Extensions of the Standard Model and Their Implications in Particle Physics and Cosmology

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Statement

As an author of the dissertation presented, I state that the dissertation represents my original work and does not include material already published, submitted for publication or presented as a PhD thesis by other authors, unless mentioned or cited in accordance with proper rules.

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11.02.2019

აბსტრაქტი

დაბალენერგეტიკული რეზონანსული ლეპტოგენეზისის სცენარის ფარგლებში კოსმოლოგიური CP ასიმეტრია შეიძლება გაჩნდეს რადიაციული შესწორებებიდან დამუხტული ლეპტონების იუკავას ბმების გათვალისწინებით. შემთხვევების უმეტესობაში, როგორც მოსალოდნელია, გადამწყვეტ როლს ამ საკითხში თამაშობს λ_τ ბმა, თუმცა ნეიტრინოს სპეციფიკური ტექსტურების განხილვისას, λ_μ-ს გათვალისწინებაც წარმოშობს CP დარღვევას ერთი მარყუჟის მიახლოებაში.

კოსმოლოგიური CP დარღვევის დასაკავშირებლად ლეპტონური CP დარღვევის δ ფაზასთან, განვიხილეთ ორი მარჯვენა ნეიტრინოთი გაფართოებული მინიმალური სუპერსიმეტრიული სტანდარტული მოდელი, იმ პირობით, რომ აღნიშნული ნეიტრინოები მაღალ ენერგეტიკულ სკალაზე მასის მიხედვით გადაგვარებულები არიან. ამასთან ერთად, განვიხილეთ ორი ტექსტურული ნულის მქონე 3×2 დირაკისეული იუკავას მატრიცები. ეს ტექსტურები, ნეიტრინოს მასების გენერირების სი-სოუ მექანიზმის გათვალისწინებით იძლევიან მსუბუქი ნეიტრინოებისთვის მასურ მატრიცებს, რომელთათვისაც ერთი, ლეპტონური რიცხვის ორი ერთეულით დამრღვევი $\Delta L = 2$ და ხუთის ტოლი განზომილების ($\mathrm{d} = 5$) ოპერატორის დამატება, იძლევა წინასწარმეტყველებების გაკეთების შესაძლებლობის მქონე ნეიტრინოს სექტორებს, გამოთვლადი CP ასიმეტრიებით. ამ უკანასკნელების გენერირება ხდება λ_τ და/ან λ_μ ბმებით ერთმარყუჟოვან დონეზე. ნაშრომში მოყვანილია ლეპტოგენეზისის დეტალური ანალიზი. გარდა ამისა, განზოგადებულია ადრე შესწავლილი ერთი ტექსტურული ნულის მქონე ზოგიერთი დირაკისეული იუკავას მატრიცა და ნაჩვენებია, რომ ნეიტრინოს მასური მატრიცებისთვის ერთი $\mathrm{d} = 5$ ოპერატორით განპირობებული წვლილის გათვალისწინება, CP ასიმეტრიებისთვის გამოთვლილ ერთმარყუჟოვან შესწორებებთან ერთად, იძლევა ექსპერიმენტთან თავსებადი ნეიტრინოს სექტორის მქონე მოდელებს და ბარიონული ასიმეტრიის სასურველი მნიშვნელობა მიიღწევა რეზონანსული ლეპტოგენეზისით მარჯვენა ნეიტრინოების მასების შედარებით დაბალი მნიშვნელობებისთვის($\sim$რამდენიმე ტევი – 10^7 გევი).

ძირითადი საძიებო სიტყვები: CP დარღვევა, რეზონანსული ლეპტოგენეზისი, ნეიტრინოს მასები და შერევა, რენორმალიზაცია.

Abstract

Within the low scale resonant leptogenesis scenario, the cosmological CP asymmetry may arise by radiative corrections through the charged lepton Yukawa couplings. While in some cases, as one expects, decisive role is played by the λ_τ coupling, we show that in specific neutrino textures only by inclusion of the λ_μ the cosmological CP violation is generated at 1-loop level.

With the purpose to relate the cosmological CP violation to the leptonic CP phase δ, we consider an extension of MSSM with two right handed neutrinos (RHN), which are degenerate in mass at high scales. Together with this, we first consider two texture zero 3×2 Dirac Yukawa matrices of neutrinos. These via see-saw generated neutrino mass matrices augmented by single $\Delta L = 2$ dimension five (d = 5) operator give predictive neutrino sectors with calculable CP asymmetries. The latter is generated through $\lambda_{\mu,\tau}$ coupling(s) at 1-loop level. Detailed analysis of the leptogenesis is performed. We also revise some one texture zero Dirac Yukawa matrices, considered earlier, and show that addition of a single $\Delta L = 2$, d = 5 entry in the neutrino mass matrices, together with newly computed 1-loop corrections to the CP asymmetries, give nice accommodation of the neutrino sector and desirable amount of the baryon asymmetry via the resonant leptogenesis even for rather low RHN masses($\sim$few TeV – 10^7 GeV).

Key Words: CP violation, resonant leptogenesis, neutrino mass and mixing, renormalization.

The main results presented in the thesis have been published in the papers:

A. Achelashvili and Z. Tavartkiladze, "Neutrino mass matrices from two zero 3×2 Yukawa textures and minimal d = 5 entries", Int. J. Mod. Phys. A **31**, no. 13, 1650077 (2016). DOI: 10.1142/S0217751X16500779

A. Achelashvili and Z. Tavartkiladze, "Calculable Cosmological CP Violation and Resonant Leptogenesis", Phys. Rev. D **96**, no. 1, 015015 (2017). DOI: 10.1103/PhysRevD.96.015015

A. Achelashvili and Z. Tavartkiladze, "Texture Zero Neutrino Models and Their Connection with Resonant Leptogenesis", Nucl. Phys. B **929**, 21 (2018). DOI: 10.1016/j.nuclphysb.2018.02.001

A. Achelashvili and Z. Tavartkiladze, "Leptonic CP violation and leptogenesis", AIP Conf.Proc. 1900 (2017) no.1, 020012. DOI: 10.1063/1.5010116

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Contents

1 Introduction

Problem of neutrino masses and generation of the baryon asymmetry of the Universe, together with the dark matter problem and naturalness issues, call for some reasonable extension(s) of the Standard Model (SM). Perhaps simplest and most elegant simultaneous resolution of the first two puzzles is by the SM extension with the right handed neutrinos (RHN). This, by the $\Delta L = 2$ lepton number violating interactions generates the neutrino masses via celebrated see-saw mechanism [2], [3], accommodating the atmospheric and solar neutrino data [4], and gives an elegant possibility for the baryogenesis through the thermal leptogenesis [5] (for reviews see Refs. [6–8]).

Motivated by these, we consider the minimal supersymmetric standard model (MSSM)[1] augmented by two degenerate RHNs. Note that the degeneracy in the RHN mass spectrum offers an elegant possibility of resonant leptogenesis [9–11] (see [12–18] for recent discussions on resonant leptogenesis). This framework, as it was shown in [1, 16, 19], with specific forms of the Yukawa couplings, allows to have highly predictive model. In particular, in [20] all possible two texture zero 3×2 Dirac type neutrino Yukawa couplings have been considered. Those, via see-saw generated neutrino mass matrices augmented by a single d = 5, $\Delta L = 2$ operator, gave consistent neutrino scenarios. As it was shown, all experimentally viable cases allowed to calculate the cosmological CP violation in terms of a single known (from the model) leptonic phase δ.[2] In the subsequent work [16], the quantum corrections, primarily due to the λ_τ Yukawa coupling, have been investigated and, confirming earlier claim of Refs. [28], it was shown that the cosmological CP asymmetry arises at 1-loop order.[3] Demonstrated on a specific fully consistent neutrino model [16], this was shown to work well and opened wide prospect for the model building for the low scale resonant leptogenesis.

Starting with the two RHN's we investigate texture zero 3×2 Dirac type Yukawa couplings, which lead to the neutrino mass matrices with zero entries. On top of this, we augment the Lagrangian couplings with a single $\Delta L = 2$ lepton number violating d = 5 operator, which allows to keep some predictions and, at the same time, makes some mass matrices experimentally acceptable.

[1]This setup with the SUSY scale $M_S \sim$ few TeV guarantees the natural stability of the EW scale.

[2]The approach with texture zeros has been put forward in [21], which successfully relates the phase δ with the cosmological CP asymmetry [1, 16, 19–27].

[3]Studies of [1] included only λ_τ's 2-loop effects in the RG of the RHN mass matrix, which give parametrically more suppressed cosmological CP violation in comparison with those evaluated in [16].

It turns out that only three Yukawa textures (out of nine) possess cosmological CP phase which we relate to neutrino CP δ phase. All experimentally viable neutrino mass matrices lead to interesting predictions, which we investigate in detail.

Next, we give detailed and conscious derivation of the loop induced leptonic cosmological CP violation showing the necessity of inclusion of the charged lepton Yukawa couplings. Proof includes analytical expressions and is extended by inclusion of the λ_μ coupling which as it turns out in specific neutrino scenarios is the only relevant source of the cosmological CP violation within considered scenarios with the RHN masses $\lesssim 10^7$ GeV. We apply obtained result to specific neutrino textures. While in Refs. [1, 19, 21–27] the textures relating the cosmological CP violation to the leptonic δ phase (being still undetermined from the construction) have been discussed, in [20] we have proposed models, which not only give such relations, but also predict the values of the δ (the leptonic Dirac phase) and $\rho_{1,2}$ (two leptonic Majorana phases) and consequently the cosmological CP violation. From the constructions of [20] we consider viable neutrino models built by two texture zero 3×2 Yukawa coupling generated see-saw neutrino mass matrices augmented by the single $\Delta L = 2$, $\mathrm{d} = 5$ operator. For all these neutrino models, applying obtained all relevant corrections, we investigate the resonant leptogenesis process based on the procedure first described and performed in [17, 18]. Along with the cases where crucial is λ_τ coupling, we have ones for which the leptonic asymmetry originates due to the λ_μ Yukawa coupling. For the first time such possibility was presented in [17, 18]. We also revise textures of [1] and consider their improved versions by addition of single $\mathrm{d} = 5$ entry to the neutrino mass matrix, making them consistent and also viable for the baryogenesis. The details of the calculation of the contribution to the leptonic asymmetry from the right handed sneutrino decays are given as well. These include new corrections corresponding to the muon lepton soft SUSY breaking terms. Also, refined and more accurate expressions for the decay widths and absorptive parts, relevant for the CP asymmetries, are used.

Although in this work we are using the results of the loop induced cosmological CP violation (summarized in section 3.1 and in Appendixes A, B) for specific texture zero models, the application can be extended to any model with two (quasi) degenerate RHNs.

The thesis is organized as follows. In section 2.1, after defining the setup with two degenerate RHNs, we list all possible Two Zero 3×2 Yukawa Textures and point out those with inherent, unremovable complexity. In section 2.2 using complex Yukawa textures we build Neutrino mass matrices and augment them with d_5 operator mass terms. In section 2.3 we classify and analyze experimentally viable neutrino mass matrices with one and two texture zeroes and make predictions. In section 2.4 we relate cosmological CP and δ phases in two texture zero neutrino mass models and calculate the cosmological CP violating ϕ phase in each case.

In section 3.1 we give details of the calculation of the loop induced cosmological CP violation. Mainly we follow the method of Ref. [16] proving inevitable emergence of the cosmological CP violation via charged lepton Yukawas at 1-loop level, confirming earlier result of [28] (which took into account λ_τ coupling). We also include the contribution due to the λ_μ which had not been considered before publication of [17, 18]. In section 3.2, with the updated neutrino data, we give updated results of the two texture zero neutrino mass models which are highly predictive and determine cosmological CP violating phases in terms of the δ phase. In section 3.3, applying results of the previous sections we determine cosmological CP violation for each considered model and use them for calculating of the baryon asymmetry. The latter is generated via resonant leptogenesis. We demonstrate that successful scenarios are possible for the low RHN masses (in a range few TeV – 10^7 GeV). In section 3.4 we revise textures of Ref. [1] and make model improvements of the obtained neutrino mass matrices by adding the single $\Delta L = 2$, d $= 5$ mass terms to certain non-zero entries (in a spirit of Sect. 3.2). This makes the neutrino scenarios compatible with the best fit values of the neutrino data [4] and also proves to blend well with the leptogenesis scenarios. We stress that in the P_4 neutrino texture scenario (discussed in Sect. 3.2) and also in the texture $B_2{}'$ (considered in Sect. 3.4), for successful leptogenesis to take place crucial role is played by the λ_μ Yukawa coupling which via 1-loop correction generates sufficient amount of the cosmological CP asymmetry. Such possibility had not been considered in the literature prior to [17, 18]. (The general expressions for the corresponding corrections are presented in Sect. 3.1). Sect. 3.5 includes discussion and outlook where we also summarize our results and highlight some prospects for a future work. Conclusions are given in Sect. 4. In Sect. 6 we stress significance of the main scientific results presented in the thesis, their novelty and relevance to particle physics and cosmology. Sect. 5 consists in the information provided in Sect. 6 and translated into Georgian. Appendix A includes some expressions, details related to the renormalization group (RG) studies and description of calculation procedures we are using. In Appendix B the contribution to the net baryon asymmetry from the decays of the scalar components (RHS) of the RHN superfields is considered in detail. These analyses also include new corrections due to λ_μ and corresponding soft SUSY breaking trilinear A_μ coupling (besides λ_τ, A_τ and other relevant couplings). In Appendix C we highlight and discuss some key concepts of Baryogenesis.

