|}/2,\ \mu_{0}^{i}=\frac{1}{2}J_{\|}+h_{0}^{i},\ \mu_{1}^{i}(n)=h_{1}^{i}(n)$.

**The onset and saturation critical magnetic fields.** The lowest onset ($H_{on}$) and highest saturation ($H_{sat}$) critical field corresponds to that value of the chemical potential $\mu_0$ for which the band of fermions (or holes, after the corresponding particle-hole transformation) starts to fill up. In this limit we can neglect the interaction term in (5) and obtain the model of free particles with three band spectrum. Below, in this subsection we consider only the case of "B" type of exchange modulation. Generalization of these results for the case "A" is straightforward. In this case three bands of the single particle spectrum are given by

$$\begin{aligned}E_{1}(k)&=-H+J_{\perp}^{0}+J_{\|}\sqrt{1+\gamma^{2}}\cos\phi(k),\\E_{1}(k)&=-H+J_{\perp}^{0}+J_{\|}\sqrt{1+\gamma^{2}}\cos(\phi(k)+2\pi/3),\\E_{1}(k)&=-H+J_{\perp}^{0}+J_{\|}\sqrt{1+\gamma^{2}}\cos\phi(k)+4\pi/3),\end{aligned} \tag{6}$$

where $\gamma=\delta J_{\perp}^{0}/J_{\|}$ and

$$\phi(k)=\frac{1}{3}\arccos\left(\frac{\cos(3k)+\gamma^{3}}{\sqrt{\left(1+\gamma^{2}\right)^{3}}}\right), \tag{7}$$

and $-\pi/3<k\leq\pi/3$. This gives

$$H_{on}=J_{\perp}^{0}+\frac{\gamma}{6}+J_{\|}\sqrt{1+\gamma^{2}}\cos\left(\phi(\pi/3)+4\pi/3\right), \tag{8a}$$

in the case of antiferromagnetic legs ($J_{\|}>0$) and

$$H_{on}=J_{\perp}^{0}+\frac{\gamma}{6}-J_{\|}\sqrt{1+\gamma^{2}}\cos\phi(0), \tag{8b}$$

in the case of ferromagnetic legs ($J_{\|}<0$).

To estimate the critical magnetic field $H_{sat}$, which marks the transition into the phase with saturated magnetization, it is useful to make a site-dependent particle-hole transformation on the Hamiltonian of Eq.(5): $a_n^+ \to d_n$. Up to constant the new Hamiltonian reads

$$\hat{H}_{sf} = \sum_n [-t(d_n^+ d_{n+1} + h.c.) + V \cdot d_n^+ d_n d_{n+1}^+ d_{n+1} - (\mu_0^d + \mu_1(n)) \cdot d_n^+ d_n], \tag{9}$$

where the hole chemical potential $\mu_0^d = -\mu_0 + J_{\parallel}/2$. In terms of holes, $H_{sat}$ corresponds to the chemical potential where the band starts to fill up, and one can neglect again the interaction term. However, the effect of interaction is now included in the shifted value of the chemical potential for holes. After simple transformations, we obtain

$$\begin{aligned} H_{sat} &= J_{\perp}^0 + \mathrm{J}_{\parallel}/2 + \mathrm{J}_{\parallel}\sqrt{1+\gamma^2}\cos\phi(0), \quad \text{at } \mathrm{J}_{\parallel} > 0, \\ H_{sat} &= J_{\perp}^0 + \mathrm{J}_{\parallel}/2 - \mathrm{J}_{\parallel}\sqrt{1+\gamma^2}\cos\left(\phi(\pi/3) + 4\pi/3\right), \quad \text{at } \mathrm{J}_{\parallel} < 0. \end{aligned} \tag{10}$$