2 Neutrino Mass Matrices from Two Zero 3×2 Yukawa Textures and Minimal $\mathrm{d} = 5$ Entries

2.1 Two texture zero 3×2 Yukawa matrices: $2T_0Y_{32}$'s

Let us consider the lepton sector of MSSM augmented with two right-handed neutrinos N_1 and N_2. The relevant Yukawa superpotential couplings are given by:

$$W_{lept} = W_e + W_\nu, \quad W_e = l^T Y_e^{\mathrm{diag}} e^c h_d, \quad W_\nu = l^T Y_\nu N h_u - \frac{1}{2} N^T M_N N, \tag{2.1}$$

where h_d and h_u are down and up type MSSM Higgs doublet superfields respectively. N, l, e^c denote:

$$N = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}, \quad l^T = (l_1, l_2, l_3), \quad e^{cT} = (e_1^c, e_2^c, e_3^c). \tag{2.2}$$

In the next section, upon deriving the neutrino mass matrices, together with couplings of Eq. (2.1), the single $\mathrm{d} = 5$ operator per the neutrino mass matrix will be applied. Because of this, in comparison with the approach considered in [1], more two texture zero Y_ν Yukawa matrices will be compatible with the current experiments. We will work in a basis in which the charged lepton Yukawa matrix is diagonal and real:

$$Y_e^{\mathrm{diag}} = \mathrm{Diag}(\lambda_e, \lambda_\mu, \lambda_\tau). \tag{2.3}$$

As far as the RHN mass matrix M_N is concerned, we will assume that it has the form:

$$M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M. \tag{2.4}$$

This form of M_N is crucial for our studies, since (2.4) at a tree level leads to the mass degeneracy of the RHN's, it has interesting implications for resonant leptogenesis [1, 19] and also, as we will see below, for building predictive neutrino scenarios. In a spirit of [1], here we attempt to classify specific texture zero scenarios with degenerate RHN's which lead to predictions consistent with experiments. The matrix Y_ν contains two columns. Since due to the form of M_N there is an exchange invariance $N_1 \to N_2$, $N_2 \to N_1$, it does not matter in which column of Y_ν we set elements to zero. Thus, starting with the Yukawa couplings, we consider the following nine different 3×2 Yukawa matrices with two zero entries:

$$T_1 = \begin{pmatrix} \times & 0 \\ \times & 0 \\ \times & \times \end{pmatrix}, \quad T_2 = \begin{pmatrix} \times & 0 \\ \times & \times \\ \times & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} \times & \times \\ \times & 0 \\ \times & 0 \end{pmatrix},$$
$$T_4 = \begin{pmatrix} 0 & 0 \\ \times & \times \\ \times & \times \end{pmatrix}, \quad T_5 = \begin{pmatrix} \times & 0 \\ 0 & \times \\ \times & \times \end{pmatrix}, \quad T_6 = \begin{pmatrix} \times & 0 \\ \times & \times \\ 0 & \times \end{pmatrix},$$
$$T_7 = \begin{pmatrix} \times & \times \\ 0 & 0 \\ \times & \times \end{pmatrix}, \quad T_8 = \begin{pmatrix} \times & \times \\ \times & 0 \\ 0 & \times \end{pmatrix}, \quad T_9 = \begin{pmatrix} \times & \times \\ \times & \times \\ 0 & 0 \end{pmatrix}, \qquad (2.5)$$

where "$\times$"s stand for non-zero entries. Next, we factor out phases from these textures, in such a way as to make maximal number of entries be real. As it turns out only T_4, T_7 and T_9 will have unfactorable phases. The latter should be relevant to the lepton asymmetry.

TEXTURE T_1

Starting with T_1 Yukawa matrix, we parameterize it and write in a form of factored out phases:

$$T_1 = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ a_2 e^{i\alpha_2} & 0 \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & 0 \\ a_2 & 0 \\ a_3 & b_3 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \qquad (2.6)$$

with

$$\omega = \rho + \alpha_3 - \beta_3, \quad x = \alpha_1 + \beta_3 - \alpha_3 - \rho, \quad y = \alpha_2 + \beta_3 - \alpha_3 - \rho, \quad z = \beta_3 - \rho. \qquad (2.7)$$

where a_i, b_3 and all phases are real. Below, in a similar way, we write down the remaining Yukawa textures given in Eq.(2.5).

TEXTURE T_2

$$T_2 = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & 0 \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ a_3 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.8}$$

with

$$\omega = \rho + \alpha_2 - \beta_2, \quad x = \alpha_1 + \beta_2 - \alpha_2 - \rho, \quad y = \beta_2 - \rho, \quad z = \alpha_3 + \beta_2 - \alpha_2 - \rho. \tag{2.9}$$

TEXTURE T_3

$$T_3 = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & 0 \\ a_3 e^{i\alpha_3} & 0 \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & 0 \\ a_3 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.10}$$

with

$$\omega = \rho + \alpha_1 - \beta_1, \quad x = \beta_1 - \rho, \quad y = \alpha_2 - \alpha_1 + \beta_1 - \rho, \quad z = \alpha_3 - \alpha_1 + \beta_1 - \rho. \tag{2.11}$$

TEXTURE T_4

$$T_4 = \begin{pmatrix} 0 & 0 \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ a_2 & b_2 \\ a_3 & b_3 e^{i\phi} \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.12}$$

with

$$\omega = \alpha_2 - \beta_2 + \rho, \quad y = \beta_2 - \rho, \quad z = \alpha_3 - \alpha_2 + \beta_2 - \rho, \quad \phi = \alpha_2 - \alpha_3 + \beta_3 - \beta_2. \tag{2.13}$$

TEXTURE T_5

$$T_5 = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ 0 & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & 0 \\ 0 & b_2 \\ a_3 & b_3 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.14}$$

with

$$\omega = \rho + \alpha_3 - \beta_3, \quad x = \alpha_1 + \beta_3 - \alpha_3 - \rho, \quad y = \beta_2 - \rho, \quad z = \beta_3 - \rho. \tag{2.15}$$

TEXTURE T_6

$$T_6 = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ 0 & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ 0 & b_3 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.16}$$

with

$$\omega = \rho + \alpha_2 - \beta_2, \quad x = \alpha_1 + \beta_2 - \alpha_2 - \rho, \quad y = \beta_2 - \rho, \quad z = \beta_3 - \rho. \tag{2.17}$$

TEXTURE T_7

$$T_7 = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ 0 & 0 \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ 0 & 0 \\ a_3 & b_3 e^{i\phi} \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.18}$$

with

$$\omega = \rho + \alpha_1 - \beta_1, \quad x = \beta_1 - \rho, \quad z = \alpha_3 - \alpha_1 + \beta_1 - \rho, \quad \phi = \alpha_1 - \alpha_3 - \beta_1 + \beta_3. \tag{2.19}$$

TEXTURE T_8

$$T_8 = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & 0 \\ 0 & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & 0 \\ 0 & b_3 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.20}$$

with

$$\omega = \rho + \alpha_1 - \beta_1, \quad x = \beta_1 - \rho, \quad y = \alpha_2 - \alpha_1 + \beta_1 - \rho, \quad z = \beta_3 - \rho. \tag{2.21}$$

TEXTURE T_9

$$T_9 = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 e^{i\phi} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix}, \tag{2.22}$$

with

$$\omega = \alpha_1 - \beta_1 + \rho, \quad x = \beta_1 - \rho, \quad y = \alpha_2 - \alpha_1 + \beta_1 - \rho, \quad \phi = \alpha_1 - \beta_1 - \alpha_2 + \beta_2. \tag{2.23}$$

The phases x, y and z can be eliminated by proper redefinition of the states l and e^c. As far as the phases ω and ρ are concerned, because of the form of the M_N matrix (2.4), also they will turn out to be non-physical. This is the one main difference of our construction from the scenarios considered earlier [26]. As we see, in textures T_4, T_7 and T_9 there remains one unremovable phase ϕ (i.e. in the second matrices of the r.h.s of Eqs. (2.12) (2.18) and (2.22) respectively). This physical phase ϕ is relevant to the leptogenesis [1] and also, as we will see below, it will be related to phase δ, determined from the neutrino sector.

2.2 Neutrino mass matrices derived from $2T_0Y_{32}$'s and one d $= 5$ operator

Integrating the RHN's, from the superpotential couplings of Eq. (2.1), using the see-saw formula, we get the following contribution to the light neutrino mass matrix:

$$M_\nu^{ss} = -\langle h_u^0\rangle^2 Y_\nu M_N^{-1} Y_\nu^T . \tag{2.24}$$

For Y_ν in (2.24) the textures T_i listed in the previous section should be used in turn. All obtained matrices M_ν^{ss}, if identified with light neutrino mass matrices, will give experimentally unacceptable results. The reason is the number of texture zeros which we have in T_i and M_N matrices. In order to overcome this difficulty we include the following d $= 5$ operator:

$$\mathcal{O}_{ij}^5 \equiv \frac{\tilde{d}_5 e^{ix_5}}{2M_*} l_i l_j h_u h_u \tag{2.25}$$

where $\tilde{d}_5$, x_5 and M_* are real parameters. For each case, we will include a single term of the type of Eq. (2.25). The latter, together with (2.24) will contribute to the neutrino mass matrix. This will allow to have viable models and, at the same time because of the minimal number of the additions, we will still have predictive scenarios. The operators (2.25) can be obtained by another sector in such a way as to not affect the forms of T_i and M_N matrices. We comment about this in Sect. 3.5. Here, we just consider operators (2.25) without specifying their origin and investigate their implications. Recall that, in the previous section, we have written the Yukawa textures in the form:

$$Y_\nu = \mathcal{P}_1 Y_\nu^R \mathcal{P}_2, \tag{2.26}$$

where $\mathcal{P}_1, \mathcal{P}_2$ are diagonal phase matrices and Y_ν^R is either a real matrix or contains only one phase (for T_4, T_7 and T_9). Making the field phase redefinitions:

$$l' = \mathcal{P}_1 l, \quad N' = \mathcal{P}_2 N, \quad (e')^c = \mathcal{P}_1^* e^c \ \text{ with } \ \mathcal{P}_1 = \mathrm{Diag}(e^{ix}, e^{iy}, e^{iz}), \quad \mathcal{P}_2 = \mathrm{Diag}(e^{i\omega}, e^{i\rho}) \tag{2.27}$$

the superpotential coupling will become:

$$W_e = (l')^T Y_e^{\rm diag}(e')^c h_d, \quad W_\nu = (l')^T Y_\nu^R N' h_u - \frac{1}{2}(N')^T M_N' N' \tag{2.28}$$

with:

$$M_N' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tilde{M}, \quad \tilde{M} = e^{-i(\omega+\rho)} M. \tag{2.29}$$

Now, for simplification of the notations, we will get rid of the primes (i.e. perform $l' \to l$, $e^{c\prime} \to e^c$,...) and in Eq. (2.24) using Y_ν^R instead of Y_ν, from different T_i textures we get corresponding M_ν^{ss}, and then adding the operator (2.25), obtain the final neutrino mass matrix.

From textures $T_{1,2,3}$ we obtain:

$$M_{T_1} = \begin{pmatrix} 0 & 0 & a_1 b_3 \\ 0 & 0 & a_2 b_3 \\ a_1 b_3 & a_2 b_3 & 2a_3 b_3 \end{pmatrix} \bar{m}, \quad M_{T_2} = \begin{pmatrix} 0 & a_1 b_2 & 0 \\ a_1 b_2 & 2a_2 b_2 & a_3 b_2 \\ 0 & a_3 b_2 & 0 \end{pmatrix} \bar{m}, \quad M_{T_3} = \begin{pmatrix} 2a_1 b_1 & a_2 b_1 & a_3 b_1 \\ a_2 b_1 & 0 & 0 \\ a_3 b_1 & 0 & 0 \end{pmatrix} \bar{m}, \tag{2.30}$$

where $\bar{m} = -\langle h_u^0 \rangle^2 / \tilde{M}$. It is easy to verify that adding one d = 5 operator mass term to any entry of these mass matrices will not make them experimentally acceptable. Thus, discarding them we move to the remaining textures.

From texture T_4:

$$M_{T_4} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2a_2 b_2 & a_3 b_2 + a_2 b_3 e^{i\phi} \\ 0 & a_3 b_2 + a_2 b_3 e^{i\phi} & 2a_3 b_3 e^{i\phi} \end{pmatrix} \bar{m}. \tag{2.31}$$

Adding the d = 5 operators to zero entries of this matrix, we will get three different neutrino mass matrices. Therefore, addition of (2.25) type term will be performed in the (1,1), (1,2) and (1,3) entries respectively. Since the phase x in Eqs. (2.12), (2.13) is undetermined, we can shift the phase of state l_1 in such a way as to match the phase of the (2.25) operator with the phase of $\bar{m}$. Thus, this addition will not introduce additional phases inside the neutrino mass matrices. They will have forms:

$$M_{T_4}^{(11)} = \begin{pmatrix} d_5 & 0 & 0 \\ 0 & 2a_2b_2 & a_3b_2 + a_2b_3e^{i\phi} \\ 0 & a_3b_2 + a_2b_3e^{i\phi} & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}, \tag{2.32}$$

$$M_{T_4}^{(12)} = \begin{pmatrix} 0 & d_5 & 0 \\ d_5 & 2a_2b_2 & a_3b_2 + a_2b_3e^{i\phi} \\ 0 & a_3b_2 + a_2b_3e^{i\phi} & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}, \tag{2.33}$$

$$M_{T_4}^{(13)} = \begin{pmatrix} 0 & 0 & d_5 \\ 0 & 2a_2b_2 & a_3b_2 + a_2b_3e^{i\phi} \\ d_5 & a_3b_2 + a_2b_3e^{i\phi} & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}, \tag{2.34}$$

where d_5 is a real parameter: $d_5 = \tilde{d}_5\tilde{M}/M_*$. By similar way, we will get the other neutrino mass matrices using the remaining Yukawa textures. Also, one can make sure that for those remaining cases there are undetermined phases [see Eqs: (2.14)-(2.23)] and their proper shift can match the phase of the term (2.25) with $\bar{m}$. Therefore, below, without loss of any generality we can take the parameter d_5 (in the neutrino mass matrices) to be real.