The spectrum of the system in the free fermion limit (6) allows also to determine two other important parameters which characterize the values of magnetization in the magnetization curve of the system in which the additional plateaus appear and the values of magnetic field which correspond to the center of each plateau. Below we consider only the case $J_{\parallel} > 0$, however extension to the case $J_{\parallel} < 0$ is straightforward. At 1/3-rd band-filling, all states in the lower band $E_3(k)$ are filled and separated from the empty at $E_2(k)$ band by the energy gap $2\Delta_1^0 = E_2(0) - E_3(0)$. Therefore, the first magnetization plateau will appear at magnetization equal to 1/3 of the saturation value. The magnetic field at the center of the plateau is given by $H_{c1} = J_{\perp}^0 + E_3(0) + \Delta_1^0$. Analogously at 2/3-rd band-filling, all states in the lower bands $E_3(k)$ and $E_2(k)$ are filled and separated from the empty $E_1(k)$ band by the energy $2\Delta_2^0 = E_1(\pi/3) - E_2(\pi/3)$. Therefore, the second magnetization plateau appears at magnetization equal to 2/3 of the saturation value and the magnetic field at the center of plateau is given by $H_{c2} = J_{\perp}^0 + E_3(0) + \Delta_2^0$. Since at finite band-filling the effect of interaction between spinless fermions cannot be ignored the width of the plateau differs from its bare value $2\Delta_j^0, j = 1, 2$. In the forthcoming section we use the continuum limit bosonization treatment of the effective spin-chain model (5) to determine parameters characterizing the appearance and scale of the magnetization plateaus.

**Critical magnetic fields characterizing magnetization plateaus $H_{c1}^{\pm}$ and $H_{c2}^{\pm}$.** To determine the critical fields which mark borders of the magnetization plateaus we use the continuum-limit bosonization approach. Following the usual procedure in the low energy limit, we bosonize the spin degrees of freedom at fixed magnetization per site *m* as follows [28]

$$\tau_n^z = m + \sqrt{\frac{K}{\pi}}\partial_x \phi(x) + \frac{A}{\pi}\sin\left(\sqrt{4\pi K}\phi(x) + (2m+1)\pi n\right), \tag{11a}$$

$$\tau_n^+ = e^{-i\sqrt{\pi/K}\theta}\left[1 + \frac{B}{\pi}\cdot\sin\left(\sqrt{4\pi K}\phi(x) + (2m+1)\pi n\right)\right], \tag{11b}$$

where *A* and *B* are non-universal real constants of the order of unity [29], $\phi(x)$ and $\theta(x)$ are dual bosonic fields $\partial_x\phi = \partial_t\theta$ and $K(\lambda,m)$ is the spin stiffness parameter for spin chain with anisotropy $\lambda$ and magnetization *m*. At zero magnetization $K(\lambda,0) = \pi/2(1-\arccos\lambda)$ and therefore for a ladder with isotropic antiferromagnetic legs $K(1/2,0) = 0.75$, while for a ladder with ferromagnetic legs $K(-1/2,0) = 1.5$. At the transition line into the ferromagnetic phase, where the magnetization reaches its saturation value $m_{sat} = 0.5$, the spin stiffness parameter takes the universal value $K(0.5,\lambda) = 1$ [28]. Respectively for finite magnetization, at $0 < m < m_{sat}$ and $J_{\parallel} > 0$ ($J_{\parallel} < 0$) the function $K(m,\lambda)$ monotonically increases (decreases) with increasing *m* and reaches its maximum (minimum) value at saturation magnetization $K(m_{sat},\lambda) = 1$.

Using (11) we obtain the following continuum-limit bosonized Hamiltonian to describe infrared properties of the system at magnetization *m*, $H^i_{Bos} = H_0 + H^i_m$, where

$$H^i_0 = \int dx\left[\tfrac{1}{2}(\partial_x\phi)^2 + \tfrac{1}{2}(\partial_x\theta)^2 - h^i_0\sqrt{\frac{K}{\pi}}\partial_x\phi\right], \quad i = A,B \tag{12a}$$

$$H^A_m = \frac{h^A_1}{2\pi a_0}\int dx\left[\cos(\sqrt{4\pi K}\phi + 2\pi(m+\tfrac{1}{6})x) - \cos(\sqrt{4\pi K}\phi + 2\pi(m+\tfrac{5}{6})x)\right], \tag{12b}$$

$$H^B_m = \frac{h^B_1}{2\pi a_0}\int dx\left[\sin(\sqrt{4\pi K}\phi + 2\pi(m+\tfrac{1}{6})x) + \sin(\sqrt{4\pi K}\phi + 2\pi(m+\tfrac{5}{6})x)\right]. \tag{12c}$$

Away from the commensurate values of effective chain magnetization $m = -1/6$ and $m = 1/6$ corresponding to the ladder magnetizations $M = M_{sat}/3$ and $M = 2M_{sat}/3$, respectively the sine terms in (12) are strongly oscillating and have to be neglected. In this case the system is described by the gapless Gaussian bose-field and the spin system is in the gapless paramagnetic phase with finite magnetization, which continuously increases with increasing magnetic field, till it reaches the commensurate values $m = \pm 1/6$.