From texture T_5:

$$M_{T_5} = \begin{pmatrix} 0 & a_1b_2 & a_1b_3 \\ a_1b_2 & 0 & a_3b_2 \\ a_1b_3 & a_3b_2 & 2a_3b_3 \end{pmatrix} \bar{m}. \tag{2.35}$$

$$M_{T_5}^{(11)} = \begin{pmatrix} d_5 & a_1b_2 & a_1b_3 \\ a_1b_2 & 0 & a_3b_2 \\ a_1b_3 & a_3b_2 & 2a_3b_3 \end{pmatrix} \bar{m}, \quad M_{T_5}^{(22)} = \begin{pmatrix} 0 & a_1b_2 & a_1b_3 \\ a_1b_2 & d_5 & a_3b_2 \\ a_1b_3 & a_3b_2 & 2a_3b_3 \end{pmatrix} \bar{m}. \tag{2.36}$$

From texture T_6:

$$M_{T_6} = \begin{pmatrix} 0 & a_1b_2 & a_1b_3 \\ a_1b_2 & 2a_2b_2 & a_2b_3 \\ a_1b_3 & a_2b_3 & 0 \end{pmatrix} \bar{m}. \tag{2.37}$$

$$M_{T_6}^{(33)} = \begin{pmatrix} 0 & a_1b_2 & a_1b_3 \\ a_1b_2 & 2a_2b_2 & a_2b_3 \\ a_1b_3 & a_2b_3 & d_5 \end{pmatrix} \bar{m}, \quad M_{T_6}^{(11)} = \begin{pmatrix} d_5 & a_1b_2 & a_1b_3 \\ a_1b_2 & 2a_2b_2 & a_2b_3 \\ a_1b_3 & a_2b_3 & 0 \end{pmatrix} \bar{m}. \tag{2.38}$$

From texture T_7:

$$M_{T_7} = \begin{pmatrix} 2a_1b_1 & 0 & a_3b_1 + a_1b_3e^{i\phi} \\ 0 & 0 & 0 \\ a_3b_1 + a_1b_3e^{i\phi} & 0 & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}. \tag{2.39}$$

$$M_{T_7}^{(22)} = \begin{pmatrix} 2a_1b_1 & 0 & a_3b_1 + a_1b_3e^{i\phi} \\ 0 & d_5 & 0 \\ a_3b_1 + a_1b_3e^{i\phi} & 0 & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}, \tag{2.40}$$

$$M_{T_7}^{(12)} = \begin{pmatrix} 2a_1b_1 & d_5 & a_3b_1 + a_1b_3e^{i\phi} \\ d_5 & 0 & 0 \\ a_3b_1 + a_1b_3e^{i\phi} & 0 & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}, \tag{2.41}$$

$$M_{T_7}^{(23)} = \begin{pmatrix} 2a_1b_1 & 0 & a_3b_1 + a_1b_3e^{i\phi} \\ 0 & 0 & d_5 \\ a_3b_1 + a_1b_3e^{i\phi} & d_5 & 2a_3b_3e^{i\phi} \end{pmatrix} \bar{m}. \tag{2.42}$$

From texture T_8:

$$M_{T_8} = \begin{pmatrix} 2a_1b_1 & a_2b_1 & a_1b_3 \\ a_2b_1 & 0 & a_2b_3 \\ a_1b_3 & a_2b_3 & 0 \end{pmatrix} \bar{m}. \tag{2.43}$$

$$M_{T_8}^{(22)} = \begin{pmatrix} 2a_1b_1 & a_2b_1 & a_1b_3 \\ a_2b_1 & d_5 & a_2b_3 \\ a_1b_3 & a_2b_3 & 0 \end{pmatrix} \bar{m}, \quad M_{T_8}^{(33)} = \begin{pmatrix} 2a_1b_1 & a_2b_1 & a_1b_3 \\ a_2b_1 & 0 & a_2b_3 \\ a_1b_3 & a_2b_3 & d_5 \end{pmatrix} \bar{m}. \tag{2.44}$$

From texture T_9:

$$M_{T_9} = \begin{pmatrix} 2a_1b_1 & a_2b_1 + a_1b_2e^{i\phi} & 0 \\ a_2b_1 + a_1b_2e^{i\phi} & 2a_2b_2e^{i\phi} & 0 \\ 0 & 0 & 0 \end{pmatrix} \bar{m}. \tag{2.45}$$

$$M_{T_9}^{(13)} = \begin{pmatrix} 2a_1b_1 & a_2b_1 + a_1b_2e^{i\phi} & d_5 \\ a_2b_1 + a_1b_2e^{i\phi} & 2a_2b_2e^{i\phi} & 0 \\ d_5 & 0 & 0 \end{pmatrix} \bar{m}, \tag{2.46}$$

$$M_{T_9}^{(23)} = \begin{pmatrix} 2a_1b_1 & a_2b_1 + a_1b_2e^{i\phi} & 0 \\ a_2b_1 + a_1b_2e^{i\phi} & 2a_2b_2e^{i\phi} & d_5 \\ 0 & d_5 & 0 \end{pmatrix} \bar{m}, \tag{2.47}$$

$$M^{(33)}_{T_9} = \begin{pmatrix} 2a_1b_1 & a_2b_1 + a_1b_2e^{i\phi} & 0 \\ a_2b_1 + a_1b_2e^{i\phi} & 2a_2b_2e^{i\phi} & 0 \\ 0 & 0 & d_5 \end{pmatrix} \bar{m}. \tag{2.48}$$

We have shown that only T_4, T_7 and T_9 $2T_0Y_{32}$'s give rise to complex mass matrices and that complexity, i.e. phase δ in the lepton mixing matrix, arises through (2.24) — from complex $2T_0Y_{32}$'s — and not from an x_5 phase.

2.3 Analyzing neutrino mass matrices

Since we are working in a basis of a diagonal charged lepton mass matrix, lepton mixing matrix U entirely comes from the neutrino sector. Therefore, the following equality holds:

$$M_\nu = PU^*P' M_\nu^{\text{diag}} U^+ P \tag{2.49}$$

where

$$M_\nu^{\text{diag}} = (m_1, m_2, m_3), \quad P = \text{Diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3}), \quad P' = \text{Diag}(1, e^{i\rho_1}, e^{i\rho_2}) \tag{2.50}$$

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.51}$$

where m_i denote neutrino masses. U given in Eq. (2.51) is the standard parametrization used in the literature (see for instance [29, 30]). The relation (2.49) turns out to be convenient and useful for neutrino mass matrix analysis. Numerical values of oscillation parameters both, for normal (NH) and inverted (IH) hierarchies can be found in [31]. Thus, for these mass orderings we will use the following notations:

For normal hierarchy (NH):

$$\Delta m^2_{sol} = m_2^2 - m_1^2, \quad \Delta m^2_{atm} = m_3^2 - m_2^2, \quad m_1 = \sqrt{m_3^2 - \Delta m^2_{atm} - \Delta m^2_{sol}}, \quad m_2 = \sqrt{m_3^2 - \Delta m^2_{atm}} \tag{2.52}$$

For inverted hierarchy (IH)

$$\Delta m^2_{atm} = m_2^2 - m_3^2, \quad \Delta m^2_{sol} = m_2^2 - m_1^2, \quad m_1 = \sqrt{m_3^2 + \Delta m^2_{atm} - \Delta m^2_{sol}}, \quad m_2 = \sqrt{m_3^2 + \Delta m^2_{atm}} \tag{2.53}$$

2.3.1 Types of neutrino mass matrices

Complex 3×3 Majorana type neutrino mass matrices with more than two independent zero entries are all excluded by current experiments. As it turns out, experimental data also exclude the possibility of real neutrino mass matrices with two independent zero entries. This was noticed earlier upon studies of the texture zero neutrino mass matrices [21,32–34]. Therefore, experimentally viable neutrino mass matrices, from our 3×2 Yukawa textures (listed in Sect. 2.1) should be produced by $T_4, ..., T_9$ giving either neutrino mass matrices with two independent zero entries and the complex phase, or the one zero entry real neutrino mass matrices (via textures T_5, T_6, T_8 and one d=5 operator). Two zero entry complex neutrino mass matrices (we have obtained) have forms:

$$P_1 = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad P_3 = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad P_4 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}. \tag{2.54}$$

These types of textures correspond to the following mass matrices, we have obtained:

P_1-type: $M_{T_4}^{(12)}$, P_2-type: $M_{T_4}^{(13)}$, P_3-type: $M_{T_7}^{(23)}$, P_4-type: $M_{T_9}^{(23)}$

As far as the one zero entry neutrino mass matrices are concerned we are getting the following types of real mass matrices:

$$P_5 = \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad P_6 = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad P_7 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}. \tag{2.55}$$

Also here, we indicate the correspondence of $P_{5,6,7}$ textures to the appropriate neutrino mass matrices we have obtained: P_5-type: $M_{T_5}^{(22)}$, $M_{T_6}^{(33)}$, P_6-type: $M_{T_5}^{(11)}$, $M_{T_8}^{(33)}$ and P_7-type: $M_{T_6}^{(11)}$, $M_{T_8}^{(22)}$.

2.3.2 Predictions from $P_{1,2,3,4}$ type neutrino mass matrices

Here we analyze neutrino mass matrices with two independent zero entries. As we will see, for each case we will get several predictions.

TYPE P_1

Structure of the P_1 in Eq.(2.54) imposes the following conditions: $M_\nu^{(1,1)}=0$ and $M_\nu^{(1,3)}$=0, which taking into account (2.49)-(2.51) give the following relations:

$$\frac{m_1}{m_3}c_{12}^2+\frac{m_2}{m_3}s_{12}^2e^{i\rho_1}=-t_{13}^2e^{i(\rho_2+2\delta)} \tag{2.56}$$

and

$$-\left(\frac{m_1}{m_3}-\frac{m_2}{m_3}e^{i\rho_1}\right)t_{23}s_{12}c_{12}-s_{13}e^{i(\rho_2+\delta)}+s_{13}e^{-i\delta}\left(\frac{m_1}{m_3}c_{12}^2+\frac{m_2}{m_3}s_{12}^2e^{i\rho_1}\right)=0 \tag{2.57}$$

Using (2.56) in the last term of (2.57) we obtain:

$$\left(\frac{m_1}{m_3}-\frac{m_2}{m_3}e^{i\rho_1}\right)t_{23}s_{12}c_{12}+s_{13}e^{i(\rho_2+\delta)}+s_{13}t_{13}^2e^{i(\rho_2+\delta)}=0 \tag{2.58}$$

which gives:

$$m_3s_{13}(1+t_{13}^2)=|m_1-m_2e^{i\rho_1}|t_{23}s_{12}c_{12} \tag{2.59}$$

while from Eq. (2.56) we have:

$$m_3t_{13}^2=|m_1c_{12}^2+m_2s_{12}^2e^{i\rho_1}|. \tag{2.60}$$

We can exclude phase ρ_1 from (2.59) and (2.60) to obtain:

$$m_3^2(t_{13}^4+s_{13}^2\cot_{23}^2(1+t_{13}^2)^2)=m_1^2c_{12}^2+m_2^2s_{12}^2 \tag{2.61}$$

From which, based on recent experimental data [31] inverted hierarchical pattern (IH) is excluded. For normal hierarchical neutrinos from (2.61), with (2.52) we get

$$m_3^2=\frac{\Delta m_{atm}^2+\Delta m_{sol}^2c_{12}^2}{1-s_{13}^2\cot_{23}^2(1+t_{13}^2)^2-t_{13}^4}. \tag{2.62}$$

Using $\sin^2\theta_{23}=0.49$, the best fit values for the remaining mixing angles [31] and also the best fit values for the atmospheric and solar neutrino mass squared differences:

$$\Delta m_{atm}^2=0.002382\ \text{eV}^2,\quad \Delta\text{m}_{\text{sol}}^2=7.5\times10^{-5}\ \text{eV}^2 \tag{2.63}$$

from (2.62) we obtain for NH:

$$m_1=0.00613\ \text{eV},\quad \text{m}_2=0.0106\ \text{eV},\quad \text{m}_3=0.0499\ \text{eV}. \tag{2.64}$$

Using these, from (2.60) we predict:

$$\cos\rho_1 = \frac{m_3^2 t_{13}^4 - m_1^2 c_{12}^4 - m_2^2 s_{12}^4}{2m_1 m_2 c_{12}^2 s_{12}^2} \Rightarrow \rho_1 = \pm 3.036, \tag{2.65}$$

while from (2.56) and (2.58) we have:

$$\delta = \arg[m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\rho_1}] - \arg[m_1 - m_2 e^{i\rho_1}],$$

$$\rho_2 = \pm\pi - \arg[m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\rho_1}] + 2\arg[m_1 - m_2 e^{i\rho_1}]. \tag{2.66}$$

With numbers given in (2.64) and (2.65), from (2.66) we obtain:

$$\delta = \pm 0.378, \quad \rho_1 = \pm 3.036, \quad \rho_2 = \pm 2.696, \quad m_{\beta\beta} = 0, \tag{2.67}$$

where the neutrino-less double beta decay parameter $m_{\beta\beta}$ is determined as: $m_{\beta\beta} = |m_1 c_{12}^2 c_{13}^2 + m_2 e^{i\rho_1} c_{13}^2 s_{12}^2 + m_3 e^{i\rho_2} s_{13}^2 e^{2i\delta}|$. We summarize our results in Table 1.

δ	ρ_1	ρ_2	works with
$\delta = \pm 0.378$	$\rho_1 = \pm 3.036$	$\rho_2 = \pm 2.696$	NH, $\sin^2\theta_{23} = 0.49$ and best fit values for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.00613, 0.0106, 0.0499)$, $m_{\beta\beta} = 0$

Table 1: Results from P_1 type texture. Masses are given in eVs.

TYPE P_2

In this case $M_\nu^{(1,1)} = 0$ and $M_\nu^{(1,2)}=0$ and together with Eq.(2.56), the following relation holds:

$$-\left(\frac{m_1}{m_3} - \frac{m_2}{m_3} e^{i\rho_1}\right) s_{12} c_{12} + s_{13} t_{23} e^{i(\rho_2+\delta)} - s_{13} t_{23} e^{-i\delta} \left(\frac{m_1}{m_3} c_{12}^2 + \frac{m_2}{m_3} s_{12}^2 e^{i\rho_1}\right) = 0. \tag{2.68}$$

Using (2.56) in the last term of (2.68) we obtain:

$$-\left(\frac{m_1}{m_3} - \frac{m_2}{m_3} e^{i\rho_1}\right) s_{12} c_{12} + s_{13} t_{23} e^{i(\rho_2+\delta)} + s_{13} t_{23} t_{13}^2 e^{i(\rho_2+\delta)} = 0 \tag{2.69}$$

which gives:

$$m_3 s_{13} t_{23} (1 + t_{13}^2) = |m_1 - m_2 e^{i\rho_1}| s_{12} c_{12}. \tag{2.70}$$

Excluding phase ρ_1 from Eqs. (2.70) and (2.60)[which is derived from Eq.(2.56), i.e. the condition $M_\nu^{(1,1)} = 0$] we obtain:

$$m_3^2 (t_{13}^4 + s_{13}^2 t_{23}^2 (1 + t_{13}^2)^2) = m_1^2 c_{12}^2 + m_2^2 s_{12}^2 \tag{2.71}$$

Last relation makes obvious that the IH case is excluded. On the other hand, for NH neutrinos, from (2.71), with (2.52) we get:

$$m_3^2 = \frac{\Delta m_{atm}^2 + \Delta m_{sol}^2 c_{12}^2}{1 - s_{13}^2 t_{23}^2 (1 + t_{13}^2)^2 - t_{13}^4}. \tag{2.72}$$

After finding the value of m_3 and remaining masses,

$$(m_1, m_2, m_3) = (0.00501, 0.01, 0.04982) \text{ eV}. \tag{2.73}$$

Eqs. (2.68) and (2.69) allow to calculate the phases:

$$\cos\rho_1 = \frac{m_3^2 t_{13}^4 - m_1^2 c_{12}^4 - m_2^2 s_{12}^4}{2 m_1 m_2 c_{12}^2 s_{12}^2} \Rightarrow \rho_1 = \mp 2.828, \tag{2.74}$$

$$\delta = \pm\pi + \arg[m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\rho_1}] - \arg[m_1 - m_2 e^{i\rho_1}],$$

$$\rho_2 = \mp\pi - \arg[m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\rho_1}] + 2\arg[m_1 - m_2 e^{i\rho_1}]. \tag{2.75}$$

Using the best fit values of measured parameters [31] for NH we obtain results

$$\delta = \pm 1.924, \quad \rho_1 = \mp 2.828, \quad \rho_2 = \mp 1.715, \quad m_{\beta\beta} = 0, \tag{2.76}$$

which are summarized in Table 2:

δ	ρ_1	ρ_2	works with
$\delta = \pm 1.924$	$\rho_1 = \mp 2.828$	$\rho_2 = \mp 1.715$	NH and best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.00501, 0.01, 0.04982)$, $m_{\beta\beta} = 0$

Table 2: Results from P_2 type texture. Masses are given in eVs.

P_1 and P_2 neutrino textures were studied in [33–39]. Our analytical expressions, allowing thorough investigations, are compact and exact. To analyze the textures P_3 and P_4 it is convenient to note, that equation $M_\nu^{(i,j)} = 0$ can be written as: $A_2 \times m_2 e^{i\rho_1} + A_3 \times m_3 e^{i\rho_2} = A_1 \times m_1$. When two mass matrix elements are equal to zero we have a pair of similar equations which we write in a matrix form:

$$\begin{pmatrix} A_2 & A_3 \\ B_2 & B_3 \end{pmatrix} \begin{pmatrix} m_2 e^{i\rho_1} \\ m_3 e^{i\rho_2} \end{pmatrix} = \begin{pmatrix} A_1 m_1 \\ B_1 m_1 \end{pmatrix}. \tag{2.77}$$

From these equations we have:

$$m_2 e^{i\rho_1} = \frac{1}{A_2B_3 - A_3B_2}(B_3A_1 - A_3B_1)m_1, \quad m_3 e^{i\rho_2} = \frac{1}{A_2B_3 - A_3B_2}(A_2B_1 - B_2A_1)m_1 \tag{2.78}$$

or,

$$m_2^2 = \frac{|B_3A_1 - A_3B_1|^2}{|A_2B_3 - A_3B_2|^2}m_1^2, \quad m_3^2 = \frac{|A_2B_1 - B_2A_1|^2}{|A_2B_3 - A_3B_2|^2}m_1^2 \tag{2.79}$$

and

$$\frac{\Delta m^2_{sol}}{\pm\Delta m^2_{atm}} = \frac{|B_3A_1 - A_3B_1|^2 - |A_2B_3 - A_3B_2|^2}{|A_2B_1 - B_2A_1|^2 - |B_3A_1 - A_3B_1|^2}, \tag{2.80}$$

where "+" and "-" signs correspond to normal and inverted hierarchies respectively. Eq. (2.80) is the relation for calculating the value of δ. At the same time (after knowing the δ), from Eq. (2.79) and (2.52)/(2.53) the neutrino masses can be calculated. After these, with relations in Eq. (2.78) the phases ρ_1 and ρ_2 can be found. Below, we use this procedure for the textures P_3 and P_4.

TYPE P_3

For this case we have:

$$A_1 = -U^*_{11}U^\dagger_{12}, \quad A_2 = U^*_{12}U^\dagger_{22}, \quad A_3 = U^*_{13}U^\dagger_{32}, \quad B_1 = -U^*_{21}U^\dagger_{12}, \quad B_2 = U^*_{22}U^\dagger_{22}, \quad B_3 = U^*_{23}U^\dagger_{32}$$

and using these in Eqs. (2.78)-(2.80), for NH and IH neutrino mass ordering, we get results which are summarized in Table 3.

δ	ρ_1	ρ_2	works with
$\delta = \pm 1.547$	$\rho_1 = \pm 0.0615$	$\rho_2 = \mp 3.098$	NH and best fit values of oscillation parameters, $(m_1, m_2, m_3) =$ $(0.07213, 0.07265, 0.08752)$, $m_{\beta\beta} = 0.0726$
$\delta = \pm 1.579$	$\rho_1 = \mp 0.0998$	$\rho_2 = \pm 3.0726$	IH and best fit values of oscillation parameters, $(m_1, m_2, m_3) =$ $(0.07195, 0.07247, 0.05294)$, $m_{\beta\beta} = 0.0716$

Table 3: Results from P_3 type texture. Masses are given in eVs.

TYPE P_4

For this case we have:

$$A_1 = -U^*_{11}U^\dagger_{13}, \quad A_2 = U^*_{12}U^\dagger_{23}, \quad A_3 = U^*_{13}U^\dagger_{33}, \quad B_1 = -U^*_{31}U^\dagger_{13}, \quad B_2 = U^*_{32}U^\dagger_{23}, \quad B_3 = U^*_{33}U^\dagger_{33}.$$

For this case NH works with $\sin^2\theta_{23}$ larger by 1σ from the best fit value. However, IH case requires a lower value of $\sin^2\theta_{23}$. Using above relations in Eqs. (2.78)-(2.80), for NH and IH cases we get results which are summarized in Table 4.

δ	ρ_1	ρ_2	works with
$\delta = \pm 1.575$	$\rho_1 = \mp 0.0127$	$\rho_2 = \pm 3.133$	NH and $\sin^2\theta_{23} = 0.51$ and best fit values for remaining oscillation parameters, $(m_1, m_2, m_3) =$ $(0.171701, 0.171919, 0.1787)$, $m_{\beta\beta} = 0.1719$
$\delta = \pm 1.5705$	$\rho_1 = \pm 0.00622$	$\rho_2 = \mp 3.137$	IH and $\sin^2\theta_{23} = 0.495$ and best fit values for remaining oscillation parameters, $(m_1, m_2, m_3) =$ $(0.2513, 0.25145, 0.2465)$, $m_{\beta\beta} = 0.2512$

Table 4: Results from P_4 type texture. Masses are given in eVs.

Our results for the textures P_3 and P_4 are compatible with ones [33–40], obtained before.[4]

2.3.3 Predictions from real one zero entry neutrino textures - $P_{5,6,7}$

Now we turn to the analysis of the one texture zero neutrino mass matrices we have obtained in Section 2.2. They fall in the category of the $P_{5,6,7}$ type mass matrices given in Eq. (2.55). One texture zero neutrino mass matrices were investigated in [41–45]. In our construction, these mass matrices are real. This makes them more predictive.

TYPE P_5

[4]Some of these works used the earlier experimental data. We have made sure, that with those inputs, we would get similar results.

In this case, our construction implies ϕ=0 and all elements of the lepton mixing matrix are real (i.e. δ=0 or π). Therefore, together with $M_\nu^{(1,1)}$=0 we have to match phases of both sides of Eq.(2.49). This turns out to be impossible for ρ_1, ρ_2 not equal to either 0 or π , because we have only three free phases $\omega_{1,2,3}$. Thus, it turns out that only normal hierarchical scenario will be allowed with $\delta = 0$ or π. With these, and from the condition $M_\nu^{(1,1)}$=0, we get

$$\tan\theta_{13} = \left(-c_1c_2s_{12}^2\frac{m_2}{m_3} - c_2c_{12}^2\frac{m_1}{m_3}\right)^{\frac{1}{2}}, \tag{2.81}$$

where c_1 and c_2 stand for $\cos\rho_1$ and $\cos\rho_2$ respectively. This relation can be satisfied by special selection of the neutrino masses and $\rho_{1,2} = 0$ or π. Since two mass square differences are fixed from the neutrino data, only one free mass is available, which we choose to be m_3. The latter is tightly constrained via Eq.(2.81). Thus, the model predicts three neutrino masses and the phases. For the best fit values of the oscillation parameters [31] for NH we obtain solutions:

$$m_1 = 0.002268 \text{ eV}, \quad \mathrm{m}_2 = 0.008952 \text{ eV}, \quad \mathrm{m}_3 = 0.04962 \text{ eV},$$
$$\text{with} \quad m_{\beta\beta} = 0, \quad \delta = 0 \quad \text{or} \quad \pi, \quad \rho_1 = \pi, \quad \rho_2 = 0 \tag{2.82}$$

and

$$m_1 = 0.010677 \text{ eV}, \quad \mathrm{m}_2 = 0.006245 \text{ eV}, \quad \mathrm{m}_3 = 0.04996 \text{ eV},$$
$$\text{with} \quad m_{\beta\beta} = 0, \quad \delta = 0 \quad \text{or} \quad \pi, \quad \rho_1 = \pi, \quad \rho_2 = \pi. \tag{2.83}$$

By the similar analysis, we can easily make sure that inverted hierarchy is not allowed within our construction for this P_5 type texture.

TYPE P_6

For this case, the condition $M_\nu^{(2,2)} = 0$ gives the following expression for θ_{12}:

$$\tan\theta_{12} = \frac{c_{23}s_{23}\hat{s}_{13}(m_2c_1 - m_1)}{m_1c_{23}^2 + m_2s_{23}^2s_{13}^2c_1 + m_3s_{23}^2c_{13}^2c_2}$$
$$\pm \frac{\sqrt{c_{23}^2s_{23}^2s_{13}^2(m_2c_1 - m_1)^2 - (m_1c_{23}^2 + m_2s_{23}^2s_{13}^2c_1 + m_3s_{23}^2c_{13}^2c_2)(m_1s_{23}^2s_{13}^2 + m_2c_{23}^2c_1 + m_3s_{23}^2c_{13}^2c_2)}}{m_1c_{23}^2 + m_2s_{23}^2s_{13}^2c_1 + m_3s_{23}^2c_{13}^2c_2} \tag{2.84}$$

where, c_1 and c_2 stand for $\cos\rho_1$ and $\cos\rho_2$ respectively. $\hat{s}_{13} = \pm s_{13}$ and a "+" corresponds to $\delta = 0$ and a "-" sign to $\delta = \pi$. So, this equation will include all cases. Some cases work with the best fit values (BFV) of the oscillation parameters [31], while some cases work only with deviations from the BFV. We will allow some of these parameters to vary within a 3σ range. Results are summarized in Table 5.

δ	p	ρ_1	ρ_2	works with
0	-	0	π	IH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.07613, 0.07662, 0.0585)$, $m_{\beta\beta} = 0.0733$
π	-	0	π	IH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.07635, 0.07684, 0.05878)$, $m_{\beta\beta} = 0.07354$
0	-	0	π	NH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.06353, 0.06412, 0.08058)$, $m_{\beta\beta} = 0.06056$
π	-	0	π	NH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.06315, 0.06374, 0.08028)$, $m_{\beta\beta} = 0.0602$
π	$+$	π	0	IH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.05735, 0.058, 0.03024)$, $m_{\beta\beta} = 0.02246$
0	$+$	π	0	IH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.04879, 0.04955, 0.002516)$, $m_{\beta\beta} = 0.0185$
π	$+$	π	0	NH, $\sin^2\theta_{13} = 0.0218$, $\sin^2\theta_{23} \in [0.382, 0.4]$, $m_3 \in [0.12, 0.3]$, $\sin^2\theta_{12} = [0.27, 0.297]$, $m_{\beta\beta} \in [0.052, 0.14]$, $\sum m_i \in [0.34, 0.9]$
0	$+$	π	π	IH, $\sin^2\theta_{13} = 0.0218$, $\sin^2\theta_{23} \in [0.552, 0.644]$, $m_3 \in [0, 0.002]$, $\sin^2\theta_{12} = [0.313, 0.344]$, $m_{\beta\beta} \in [0.0146, 0.0176]$

Table 5: Results from P_6 type texture. "p" stands for a sign of a square root in (2.84). Masses are given in eVs.

TYPE P_7

For this case, the condition $M_\nu^{(3,3)} = 0$ gives:

$$\tan\theta_{12} = \frac{c_{23}s_{23}\hat{s}_{13}(m_1 - m_2c_1)}{m_1s_{23}^2 + m_2c_{23}^2s_{13}^2c_1 + m_3c_{23}^2c_{13}^2c_2}$$

$$\pm\frac{\sqrt{c_{23}^2 s_{23}^2 s_{13}^2 (m_1 - m_2 c_1)^2 - (m_1 s_{23}^2 + m_2 c_{23}^2 s_{13}^2 c_1 + m_3 c_{23}^2 c_{13}^2 c_2)(m_1 c_{23}^2 s_{13}^2 + m_2 s_{23}^2 c_1 + m_3 c_{23}^2 c_{13}^2 c_2)}}{m_1 s_{23}^2 + m_2 c_{23}^2 s_{13}^2 c_1 + m_3 c_{23}^2 c_{13}^2 c_2} \tag{2.85}$$

Notations here are similar to those for case P_6 [see comment after Eq. (2.84)]. Results are summarized in Table 6. As above, we have used data from Ref. [31].

δ	p	ρ_1	ρ_2	works with
0	+	π	0	IH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.9997, 0.10034, 0.08729)$, $m_{\beta\beta} = 0.04$
0	-	0	π	IH, $\sin^2\theta_{23} \in [0.389, 0.487]$, and bfv for remaining osc. parameters, $m_3 \in [0.04496, 0.4138]$, $m_{\beta\beta} \in [0.064, 0.398]$, $\sum m_i \in [0.178, 1.25]$
π	+	π	0	IH, by best fit values of oscillation parameters, $(m_1, m_2, m_3) = (0.05004, 0.05078, 0.01142)$, $m_{\beta\beta} = 0.019$
π	+	π	π	IH, $\sin^2\theta_{23} \in [0.389, 0.448]$, $\sin^2\theta_{12} = [0.325, 0.344]$ and bfv for remaining osc. parameters, $m_3 \in [0, 0.001379]$, $m_{\beta\beta} \in [0.0146, 0.0165]$
π	-	0	π	IH, $\sin^2\theta_{23} \in [0.389, 0.488]$, and bfv for remaining osc. parameters, $m_3 \in [0.04473, 0.6183]$, $m_{\beta\beta} \in [0.064, 0.59]$, $\sum m_i \in [0.178, 1.86]$
0	+	π	0	NH, $\sin^2\theta_{23} \in [0.621, 0.643]$, and bfv for remaining osc. parameters, $m_3 \in [0.1246, 0.5928]$, $m_{\beta\beta} \in [0.046, 0.24]$, $\sum m_i \in [0.354, 1.77]$
0	-	0	π	NH, $\sin^2\theta_{23} \in [0.49, 0.643]$, and bfv for remaining oscillation parameters, $m_3 \in [0.05803, 0.5187]$, $m_{\beta\beta} \in [0.0286, 0.4938]$, $\sum m_i \in [0.1196, 1.551]$
π	-	0	π	NH, $\sin^2\theta_{23} \in [0.49, 0.643]$, and bfv for remaining oscillation parameters, $m_3 \in [0.05821, 0.5209]$, $m_{\beta\beta} \in [0.02895, 0.4959]$, $\sum m_i \in [0.1205, 1.558]$

Table 6: Results from P_7 type texture. "p" stands for a sign of a square root in (2.85). Masses are given in eVs.

2.4 Relating cosmological CP and δ

As we have already seen, from certain $2T_0Y_{32}$'s complex phases cannot be factored out. Such couplings are: T_4, T_7, T_9 and they give rise to complex mass matrices. Here we calculate phase ϕ in terms of the CP phase entering in neutrino oscillation. Recall that the δ is predicted from the neutrino mass matrices (2.33),(2.34),(2.42),(2.47), which we have considered. Keeping in mind (2.54), we use (2.49) and (2.50) to find the numerical value of phase ϕ in each case.