At $m = -1/6$ ($m = 1/6$) the first (second) sine term in Eqs. (12.b-c) is non-oscillating and has to be retained. Up to an irrelevant shift on constant of the bosonic field, the general bosonic Hamiltonoian which describes infrared properties of the ladder for both “A” and “B” type of trimer rung exchange modulation is given by

$$H^i_0 = \int dx\left[\tfrac{1}{2}(\partial_x\phi)^2 + \tfrac{1}{2}(\partial_x\theta)^2 - h^i_0\sqrt{\frac{K}{\pi}}\partial_x\phi + \frac{\Delta^i_0}{2\pi a_0}\cos\sqrt{4\pi K}\phi\right], \quad i = A,B \tag{13}$$

where $\Delta^A_0 = \delta J^0_{\perp}/\sqrt{3}$ and $\Delta^B_0 = 2\delta J^0_{\perp}/3$. The Hamiltonian (13) can be easily recognized as the standard Hamiltonian for the commensurate-incommensurate [30] transition which has been intensively studied using bosonization approach [31] and the Bethe ansatz technique [32]. Below we use the results obtained in [31, 32] to describe the magnetization plateau and the transitions from a gapped (plateau) to gapless paramagnetic phases.

Let us first consider $h^i_0 = 0$. In this case the continuum theory of the initial ladder model in the magnetic field $H = J_{\perp} + J_{\parallel}/2$ is given by the quantum sine-Gordon (SG) model with a massive term $\approx h^1_{eff}\sin(\sqrt{4\pi K}\phi)$. From the exact solution of the SG model [33] it is known that the excitation spectrum of the model (13) is gapped and the value of the renormalized spin gap $M_{sol}$ scales with its bare value as [34]

$M_{sol} \approx J_{\|}(\Delta / J_{\|})^{1/(2-K)}$. Thus for $h_{eff}^0 = 0$ the low-energy behavior of the system is determined by the strongly relevant modulated magnetic field (i.e. modulated part of the rung exchange), represented by the term $h_1^i \sin(\sqrt{4\pi K}\phi)$. In the ground state the field $\varphi$ is pinned in one of the minima of the potential $<\sin(\sqrt{4\pi K}\phi)|>=-1$. In view of (7a) we conclude that this state corresponds to a long-range-ordered antiferromagnetic phase of the effective Heisenberg chain (2), i.e. to a phase of the initial ladder system, where spins on weak rungs which have a dominant triplet character and spins on strong rungs with ($n = 3m \pm 1$) predominantly singlets. At $h_{eff}^0 \neq 0$ the very presence of the gradient term in the Hamiltonian (12) makes it necessary to consider the ground state of the SG model in sectors with nonzero topological charge. The effective chemical potential $\approx h_{eff}^0 \partial_x \varphi$ tends to change the number of particles in the ground state i.e. to create finite and uniform density solitons. However, this implies that the vacuum distribution of the field $\varphi$ will be shifted with respect to the corresponding minima. This competition between contributions of the smooth and staggered components of the magnetic field is resolved as a continuous phase transition from a gapped state at $h_{eff}^0 < M_{sol}$ to a gapless (paramagnetic) phase at $h_{eff}^0 > M_{sol}$ [25]. The condition $h_{eff}^0 = M_{sol}$ gives two additional critical values of the magnetic field $H_{c1}^{\pm} = J_{\perp}^0 + \frac{1}{2}J_{\|} \pm J_{\|}\left(\delta J_{\perp}^0 / J_{\|}\right)^{1/(2-K)}$. Correspondingly the width of each magnetization plateau is given by $H_{c1}^{+} - H_{c1}^{-} = 2C_0 J_{\|}\left(\delta J_{\perp}^0 / J_{\|}\right)^{1/(2-K)}$. To estimate the numerical value of the spin stiffness parameter $K$ at magnetization $m$ and anisotropy $\lambda$, we use the following ansatz [35]

$$K(m,\lambda) = K(0,\lambda) + \frac{|m|}{m_{sat}}[1 - K(0,\lambda)]. \tag{13}$$

This ansatz gives that $K \cong 0.87$ at $J_{\|} > 0$ and $K \cong 1.335$ at $J_{\|} < 0$. It is straightforward to get that in the antiferromagnetic case the width of the magnetization plateau scales as $\delta^{1.13}$, while in the case of a chain with ferromagnetic legs it scales as $\delta^{3/2}$.