Case of $M_{T_4}^{(12)}$ (Texture P_1):
Equating (2,2), (3,3) and (2,3) matrix elements of both sides in Eq. (2.49), we get the relations:

$$2a_2b_2|\bar{m}|e^{i\phi_{\bar{m}}}=e^{2i\omega_2}\mathcal{A}_{22},\quad 2a_3b_3e^{i\phi}|\bar{m}|e^{i\phi_{\bar{m}}}=e^{2i\omega_3}\mathcal{A}_{33},\quad (a_3b_2+a_2b_3e^{i\phi})|\bar{m}|e^{i\phi_{\bar{m}}}=e^{i(\omega_2+\omega_3)}\mathcal{A}_{23}, \tag{2.86}$$

with

$$\mathcal{A}_{ij}=U_{i1}^*U_{j1}^*m_1+U_{i2}^*U_{j2}^*m_2e^{i\rho_1}+U_{i3}^*U_{j3}^*m_3e^{i\rho_2}. \tag{2.87}$$

Note, that from the neutrino sector all $\mathcal{A}_{ij}$ numbers are determined. Dividing the last relation in (2.86) in turn on the 1-st and 2-nd relations and then multiplying resulting two equations, we get the following relation:

$$xe^{i\phi}=\left(\frac{\mathcal{A}_{23}}{\sqrt{\mathcal{A}_{22}\mathcal{A}_{33}}}\pm\sqrt{\frac{\mathcal{A}_{23}^2}{\mathcal{A}_{22}\mathcal{A}_{33}}-1}\right)^2,\quad x\equiv\frac{a_2b_3}{a_3b_2}. \tag{2.88}$$

Therefore, we have:

$$\phi=\mathrm{Arg}\left[\left(\frac{\mathcal{A}_{23}}{\sqrt{\mathcal{A}_{22}\mathcal{A}_{33}}}\pm\sqrt{\frac{\mathcal{A}_{23}^2}{\mathcal{A}_{22}\mathcal{A}_{33}}-1}\right)^2\right]. \tag{2.89}$$

From here, using results given in Table 1, we find numerical value of ϕ:

$$\phi=\pm1.287. \tag{2.90}$$

In a pretty similar way, for remaining three neutrino mass matrices (2.34),(2.42),(2.47), for the phase ϕ we get:

$$\phi=\mathrm{Arg}\left[\left(\frac{\mathcal{A}_{23}}{\sqrt{\mathcal{A}_{22}\mathcal{A}_{33}}}\pm\sqrt{\frac{\mathcal{A}_{23}^2}{\mathcal{A}_{22}\mathcal{A}_{33}}-1}\right)^2\right],\quad \phi=\mathrm{Arg}\left[\left(\frac{\mathcal{A}_{13}}{\sqrt{\mathcal{A}_{11}\mathcal{A}_{33}}}\pm\sqrt{\frac{\mathcal{A}_{13}^2}{\mathcal{A}_{11}\mathcal{A}_{33}}-1}\right)^2\right], \tag{2.91}$$

$$\phi=\mathrm{Arg}\left[\left(\frac{\mathcal{A}_{12}}{\sqrt{\mathcal{A}_{11}\mathcal{A}_{22}}}\pm\sqrt{\frac{\mathcal{A}_{12}^2}{\mathcal{A}_{11}\mathcal{A}_{22}}-1}\right)^2\right], \tag{2.92}$$

which yield

$$\phi = \pm 1.169, \quad \phi^{\rm NH} = \pm 2.957 \quad \text{and} \quad \phi^{\rm IH} = \pm 3.124,$$

$$\phi^{\rm NH} = \pm 3.058 \quad \text{and} \quad \phi^{\rm IH} = \pm 3.136 \tag{2.93}$$

respectively. For these we have used results given in Tables: 2, 3 and 4 resp. Note, that ϕ phases in all four cases have been found for the reason that with a predictive neutrino sector there is no undetermined parameter. This makes the whole scenario really attractive to study the baryon asymmetry via the leptogenesis (for similar studies see: [1,19,21,26,27,32,46]). As mentioned, since the ϕ participates in the coupling of RHN states with l and h_u (2.1) it will control CP asymmetric decays of the N states. Thus, it is interesting to look into the details of the leptogenesis within the scenarios we have considered here.

3 Texture Zero Neutrino Models and Their Connection with Resonant Leptogenesis

3.1 Loop Induced Calculable Cosmological CP Violation

The setup considered in this section is the same as the one presented in the previous section and is given by formulas: (2.1), (2.2), (2.3) and (2.4). Moreover, we assume that the RHN mass matrix M_N is strictly degenerate at the GUT scale, which will be taken to be $M_G \simeq 2 \cdot 10^{16}$ GeV. [5] To stress scale dependence of $M(\mu)$ we rewrite (2.4) as:

$$\text{at} \;\; \mu = M_G: \quad M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(M_G). \tag{3.1}$$

Although it is interesting and worth to study, we do not attempt here to justify the form of M_N (and of the textures considered below) by symmetries. Our approach here is rather phenomenological aiming to investigate possibilities, outcomes and implications of the textures we consider. Since (3.1) at a tree level leads to the mass degeneracy of the RHN's, it has interesting implications for resonant leptogenesis [1, 19, 24] and also, as we will see below, for building predictive neutrino scenarios [1, 20].

[5] Degeneracy of M_N can be guaranteed by some symmetry at high energies. For concreteness, we assume this energy interval to be $\geq M_G$ (although the degeneracy at lower energies can be considered as well).

For the leptogenesis scenario two necessary conditions need to be satisfied. First of all, at the scale $\mu = M_{N_{1,2}}$ the degeneracy between the masses of N_1 and N_2 has to be lifted. And, at the same scale, the neutrino Yukawa matrix $\hat{Y}_\nu$ - written in the mass eigenstate basis of M_N, must be such that $\mathrm{Im}[(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}]^2 \neq 0$. [These can be seen from Eq. (3.27) with a demand $\epsilon_{1,2} \neq 0$.] Below we show that both of them are realized by radiative corrections and needed effect already arises at 1-loop level, with a dominant contribution due to the Y_e Yukawa couplings (in particular from λ_τ and in some cases from λ_μ) in the RG.

As it was shown [1, 16, 28], within considered setup, radiative corrections are crucial for generating cosmological CP violation. In particular, the needed asymmetry is generated at 1-loop level due to λ_τ Yukawa coupling provided that the condition $(Y_\nu)_{31}(Y_\nu)_{32} \neq 0$ is satisfied [16]. Here, to be more generic and to not limit the class of the models, we also include the effects of the λ_μ Yukawa coupling in the calculation.[6] Thus, in this section we present details of these calculations. We will start with radiative corrections to the M_N matrix. RG effects cause lifting of the mass degeneracy and, as we will see, are important also for the phase misalignment (explained below).

At the GUT scale, the M_N has off-diagonal form with $(M_N)_{11} = (M_N)_{22} = 0$ [see Eq. (3.1)]. However, at low energies, RG corrections generate these entries. Thus, we parameterize the matrix M_N at scale μ as:

$$M_N(\mu) = \begin{pmatrix} \delta_N^{(1)}(\mu) & 1 \\ 1 & \delta_N^{(2)}(\mu) \end{pmatrix} M(\mu). \tag{3.2}$$

While all entries of the matrix M_N run, for our studies will be relevant the ratios $\frac{(M_N)_{11}}{(M_N)_{12}} = \delta_N^{(1)}$ and $\frac{(M_N)_{22}}{(M_N)_{12}} = \delta_N^{(2)}$ (obeying the RG equations investigated below). That's why M_N was parametrized in a form given in Eq. (3.2). With $|\delta_N^{(1,2)}| \ll 1$, the M (at scale $\mu = M$) will determine the masses of RHNs M_1 and M_2, while $\delta_N^{(1,2)}$ will be responsible for their splitting and for complexity in M_N (the phase of the overall factor M does not contribute to the physical CP). As will be shown below:

$$\delta_N^{(1)} = (\delta_N^{(2)})^* \equiv -\delta_N. \tag{3.3}$$

Therefore, M_N is diagonalized by the transformation

$$U_N^T M_N U_N = M_N^{Diag} = \mathrm{Diag}\,(M_1, M_2)\ , \qquad \text{with}\ \ U_N = P_N O_N P_N{}'\ ,$$

[6] In Sections 3.3 and 3.4, among other neutrino scenarios, we consider ones for which such corrections are crucial for generation of the needed amount of Baryon asymmetry.

$$M_1 = |M|\,(1-|\delta_N|)\ , \qquad M_2 = |M|\,(1+|\delta_N|)\ , \tag{3.4}$$

where

$$P_N = \mathrm{Diag}\left(e^{-i\eta/2}, e^{i\eta/2}\right), \quad O_N = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad P_N{}' = \mathrm{Diag}\left(e^{-i\phi_M/2}, ie^{-i\phi_M/2}\right),$$

$$\text{with} \quad \eta = \mathrm{Arg}\,(\delta_N)\ , \qquad \phi_M = \mathrm{Arg}\,(M)\,. \tag{3.5}$$

In the N's mass eigenstate basis, the Dirac type neutrino Yukawa matrix will be $\hat{Y}_\nu = Y_\nu U_N$. In the CP asymmetries, the components $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}$ and $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$ appear [see Eq. (3.27)]. From (3.4) and (3.5) we have

$$\left[(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}\right]^2 = -\left[(O_N^T P_N^* Y_\nu^\dagger Y_\nu P_N O_N)_{21}\right]^2, \quad \left[(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}\right]^2 = -\left[(O_N^T P_N^* Y_\nu^\dagger Y_\nu P_N O_N)_{12}\right]^2. \tag{3.6}$$

Therefore, the CP violation should come from $P_N^* Y_\nu^\dagger Y_\nu P_N$, which in a matrix form is:

$$P_N^* Y_\nu^\dagger Y_\nu P_N = \begin{pmatrix} (Y_\nu^\dagger Y_\nu)_{11} & \left|(Y_\nu^\dagger Y_\nu)_{12}\right| e^{i(\eta-\eta')} \\ |(Y_\nu^\dagger Y_\nu)_{21}| e^{i(\eta'-\eta)} & (Y_\nu^\dagger Y_\nu)_{22} \end{pmatrix}, \quad \text{with} \quad \eta' = \mathrm{Arg}[(Y_\nu^\dagger Y_\nu)_{21}]\ . \tag{3.7}$$

We see that $\eta' - \eta$ difference (mismatch) will govern the CP asymmetric decays of the RHNs. Without including the charged lepton Yukawa couplings in the RG effects we will have $\eta' \simeq \eta$ with a high accuracy. It was shown in Ref. [14] that by ignoring Y_e Yukawas no CP asymmetry emerges at $\mathcal{O}(Y_\nu^4)$ order and non-zero contributions start only from $\mathcal{O}(Y_\nu^6)$ terms [15]. Such corrections are extremely suppressed for $Y_\nu \lesssim 1/50$. Since in our consideration we are interested in cases with $M_{1,2} \lesssim 10^7$ GeV leading to $|(Y_\nu)_{ij}| < 7 \cdot 10^{-4}$ (well fixed from the neutrino sector and the desired value of the baryon asymmetry), these effects (i.e. order $\sim Y_\nu^6$ corrections) will not have any relevance. In Ref. [1] in the RG of M_N the effect of Y_e, coming from 2-loop corrections, was taken into account and it was shown that sufficient CP violation can emerge. Below we show that including Y_e in the Y_ν's 1-loop RG, will induce sufficient amount of CP violation. This mainly happens via λ_τ and in particular cases (which are considered below) from λ_μ Yukawa couplings. Thus, below we give detailed investigation of $\lambda_{\tau,\mu}$'s effects.

Using M_N's RG given in Eq. (A.3) (of Appendix A.1), for $\delta_N^{(1,2)}$, which are the ratios $\frac{(M_N)_{11}}{(M_N)_{12}}$ and $\frac{(M_N)_{22}}{(M_N)_{12}}$, [see parametrization in Eq. (3.2)], we can derive the following RG equations:

$$16\pi^2 \frac{d}{dt}\delta_N^{(1)} = 4(Y_\nu^\dagger Y_\nu)_{21} + 2\delta_N^{(1)}\left[(Y_\nu^\dagger Y_\nu)_{11} - (Y_\nu^\dagger Y_\nu)_{22}\right] - 2(\delta_N^{(1)})^2 (Y_\nu^\dagger Y_\nu)_{12} - 2\delta_N^{(1)}\delta_N^{(2)} (Y_\nu^\dagger Y_\nu)_{21}$$

$$
- \frac{1}{4\pi^2}(Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu)_{21} + \cdots \tag{3.8}
$$

$$
16\pi^2 \frac{d}{dt}\delta_N^{(2)} = 4(Y_\nu^\dagger Y_\nu)_{12} + 2\delta_N^{(2)}\left[(Y_\nu^\dagger Y_\nu)_{22} - (Y_\nu^\dagger Y_\nu)_{11}\right] - 2(\delta_N^{(2)})^2 (Y_\nu^\dagger Y_\nu)_{21} - 2\delta_N^{(1)}\delta_N^{(2)}(Y_\nu^\dagger Y_\nu)_{12}
$$

$$
- \frac{1}{4\pi^2}(Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu)_{12} + \cdots \tag{3.9}
$$

were in second lines of (3.8) and (3.9) are given 2-loop corrections depending on Y_e. Dots there stand for higher order irrelevant terms. From 2-loop corrections we keep only Y_e dependent terms. Remaining contributions are not relevant for us.[7] From (3.8) and (3.9) we see that dominant contributions come from the first terms of the r.h.s. and from those given in the second rows. Other terms give contributions of order $\mathcal{O}(Y_\nu^4)$ or higher and thus will be ignored. At this approximation we have

$$
\delta_N^{(1)}(t) \simeq \delta_N^{(2)*}(t) \equiv -\delta_N(t) \simeq -\frac{1}{4\pi^2}\int_t^{t_G} dt \, \left(Y_\nu^\dagger (\mathbf{1} - \frac{1}{16\pi^2} Y_e Y_e^\dagger) Y_\nu \right)_{21} \tag{3.10}
$$

where $t = \ln\mu$, $t_G = \ln M_G$ and we have used the boundary conditions at the GUT scale $\delta_N^{(1)}(t_G) = \delta_N^{(2)}(t_G) = 0$. For evaluation of the integral in (3.10) we need to know the scale dependence of Y_ν and Y_e. This is found in Appendix A.1 by solving the RG equations for Y_ν and Y_e. Using Eqs. (A.5) and (A.6), the integral of the matrix appearing in (3.10) can be written as:

$$
\int_{t_M}^{t_G} Y_\nu^\dagger (\mathbf{1} - \frac{1}{16\pi^2} Y_e Y_e^\dagger) Y_\nu dt \simeq \bar{\kappa}(M) Y_{\nu G}^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{r}_\mu(M) & 0 \\ 0 & 0 & \bar{r}_\tau(M) \end{pmatrix} Y_{\nu G} \tag{3.11}
$$

where

$$
\bar{r}_\tau(M) = \frac{\int_{t_M}^{t_G} \kappa(t) r_\tau(t)(1 - \frac{\lambda_\tau^2}{16\pi^2})dt}{\int_{t_M}^{t_G}\kappa(t)dt} \; , \;\; \bar{r}_\mu(M) = \frac{\int_{t_M}^{t_G} \kappa(t) r_\mu(t)(1 - \frac{\lambda_\mu^2}{16\pi^2})dt}{\int_{t_M}^{t_G}\kappa(t)dt} \; , \;\; \bar{\kappa}(M) = \int_{t_M}^{t_G} \kappa(t) dt \; , \tag{3.12}
$$

$$
r_\tau(\mu) = \eta_\tau^2(\mu) \; , \qquad r_\mu(\mu) = \eta_\mu^2(\mu) \; , \qquad \kappa(\mu) = \eta_t^6(\mu)\eta_{g\nu}^2(\mu) \tag{3.13}
$$

and we have ignored λ_e Yukawa couplings. For the definition of η-factors see Eq. (A.6). The $Y_{\nu G}$ denotes corresponding Yukawa matrix at scale $\mu = M_G$. On the other hand, we have:

$$
(Y_\nu^\dagger Y_\nu)\big|_{\mu=M} \simeq \kappa(M) Y_{\nu G}^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_\mu(M) & 0 \\ 0 & 0 & r_\tau(M) \end{pmatrix} Y_{\nu G} \; . \tag{3.14}
$$

[7] Omitted terms are either strongly suppressed or do not give any significant contribution to either the CP violation or the RHN mass splittings.