**Conclusions.** We have studied the effect of period three modulation of the strong antiferromagnetic rung exchange on the ground state magnetic phase diagram of a spin-1/2 ladder.

In the limit commensurate with rung exchange uniform magnetic field, we map the model to an effective XXZ Heisenberg chain in the presence of uniform and spatially modulated longitudinal magnetic fields. The anisotropy parameter of the effective chain $\lambda = 0.5$ ($\lambda = -0.5$) for a ladder with isotropic antiferromagnetic (ferromagnetic) legs, while the amplitude of the effective magnetic field modulation is proportional to the amplitude of the rung-exchange modulation $\delta$. Using the continuum-limit bosonization treatment of the effective spin-chain model, we have shown that modulation of rung exchange leads to generation of two gaps in the excitation spectrum of the system at magnetization equal to the 1/3 and 2/3 of its saturation value. As a result, the magnetization curve of the system $M(H)$ exhibits two plateaus at $M = M_{sat}/3$ and $M = 2M_{sat}/3$. The width of the plateaus, is proportional to the excitation gap and scales as $\delta^{\nu}$, where the critical exponent $\nu = 1.13$ in the case of a AF legs and $\nu = 1.50$ in the case of ladder with ferromagnetic legs.

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*ფიზიკა*

# დამაგნიტებულობის პლატოები კიბის სტრუქტურის მქონე S=1/2 სპინურ ჯაჭვში "საფეხურის" გასწვრივ გაცვლის ტრიმერული მოდულაციის შემთხვევაში

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**შესწავლილია სპინი $S=1/2$ ორჯაჭვიანი, "კიბის" სტრუქტურის მქონე, იზოტროპული სისტემის მაგნიტური ფაზური დიაგრამა კიბის შემადგენელ ჯაჭვებს შორისი „საფეხურის" გასწვრივი გაცვლითი ურთიერთქმედების ტრიმერული მოდულაციის შემთხვევაში. მოდელი განხილულია ძლიერი „საფეხუროვანი" გაცვლისა და (ერთგვაროვანი) მაგნიტური ველის ზღვარში, რომელშიც საწყისი ორჯაჭვიანი ამოცანა ეფექტური თავისუფლების ხარისხების ტერმინებში დაიყვანება სპინი $\tau=1/2$ ანიზოტროპული ჰაიზენბერგის ჯაჭვის ამოცანაზე გარე არაერთგვაროვან მაგნიტურ ველში. ეფექტური სპინური ჯაჭვის ანიზოტროპიის პარამეტრი $\lambda=0.5$ ($\lambda=-0.5$) კიბის შემადგენელი იზოტროპული ანტიფერომაგნიტური (ფერომაგნიტური) ჯაჭვების შემთხვევაში, ხოლო ეფექტური მაგნიტური ველის მოდულაცია განისაზღვრება კიბის „საფეხურის" გასწვრივი გაცვლის მოდულაციით. ნაჩვენებია, რომ გაცვლის მოდულაცია განაპირობებს სისტემის აღგზნებათა სპექტრში ღრეჭოების გაჩენას და ამის შედეგად ე.წ. დამაგნიტებულობის პლატოს წარმოქმნას მაგნიტური ველის იმ მნიშვნელობისას, როდესაც სისტემის დამაგნიტებულობა თავისი ნაჯერი მნიშვნელობის 1/3 და 2/3 აღწევს. დამაგნიტებულობის პლატოთა სიგანე შესაბამისი სპექტრალური ღრეჭოს პროპორციულია, ხოლო მისი მოდულაციის ამპლიტუდაზე ($\delta$) დამოკიდებულება მოიცემა ხარისხობრივი კანონით $\delta^{\nu}$, სადაც $\nu=1.13$ ($\nu=1.50$) ანტიფერომაგნიტული (ფერომაგნიტული) ჯაჭვების შემთხვევაში.**

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