(Derivations are given in Appendix A.1.)

Comparing (3.11) with (3.14) we see that difference in these matrix structures (besides overall flavor universal RG factors) is in the RG factors $r_{\tau,\mu}(M)$ and $\bar{r}_{\tau,\mu}(M)$. Without the $\lambda_{\tau,\mu}$ Yukawa couplings these factors are equal and there is no mismatch between the phases η and η' [defined in Eqs. (3.5) and (3.7)] of these matrices. Non zero $\eta' - \eta$ will be due to the deviations, which we parameterize as

$$\xi_\tau = \frac{\bar{r}_\tau(M)}{r_\tau(M)} - 1, \qquad \xi_\mu = \frac{\bar{r}_\mu(M)}{r_\mu(M)} - 1 \ . \tag{3.15}$$

The values of ξ_μ and ξ_τ can be computed numerically by evaluation of the appropriate RG factors. Approximate expressions can be derived for $\xi_{\tau,\mu}$, which are given by:

$$\xi_\tau \simeq \left[\frac{\lambda_\tau^2(M)}{16\pi^2}\ln\frac{M_G}{M} + \frac{1}{3}\frac{\lambda_\tau^2(M)}{(16\pi^2)^2}\left[3\lambda_t^2 + 6\lambda_b^2 + 10\lambda_\tau^2 - (2c_e^a + c_\nu^a)g_a^2\right]_{\mu=M}\left(\ln\frac{M_G}{M}\right)^2\right]_{1-\text{loop}} - \left[\frac{\lambda_\tau^2(M)}{16\pi^2}\right]_{2-\text{loop}} , \tag{3.16}$$

$$\xi_\mu \simeq \left[\frac{\lambda_\mu^2(M)}{16\pi^2}\ln\frac{M_G}{M} + \frac{1}{3}\frac{\lambda_\mu^2(M)}{(16\pi^2)^2}\left[3\lambda_t^2 + 6\lambda_b^2 + 2\lambda_\tau^2 - (2c_e^a + c_\nu^a)g_a^2\right]_{\mu=M}\left(\ln\frac{M_G}{M}\right)^2\right]_{1-\text{loop}} - \left[\frac{\lambda_\mu^2(M)}{16\pi^2}\right]_{2-\text{loop}} , \tag{3.17}$$

where one and two loop contributions are indicated. Derivation of approximate expression of ξ_τ [Eq.(3.16)] is given in Appendix A.1 of Ref. [16]. Eq. (3.17) can be derived in a similar way. As we see, non-zero $\xi_{\tau,\mu}$ are induced already at 1-loop level [without 2-loop correction of $\frac{\lambda_{\tau,\mu}^2}{16\pi^2}$ in Eq. (3.12)]. However, inclusion of 2-loop correction can contribute to the $\xi_{\tau,\mu}$ by amount of $\sim 3-5\%$ (because of $\ln\frac{M_G}{M}$ factor suppression) and we have included it.

Now we write down quantities which have direct relevance for leptogenesis calculations. Using Eq. (3.11) in (3.10) and then applying Eq.(A.5) [for expressing $Y_{\nu G}$'s elements with corresponding entries of $Y_\nu(M)$], with definitions of Eqs. (3.13) and (3.15), we obtain:

$$|\delta_N(M)|e^{i\eta} = \frac{1}{4\pi^2}\frac{\bar{\kappa}(M)}{\kappa(M)}\left[|(Y_\nu^\dagger Y_\nu)_{21}|e^{i\eta'} + \xi_\tau|(Y_\nu)_{31}(Y_\nu)_{32}|e^{i(\phi_{31}-\phi_{32})} + \xi_\mu|(Y_\nu)_{21}(Y_\nu)_{22}|e^{i(\phi_{21}-\phi_{22})}\right]_{\mu=M} \tag{3.18}$$

where ϕ_{ij} denotes the phase of the matrix element $(Y_\nu)_{ij}$ at scale $\mu = M$. Eq. (3.18) shows well that in the limit $\xi_{\tau,\mu} \to 0$, we have $\eta = \eta'$, while the mismatch between these two phases is due to

$\xi_{\tau,\mu} \neq 0$. With $\xi_{\tau,\mu} \ll 1$, from (3.18) we derive:

$$\eta - \eta' \simeq \frac{\xi_\tau |(Y_\nu)_{31}(Y_\nu)_{32}| \sin(\phi_{31} - \phi_{32} - \eta') + \xi_\mu |(Y_\nu)_{21}(Y_\nu)_{22}| \sin(\phi_{21} - \phi_{22} - \eta')}{|(Y_\nu^\dagger Y_\nu)_{21}|} \ . \tag{3.19}$$

We stress, that the 1-loop renormalization of the Y_ν matrix plays the leading role in generation of $\xi_{\tau,\mu}$, i.e. in the CP violation.[8] [This is also demonstrated by Eq. (3.16).] When the product $(Y_\nu)_{31}(Y_\nu)_{32}$ is non-zero, the leading role for the mismatch between η and η' is played by ξ_τ. However, for the Yukawa texture, having this product zero, important will be contribution from ξ_μ. [As we will see on working examples, this will happen for T_9 of Eq. (2.5) and texture B_2 of Eq. (3.40)].

The value of $|\delta_N(M)|$, which characterizes the mass splitting between the RHN's, can be computed by taking the absolute values of both sides of (3.18):

$$|\delta_N(M)| = \frac{\kappa_N}{4\pi^2} \left|(Y_\nu^\dagger Y_\nu)_{21} + \xi_\tau (Y_\nu)_{31}(Y_\nu^*)_{32} + \xi_\mu (Y_\nu)_{21}(Y_\nu^*)_{22}\right|_{\mu=M} \ln \frac{M_G}{M} \ , \ \text{with} \ \ \kappa_N = \frac{\bar{\kappa}(M)}{\kappa(M) \ln \frac{M_G}{M}} \ . \tag{3.20}$$

These expressions can be used upon the calculation of the leptogenesis, which we will do in sections 3.3 and 3.4 for concrete models of the neutrino mass matrices.

3.2 See-Saw via Two Texture Zero 3×2 Dirac Yukawas Augmented by Single d=5 Operator. Predicting CP Violation

Within the setup with two RHNs, having at the GUT scale mass matrix of the form (3.1), we consider all two texture zero 3×2 Yukawa matrices with an unremovable complex ϕ phase. As shown in [20] and in Sect. 2.1, there are nine two texture zero 3×2 Yukawa matrices, out of which only three, namely T_4, T_7 and T_9 (given by (2.12), (2.18) and (2.22) respectively) possess unremovable complexity.

That complexity expressed through physical phase ϕ is relevant to the leptogenesis [1] and also, as it was shown in [20], it can be related to phase δ, determined from the neutrino sector. As will be shown on concrete neutrino models, this will remain true after inclusion of specific single $d = 5$ operator. Since we are interested in complex two texture zero 3×2 Yukawa matrices for Y_ν in (2.24) the textures $T_{4,7,9}$ should be used in turn.

[8] Note that since RG equations for M_N and Y_ν in non-SUSY case have similar structures (besides some group-theoretical factors) the $\xi_{\tau,\mu}$ would be generated also within non-SUSY setup.

Before switching to concrete Neutrino mass texture models, we explain our choice of numerical data used hereafter. As far as the numerical values of the oscillation parameters are concerned, since the bfv's of the works of Ref. [4] differ from each other by few %'s, we will use their mean values:

$$\sin^2\theta_{12} = 0.308, \qquad \sin^2\theta_{23} = \begin{cases} 0.432 & \text{for NH} \\ 0.591 & \text{for IH} \end{cases}, \quad \sin^2\theta_{13} = \begin{cases} 0.02157 & \text{for NH} \\ 0.0216 & \text{for IH} \end{cases},$$

$$\Delta m^2_{sol} = 7.48\cdot 10^{-5}\ \text{eV}^2, \qquad \Delta m^2_{atm} = |m_3^2 - m_2^2| = \begin{cases} 2.47\cdot 10^{-3}\ \text{eV}^2 & \text{for NH} \\ 2.54\cdot 10^{-3}\ \text{eV}^2 & \text{for IH} \end{cases}. \tag{3.21}$$

In models, which allow to do so, we use the best fit values (bfv) given in (3.21). However, in some cases we also apply the value(s) of some oscillation parameter(s) which deviate from the bfv's by several σ.

P_1 Neutrino Texture

This texture, within our scenario, can be parameterized as:

$$M_\nu(M_Z) = \begin{pmatrix} 0 & d_5 & 0 \\ d_5 & 2a_2b_2 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu3} \\ 0 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu3} & 2a_3b_3e^{i\phi}r_{\nu3}^2 \end{pmatrix}\bar{m} \tag{3.22}$$

where,

$$\bar{m} = -\frac{r_{\bar{m}}v_u^2(M_Z)}{M\cdot e^{-i(\omega+\rho)}} \tag{3.23}$$

δ	ρ_1	ρ_2	works with
± 0.0879121	± 3.11851	± 3.03949	NH, $\sin^2\theta_{23} = 0.451$, $\sin^2\theta_{12} = 0.323$ and best fit values for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.00694406, 0.0110914, 0.0509217)$, $m_{\beta\beta} = 0$

Table 7: Results from P_1 type texture. Masses are given in eVs.

and RG factors $r_{\bar{m}}$ and $r_{\nu3}$ are given in Eqs. (A.17) and (A.18) of Ref. [16]. (For notations and definitions see also Appendix A.2) The entries depending on a_i, b_j in (3.22) arise from the T_4 texture [given in (2.12)] by the see-saw mechanism. The entry d_5 comes from the (2.25) type operator $\frac{\tilde{d}_5 e^{ix_5}}{M_*} l_1 l_2 h_u h_u$. Since, as we see from Eqs. (2.12) and (2.13), the phase x is undetermined, we can select it in such a way as to set (3.22)'s d_5 entry to be real. Therefore, we still have single physical phase ϕ. It will be related to the phase δ and will govern the leptogenesis process (discussed in Sect. 3.3). Due to the texture zeros, it is possible to predict the phases and values of the neutrino masses in terms of the measured oscillation parameters. In particular, the conditions $M_\nu^{(1,1)} = 0$ and $M_\nu^{(1,3)}$=0, using (2.49)-(2.51), give

two complex equations (2.56) and (2.57), which with the input of five oscillation parameters allow to calculate all neutrino masses and predict three phases δ, ρ_1 and ρ_2. Without providing here further analytical relations [followed from Eqs. (2.56), (2.57) and given in [20]), in Table 7 we summarize the results. [Only normal hierarchical (NH) neutrino mass ordering scenario works for the P_1 type texture.]

P_2 Neutrino Texture

$$M_\nu(M_Z) = \begin{pmatrix} 0 & 0 & d_5 \\ 0 & 2a_2b_2 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu 3} \\ d_5 & (a_3b_2 + a_2b_3e^{i\phi})r_{\nu 3} & 2a_3b_3e^{i\phi}r_{\nu 3}^2 \end{pmatrix} \bar{m} \tag{3.24}$$

This texture's a_i, b_i entries are also obtained from the T_4 texture (2.12) via the see-saw mechanism and by addition of the $d = 5$ operator $\frac{\tilde{d}_5 e^{ix_5}}{M_*} l_1 l_3 h_u h_u$. By proper adjustment of the phase x [remaining undetermined in (2.12) and (2.13)], we can set d_5 entry of (3.24) to be real. The two conditions $M_\nu^{(1,1)} = 0$ and $M_\nu^{(1,2)}$=0 give relation of Eq. (2.56) and Eq. (2.68) which allow to predict neutrino masses and three phases $\delta, \rho_{1,2}$. Results are given in Table 8. For inputs the best fit values (bfv) of the oscillation parameters are taken from Eq.(3.21). For more details we refer the reader to [20].

δ	ρ_1	ρ_2	works with
± 1.71006	∓ 2.79206	∓ 1.47308	NH and bfv's of oscillation parameters, $(m_1, m_2, m_3) = (0.00471158, 0.0098488, 0.0506656)$, $m_{\beta\beta} = 0$

Table 8: Results from P_2 type texture. Masses are given in eVs.

P_3 Neutrino Texture

Using the see-saw formula (2.24) for the T_7 texture (2.18) and including the $d = 5$ operator $\frac{\tilde{d}_5 e^{ix_5}}{M_*} l_2 l_3 h_u h_u$, we obtain the P_3 neutrino texture:

$$M_\nu(M_Z) = \begin{pmatrix} 2a_1b_1 & 0 & (a_3b_1 + a_1b_3e^{i\phi})r_{\nu 3} \\ 0 & 0 & d_5 \\ (a_3b_1 + a_1b_3e^{i\phi})r_{\nu 3} & d_5 & 2a_3b_3e^{i\phi}r_{\nu 3}^2 \end{pmatrix} \bar{m} \tag{3.25}$$

Since the phase y is not fixed in (2.18) and (2.19), without loss of any generality the d_5 entry of (3.25) can be set to be real. The conditions $M_\nu^{(1,2)} = 0$ and $M_\nu^{(2,2)}$=0, similar to previous cases, allow to predict $m_{1,2,3}$ and $\delta, \rho_{1,2}$. Without giving the expressions (being lengthy and presented in Ref. [20]), we proceed to give numerical results, which for NH and inverted hierarchical (IH) neutrino mass orderings are summarized in Table 9.

δ	ρ_1	ρ_2	works with
± 1.53714	± 0.0867342	± 3.20236	NH and bfv's of oscillation parameters, $(m_1, m_2, m_3) =$ $(0.0588907, 0.0595224, 0.077543)$, $m_{\beta\beta} = 0.059436$
± 1.58066	∓ 0.114316	± 3.06301	IH and bfv's of oscillation parameters, $(m_1, m_2, m_3) =$ $(0.0696426, 0.0701776, 0.0488354)$, $m_{\beta\beta} = 0.0692588$

Table 9: Results from P_3 type texture. Masses are given in eVs.

P_4 Neutrino Texture

This texture is obtained by applying the see-saw formula (2.24) to the T_9 texture (2.22) and including the $d = 5$ operator $\frac{\tilde{d}_5 e^{ix_5}}{M_*} l_2 l_3 h_u h_u$. Doing these we obtain the P_4 neutrino texture:

$$M_\nu(M_Z) = \begin{pmatrix} 2a_1b_1 & (a_2b_1 + a_1b_2e^{i\phi}) & 0 \\ (a_2b_1 + a_1b_2e^{i\phi}) & 2a_2b_2e^{i\phi} & d_5 \\ 0 & d_5 & 0 \end{pmatrix} \bar{m} \tag{3.26}$$

In this case the phase z is not fixed [see Eqs. (2.22) and (2.23)] and we can use this phase freedom to take d_5 entry of (3.26) matrix as a real parameter. The conditions $M_\nu^{(1,3)} = M_\nu^{(3,3)} = 0$ will give two complex (i.e. four real) equations, which contain three phases $\delta, \rho_{1,2}$ and one of the neutrino masses (remember that two measured parameters $\Delta m^2_{sol} = m_2^2 - m_1^2$ and $\Delta m^2_{atm} = |m_3^2 - m_2^2|$ leave undetermined values of the neutrino masses). Therefore, as for previous cases, with input of five measured oscillation parameters (which are: $\Delta m^2_{sol}, \Delta m^2_{atm}$ and $\{\theta_{12}, \theta_{23}, \theta_{13}\}$) from the conditions given above we predict all light neutrino masses and three phases $\delta, \rho_{1,2}$. Still referring to [20], for analytical expressions, in Table 10 we give the numerical results obtained for this texture P_4 for NH and IH cases. The value of s^2_{23} we are using is deviated from the bfv, because the conditions $M_\nu^{(1,3)} = M_\nu^{(3,3)} = 0$ do not allow to use bfv's. Note that in NH, case 2 and for IH the values of s^2_{23} are less deviated from bfv, but the NH's case 1, as it turns out, is preferred for obtaining needed amount of the baryon asymmetry. Without the latter constraint, just for satisfying the neutrino data, we could have used smaller values of s^2_{23}, but this would give higher values of neutrino masses which would not satisfy the current cosmological constraint $\sum_i m_i < 0.23$ eV (the limit set by the Planck observations [47][9]). Upon leptogenesis investigation we will use NH, case 1 given in Tab.10.

	δ	ρ_1	ρ_2	works with
NH, case 1	± 1.62446	∓ 0.129186	± 3.05085	NH and $\sin^2\theta_{23} = 0.6$ and bfv's for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.044819, 0.0456458, 0.0674799)$, $m_{\beta\beta} = 0.0454757$
NH, case 2	± 1.59508	∓ 0.0647305	± 3.09629	NH and $\sin^2\theta_{23} = 0.551$ and bfv's for remaining oscillation parameters, $(m_1, m_2, m_3) = (0.0707692, 0.0712957, 0.0869084)$, $m_{\beta\beta} = 0.0712444$

[9]Tighter upper bound can be obtained by considering additional combined datasets [48]. However, bound also depends on the theoretical framework and can be relaxed (see e.g. 2$^{\text{nd}}$ Ref. of [4], where as demonstrated in Table II, the scenario with extra A_{lens} parameter yields more relaxed bounds). Thus, upon our calculations we use the constraint $\sum_i m_i < 0.23$ eV.

δ	ρ_1	ρ_2	works with
± 1.56553	± 0.0733633	± 3.19198	IH and $\sin^2\theta_{23} = 0.441$ and bfv's for remaining oscillation parameters, $(m_1, m_2, m_3) =$ $(0.0820116, 0.0824663, 0.065274)$, $m_{\beta\beta} = 0.0817407$

Table 10: Results from P_4 type texture. Masses are given in eVs.

3.3 Resonant Leptogenesis

Expression for $\delta_N(M)$ with effects of $\lambda_{\mu,\tau}$ and ignoring λ_e, is given by Eq. (3.18). The CP asymmetries ϵ_1 and ϵ_2 generated by out-of-equilibrium decays of the quasi-degenerate fermionic components of N_1 and N_2 states respectively are given by [10, 11]:[10]

$$\epsilon_1 = \frac{\mathrm{Im}[(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}]^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}} \frac{(M_2^2 - M_1^2)\, M_1 \Gamma_2}{(M_2^2 - M_1^2)^2 + M_1^2 \Gamma_2^2} \;, \qquad \epsilon_2 = \epsilon_1 (1 \leftrightarrow 2) \;. \tag{3.27}$$

Here M_1, M_2 (with $M_2 > M_1$) are the mass eigenvalues of the RHN mass matrix. These masses, within our scenario, are given in (3.4) with the splitting parameter given in Eq. (3.20). For the decay widths, here we will use more accurate expressions [6]:

$$\Gamma_{N_i} = \frac{M_i}{8\pi} (\hat{Y}^\dagger \hat{Y})_{ii} \left(\left(1 - 4\frac{M_S^2}{M_i^2}\right)^{\frac{1}{2}} + s_\beta^2 + c_\beta^2 \left(1 - \frac{M_S^2}{M_i^2}\right)^2 \right), \tag{3.28}$$

where M_S is the SUSY scale and we assume that all SUSY states have the common mass equal to this scale. s_β and c_β are short hand notations for $\sin\beta$ and $\cos\beta$ respectively. N_i decays proceed via $N_i \to h_u l_i$ and $N_i \to \tilde{h}_u \tilde{l}_i$ channels. Upon derivation of (3.28) we took into account that h_u is a linear combination of the SM Higgs doublet h_{SM} and the heavy Higgs doublet H: $h_u \simeq s_\beta h_{SM} + c_\beta H$. Mass of the h_{SM} has been ignored, while the mass of the H has been taken$\simeq M_S$. Moreover, the imaginary part of $[(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}]^2$ will be computed with help of (3.6) and (3.7) with the relevant phase given in Eq. (3.19). Using general expressions (3.19) and (3.20) for the given neutrino model we will compute $\eta - \eta'$ and $|\delta_N(M)|$. With these, since we know the possible values of the phase

[10]In Appendix B we investigate the contribution to the baryon asymmetry via decays of the scalar components of the RHN superfields. As we show, these effects are less than 3.4%.

ϕ [see Eqs. (3.31),(3.33),(3.35),(3.37)], and with the help of the relations (3.32), (3.34), (3.36), (3.38) we can compute $\epsilon_{1,2}$ in terms of $|M|$ and a_2 or a_1 (depending on the texture we are dealing with). Recalling that the lepton asymmetry is converted to the baryon asymmetry via sphaleron processes [49], with the relation $\frac{n_b^f}{s} \simeq -1.48 \times 10^{-3}({\kappa_f}^{(1)}\epsilon_1 + {\kappa_f}^{(2)}\epsilon_2)$ we can compute the baryon asymmetry. The notion n_b^f is used for the baryon asymmetry created through the decays of the fermionic components of $N_{1,2}$ superfields. The net baryon asymmetry n_b receives the contribution from the decays of the scalar components $\tilde{N}_{1,2}$. The latter contribution we denote by $\tilde{n}_b$. The computation of it (being suppressed in comparison with n_b^f) will be discussed in Appendix B. For the efficiency factors ${\kappa_f}^{(1,2)}$ we will use the extrapolating expressions [6] (see Eq. (40) in Ref. [6]), with ${\kappa_f}^{(1)}$ and ${\kappa_f}^{(2)}$ depending on the mass scales $\tilde{m}_1 = \frac{v_u^2(M)}{M_1}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}$ and $\tilde{m}_2 = \frac{v_u^2(M)}{M_2}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}$ respectively.

Within our studies we will consider the RHN masses $\simeq |M| \lesssim 10^7$ GeV. With this, we will not have the relic gravitino problem [50,51]. For simplicity, we consider all SUSY particle masses to be equal to $M_S < |M|$, with M_S identified with the SUSY scale, below which we have just SM. As it turns out, via the RG factors, the asymmetry also depends on the top quark mass.

It is remarkable that within some models the observed baryon asymmetry

$$\left(\frac{n_b}{s}\right)_{\rm exp} = (8.65 \pm 0.085) \times 10^{-11} \tag{3.29}$$

(the recent value reported by WMAP and Planck [47]), can be obtained even for low values of the MSSM parameter $\tan\beta = \frac{v_u}{v_d}$ (defined at the SUSY scale $\mu = M_S$).

Below, we perform analysis for each of these $P_{1,2,3,4}$ cases (and for revised models of Ref. [1] discussed in Sect. 3.4) in turn and present our results. As an input for the top's running mass we will use the central value, while for the SUSY scale M_S we will consider two cases:

$$m_t(m_t) = 163.48 \text{ GeV},$$

$$\text{Case } (\mathbf{I}): \quad M_S = 10^3 \text{ GeV}, \qquad \text{Case } (\mathbf{II}): \quad M_S = 2 \times 10^3 \text{ GeV}. \tag{3.30}$$

Procedure of our RG calculation and used schemes are described in Appendix A.3. As it was shown in [20], for neutrino mass matrix textures $P_{1,2,3,4}$, we will be able to relate the cosmological phase ϕ to the CP violating phase δ.

For P_1 Texture

As was shown in Sect.2.4 for this case, using the form of the M_ν [given by Eq. (3.22) and derived within our setup] in the relation (2.49) and equating appropriate matrix elements of the both sides, we will be able to calculate the phase ϕ [16, 20]:

$$\phi = \mathrm{Arg}\left[\left(\frac{\mathcal{A}_{23}}{\sqrt{\mathcal{A}_{22}\mathcal{A}_{33}}} \mp \sqrt{\frac{\mathcal{A}_{23}^2}{\mathcal{A}_{22}\mathcal{A}_{33}} - 1}\right)^2\right]. \qquad (3.31)$$

Moreover, expressing $a_3, b_{2,3}$ in terms of a_2 (taking a_2 to be an independent variable) and other known and/or predicted parameters, we will have:

$$a_3 = \frac{a_2}{r_{\nu 3}}\frac{1}{|\mathcal{A}_{22}|}\left|\mathcal{A}_{23} \pm \sqrt{\mathcal{A}_{23}^2 - \mathcal{A}_{22}\mathcal{A}_{33}}\right|, \quad b_2 = \frac{|\mathcal{A}_{22}|}{2|\bar{m}|a_2}, \quad b_3 = \frac{|\mathcal{A}_{33}|}{2|\bar{m}|a_3 r_{\nu 3}^2}. \qquad (3.32)$$

As we see from Eqs. (3.31) and (3.32), there is a pair of solutions. When for the a_3 in (3.32) we are taking the " + " sign, in (3.31) we should take the sign " − ", and vice versa. (The same applies to the cases of textures $P_{2,3,4}$.) For this case, the baryon asymmetry via the resonant leptogenesis has been investigated in Ref. [16]. Here, for the decay widths we use more refined expressions of Eq. (3.28). Because of this, the values of $\tan\beta$ (given in Table 11) are slightly different. Since in this model $(Y_\nu)_{31}$ and $(Y_\nu)_{32}$ are non-zero, according to Eq. (3.18) the mismatch $\eta - \eta'$ (e.g. CP asymmetry) is mainly arising due to ξ_τ. However, in numerical calculations we have also taken into account the contribution of ξ_μ. The results are given in Table 11 (for more explanations see also caption of this table). While in the table we vary the values of M and $\tan\beta$, the cases with **I** and **II** correspond respectively to the cases **(I)** and **(II)** of Eq. (3.30) (i.e. $M_S = 1$ and 2 TeV resp.). For the definition of the RG factors given in this table see Appendix A.2 of Ref. [16](For notations and definitions see also Appendix A.2). For finding maximal values of the Baryon asymmetries (given in Tab.11) we have varied the parameter a_2. As we see, the value of the net baryon asymmetry n_b slightly differs from n_b^f. This is due to the contribution from $\tilde{n}_b$ [coming from the right handed sneutrino (RHS) decays], which is small (less than 3.4% of n_b^f). Details of $\tilde{n}_b$'s calculations are discussed in Appendix B.

Case	$M(\mathrm{GeV})$	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^5\times\xi_\tau$	$10^{11}\times\left(\frac{n_b^f}{s}\right)_{max}$	$10^{11}\times\left(\frac{n_b}{s}\right)_{max}$
(I.1)	$3\cdot 10^3$	1.72	0.8868	0.9714	1.206	6.106	8.29	8.57
(I.2)	10^4	1.619	0.832	0.9523	1.2322	5.303	8.34	8.6
(I.3)	10^5	1.664	0.7482	0.9203	1.1807	4.821	8.36	8.6
(I.4)	10^6	1.719	0.682	0.8923	1.1345	4.381	8.37	8.6
(I.5)	10^7	1.773	0.6291	0.8676	1.0971	3.937	8.37	8.6
(II.1)	$6\cdot 10^3$	1.701	0.8689	0.9678	1.175	5.897	8.294	8.57
(II.2)	10^4	1.615	0.8464	0.9599	1.1994	5.365	8.334	8.59
(II.3)	10^5	1.625	0.7629	0.9283	1.1669	4.755	8.36	8.6
(II.4)	10^6	1.678	0.6974	0.9008	1.1243	4.321	8.36	8.6
(II.5)	10^7	1.731	0.645	0.8765	1.0894	3.887	8.36	8.6

Table 11: Texture P_1, normal hierarchy: Baryon asymmetry for various values of M and for minimal (allowed) value of $\tan\beta$. With neutrino oscillation parameters and results given in the Table 7 and computed from Eq. (3.31) $\phi = \pm 1.264$. For all cases $r_{\nu 3}\simeq 1$.

For P_2 Texture

With a pretty similar procedure, for this case we get:

$$\phi = \mathrm{Arg}\left[\left(\frac{\mathcal{A}_{23}}{\sqrt{\mathcal{A}_{22}\mathcal{A}_{33}}} \mp \sqrt{\frac{\mathcal{A}_{23}^2}{\mathcal{A}_{22}\mathcal{A}_{33}} - 1}\right)^2\right]. \tag{3.33}$$

Expressing $a_3, b_{2,3}$ in terms of a_2 and other parameters (yet known or predicted in this scenario), we will have:

$$a_3 = \frac{a_2}{r_{\nu 3}}\frac{1}{|\mathcal{A}_{22}|}\left|\mathcal{A}_{23} \pm \sqrt{\mathcal{A}_{23}^2 - \mathcal{A}_{22}\mathcal{A}_{33}}\right|, \quad b_2 = \frac{|\mathcal{A}_{22}|}{2|\bar{m}|a_2}, \quad b_3 = \frac{|\mathcal{A}_{33}|}{2|\bar{m}|a_3 r_{\nu 3}^2} \tag{3.34}$$

Results for this case are presented in Table 12.

Case	M(GeV)	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^5 \times \xi_\tau$	$10^{11}\times\left(\frac{n_b^f}{s}\right)_{max}$	$10^{11}\times\left(\frac{n_b}{s}\right)_{max}$
(I.1)	$3\cdot 10^3$	1.948	0.8908	0.9725	1.1439	7.264	8.306	8.57
(I.2)	10^4	1.833	0.8412	0.955	1.1543	6.242	8.35	8.6
(I.3)	10^5	1.881	0.7647	0.9254	1.1158	5.692	8.37	8.6
(I.4)	10^6	1.938	0.7039	0.8994	1.0821	5.182	8.36	8.6
(I.5)	10^7	1.996	0.6554	0.8766	1.0544	4.671	8.36	8.6
(II.1)	$6\cdot 10^3$	1.933	0.8728	0.9689	1.1201	7.058	8.314	8.57
(II.2)	10^4	1.836	0.8526	0.9616	1.133	6.373	8.35	8.6
(II.3)	10^5	1.843	0.7771	0.9326	1.1063	5.638	8.36	8.6
(II.4)	10^6	1.9	0.7175	0.9072	1.0748	5.14	8.37	8.6
(II.5)	10^7	1.956	0.6697	0.8848	1.049	4.632	8.37	8.6

Table 12: Texture P_2, normal hierarchy: Baryon asymmetry for various values of M and for minimal (allowed) value of $\tan\beta$. With neutrino oscillation parameters and results given in the Table 8 and computed from Eq. (3.33) $\phi = \pm 1.1$. For all cases $r_{\nu 3} \simeq 1$.

For P_3 Texture

Case	M(GeV)	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^5 \times \xi_\tau$	$10^{11}\times\left(\frac{n_b^f}{s}\right)_{max}$	$10^{11}\times\left(\frac{n_b}{s}\right)_{max}$
(I.1)	$3\cdot 10^3$	7.158	0.904	0.9761	1.0076	76.29	8.49	8.59
(I.2)	10^4	6.802	0.8717	0.9635	0.9983	64.79	8.508	8.6
(I.3)	10^5	6.922	0.82	0.9417	0.9819	59.11	8.51	8.6
(I.4)	10^6	7.074	0.7789	0.9225	0.9692	53.92	8.51	8.6
(I.5)	10^7	7.227	0.7467	0.9056	0.96	48.65	8.51	8.6
(II.1)	$6\cdot 10^3$	7.146	0.8852	0.9723	0.9986	75.06	8.5	8.6
(II.2)	10^4	6.85	0.8725	0.9672	0.9954	67.24	8.5	8.6
(II.3)	10^5	6.858	0.8229	0.946	0.9802	59.44	8.51	8.6
(II.4)	10^6	7.003	0.7835	0.9274	0.9684	54.17	8.51	8.6
(II.5)	10^7	7.151	0.7524	0.9109	0.9597	48.87	8.51	8.6

Table 13: Texture P_3, normal hierarchy: Baryon asymmetry for various values of M and for minimal (allowed) value of $\tan\beta$. With neutrino oscillation parameters and results given in the Table 9 and computed from Eq. (3.35) (for NH case) $\phi = \pm 2.92$. For all cases $r_{\nu 3} \simeq 1$.

Case	M(GeV)	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^5 \times \xi_\tau$	$10^{11} \times \left(\frac{n_b^f}{s}\right)_{max}$	$10^{11} \times \left(\frac{n_b}{s}\right)_{max}$
(I.1)	$3 \cdot 10^3$	27.11	0.905	0.9764	1.0038	1154.3	8.515	8.6
(I.2)	10^4	25.824	0.8738	0.9641	0.9938	980.4	8.52	8.6
(I.3)	10^5	26.138	0.8234	0.9427	0.9784	894.7	8.53	8.6
(I.4)	10^6	26.55	0.7833	0.9238	0.9667	815.9	8.53	8.6
(I.5)	10^7	26.96	0.7515	0.9071	0.9583	736	8.53	8.6
(II.1)	$6 \cdot 10^3$	27.1	0.886	0.9725	0.995	1135.1	8.516	8.6
(II.2)	10^4	26.061	0.8739	0.9676	0.991	1017.9	8.518	8.6
(II.3)	10^5	25.979	0.8259	0.9469	0.9766	899.4	8.52	8.6
(II.4)	10^6	26.38	0.7875	0.9285	0.9657	819.9	8.53	8.6
(II.5)	10^7	26.783	0.757	0.9123	0.9578	739.6	8.53	8.6

Table 14: Texture P_3, inverted hierarchy: Baryon asymmetry for various values of M and for minimal (allowed) value of $\tan\beta$. With neutrino oscillation parameters and results given in the Table 9 and computed from Eq. (3.35) (for IH case) $\phi = \pm 3.124$. For all cases $r_{\nu 3} \simeq 1$.

$$\phi = \mathrm{Arg}\left[\left(\frac{\mathcal{A}_{13}}{\sqrt{\mathcal{A}_{11}\mathcal{A}_{33}}} \mp \sqrt{\frac{\mathcal{A}_{13}^2}{\mathcal{A}_{11}\mathcal{A}_{33}} - 1}\right)^2\right]. \tag{3.35}$$

Expressing $a_3, b_{1,3}$ in terms of a_1 and other fixed parameters, we will have:

$$a_3 = \frac{a_1}{r_{\nu 3}} \frac{1}{|\mathcal{A}_{11}|}\left|\mathcal{A}_{13} \pm \sqrt{\mathcal{A}_{13}^2 - \mathcal{A}_{11}\mathcal{A}_{33}}\right|, \quad b_1 = \frac{|\mathcal{A}_{11}|}{2|\bar{m}|a_1}, \quad b_3 = \frac{|\mathcal{A}_{33}|}{2|\bar{m}|a_3 r_{\nu 3}^2} \tag{3.36}$$

Results for this texture for cases of NH and IH neutrinos are presented in Tables 13 and 14 respectively.

For P_4 Texture

For this case cosmological phase is given by:

$$\phi = \mathrm{Arg}\left[\left(\frac{\mathcal{A}_{12}}{\sqrt{\mathcal{A}_{11}\mathcal{A}_{22}}} \mp \sqrt{\frac{\mathcal{A}_{12}^2}{\mathcal{A}_{11}\mathcal{A}_{22}} - 1}\right)^2\right]. \tag{3.37}$$

Expressing $a_1, b_{1,2}$ in terms of a_2 and other known and/or predicted parameters, we will have:

$$a_1 = \frac{|\mathcal{A}_{11}|}{|\mathcal{A}_{12} \pm \sqrt{\mathcal{A}_{12}^2 - \mathcal{A}_{11}\mathcal{A}_{22}}|} a_2, \quad b_1 = \frac{|\mathcal{A}_{11}|}{2|\bar{m}|a_1}, \quad b_2 = \frac{|\mathcal{A}_{22}|}{2|\bar{m}|a_2} \tag{3.38}$$

In this scenario, since $(Y_\nu)_{31}$ and $(Y_\nu)_{32}$ are zero, according to Eq. (3.18) the mismatch $\eta - \eta'$ (e.g. CP asymmetry) is arising due to ξ_μ. Since the latter is suppressed by λ_μ^2, as it turns out large values of the $\tan\beta$ are required and only in NH case needed amount of the Baryon asymmetry can be generated. Results are given in Table 15.

Case	$M(\mathrm{GeV})$	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^4 \times \xi_\mu$	$10^{11} \times \left(\frac{n_b^f}{s}\right)_{max}$	$10^{11} \times \left(\frac{n_b}{s}\right)_{max}$
(I.1)	$3 \cdot 10^3$	64.639	0.9048	0.9763	1.0349	3.111	8.518	8.6
(I.2)	10^4	62.213	0.873	0.9638	1.0212	2.638	8.52	8.6
(I.3)	10^5	62.02	0.8203	0.9418	1.0059	2.416	8.53	8.6
(I.4)	10^6	62.006	0.7767	0.9218	0.994	2.213	8.53	8.6
(I.5)	10^7	62	0.7404	0.9037	0.9848	2.008	8.53	8.6
(II.1)	$6 \cdot 10^3$	65.28	0.8859	0.9725	1.0208	3.045	8.517	8.59
(II.2)	10^4	63.398	0.8735	0.9675	1.0145	2.728	8.525	8.59
(II.3)	10^5	62.548	0.8239	0.9463	0.9996	2.417	8.53	8.6
(II.4)	10^6	62.528	0.7827	0.9271	0.9886	2.211	8.53	8.6
(II.5)	10^7	62.535	0.7484	0.9097	0.9803	2.005	8.53	8.6

Table 15: Texture P_4, normal hierarchy: Baryon asymmetry for various values of M and for minimal (allowed) value of $\tan\beta$. With neutrino oscillation parameters and results given in the Table 10, NH, case 1, and ϕ computed from Eq. (3.37) (for NH case) $\phi = \pm 2.872$.

3.4 Revising Textures of Ref. [1] and Improved Versions

In this section we revise the textures considered in the work [1]. Since some of them are excluded by the current neutrino data [4](see also Eq. (3.21)), we apply $d = 5$ contributions (in a spirit of section 3.2) and achieve their compatibility with the best fit values. Together with this, we investigate resonant leptogenesis and show that one loop corrections via λ_τ and/or λ_μ are crucial. In [1], while ignoring λ_μ the two loop correction to λ_τ was taken into account and this suggested for textures A and B_1 specific low bounds on the values of $\tan\beta$. As demonstrated below, one loop effects of λ_τ (giving dominant contribution for textures A and B_1) and λ_μ (for the texture B_2) significantly change results.

In the setup of two degenerate RHNs, in Ref. [1] the following three possible one texture zero neutrino Dirac Yukawa couplings have been considered :

$$\text{Texture A}: \qquad Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix}, \tag{3.39}$$

$$\text{Texture } B_1: \; Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & 0 \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix}, \qquad \text{Texture } B_2: \; Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & 0 \end{pmatrix}, \tag{3.40}$$

where for notational consistency with the entire work, we have shown phases α_i, β_j, while assuming that the couplings a_i, b_j are real.[11] Below we will (re)investigate these textures in turn.

Texture A

The A Yukawa texture can be written as:

$$\text{Texture A}: \; Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & 0 \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & b_3 e^{i\beta_3} \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ a_3 e^{i\phi} & b_3 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix},$$

$$\text{with} \quad x = \alpha_1 - \alpha_2 + \beta_2 - \rho, \quad y = \beta_2 - \rho, \quad z = \beta_3 - \rho, \quad \omega = \alpha_2 - \beta_2 + \rho, \quad \phi = \alpha_3 - \alpha_2. \tag{3.41}$$

As we see, besides the phase ϕ all phases are factored out and have no physical relevance. With the RHN mass matrix of Eq.(2.29), via the see-saw[see expression in Eq.(2.24)] we will get the light neutrino mass matrix:

$$M_\nu^{(\mathrm{A})}(M_Z) = \begin{pmatrix} 0 & a_1 b_2 & a_1 b_3 r_{\nu 3} \\ a_1 b_2 & 2 a_2 b_2 & (a_2 b_3 + a_3 b_2 e^{i\phi}) r_{\nu 3} \\ a_1 b_3 r_{\nu 3} & (a_2 b_3 + a_3 b_2 e^{i\phi}) r_{\nu 3} & 2 a_3 b_3 e^{i\phi} r_{\nu 3}^2 \end{pmatrix} \bar{m}, \tag{3.42}$$

[11] On the contrary, in Ref. [1], without writing down the phase factors, a_i and b_j were treated as a complex parameters.

[For definitions of $\bar{m}$, $r_{\nu 3}$ and proper explanations see respectively Eq. (3.23) and also Eqs. (A.17), (A.18) of Ref. [16], and comments therein, as well as Appendix A.2] This neutrino mass texture has only two non-zero mass eigenvalues. As it was shown in [1], this for NH ($m_1 = 0$) and IH ($m_3 = 0$) neutrino mass patterns, gives respectively the predictive relations $\tan\theta_{13} = \sqrt{\frac{m_2}{m_3}} s_{12}$ and $\tan\theta_{12} = \sqrt{\frac{m_1}{m_2}}$. Both of them are in a gross conflict with the current neutrino data, which exclude this scenario.

A′ Neutrino Texture: Improved Version

The drawbacks coming from the A neutrino mass matrix (3.42) can be avoided by adding d_5 term to one of the entries. Here we consider this addition to the $(2,3)$ and $(3,2)$ elements of the light neutrino mass matrix, which would make the model viable. (We refer to this improved version of (3.42) as the A′ neutrino texture.) After this, the M_ν will have the form:

$$M_\nu^{(\mathrm{A}')}(M_Z) = \begin{pmatrix} 0 & a_1 b_2 & a_1 b_3 r_{\nu 3} \\ a_1 b_2 & 2a_2 b_2 & (a_2 b_3 + a_3 b_2 e^{i\phi}) r_{\nu 3} + d_5 \\ a_1 b_3 r_{\nu 3} & (a_2 b_3 + a_3 b_2 e^{i\phi}) r_{\nu 3} + d_5 & 2a_3 b_3 e^{i\phi} r_{\nu 3}^2 \end{pmatrix} \bar{m}. \tag{3.43}$$

With this modification, all masses are non-zero. One can check out, that with the fixed phase redefinitions [given in Eq. (3.41)], in general d_5 is a complex parameter. Thus, together with additional mass, we will have one more independent phase. As it turns out, only NH scenario is possible to realize. Therefore as additional independent parameters we take one of the mass and $\Delta\rho = \rho_1 - \rho_2$. From the condition $M_\nu^{(1,1)} = 0$ we have:

$$\cos(2\delta - \Delta\rho) = \frac{m_1^2 c_{12}^4 - m_2^2 s_{12}^4 - m_3^2 t_{13}^4}{2 m_2 m_3 s_{12}^2 t_{13}^2}, \quad \rho_1 = \pi - \mathrm{Arg}\left[\frac{m_2}{m_3} s_{12}^2 + t_{13}^2 e^{i(2\delta - \Delta\rho)}\right] \quad \text{with} \quad \Delta\rho = \rho_1 - \rho_2. \tag{3.44}$$

(Here and below we use shorthanded notations $t_{ij} \equiv \tan\theta_{ij}$.) From the first relation of (3.44) one can check that IH scenario can not be realized. As far as the NH scenario is concerned, it will work with low bound on the lightest neutrino mass m_1. In fact, the first relation of (3.44) gives the allowed range for m_1. For example, with bfv's of the oscillation parameters (3.21) we have:

$$0.00239 \text{ eV} \lesssim m_1 \lesssim 0.00641 \text{ eV}. \tag{3.45}$$

Thus, as independent parameters we will take m_1 and $\Delta\rho$. We will select them in such a way as to get desirable baryon asymmetry. For example, with the choice

$$m_1 = 0.005719 \text{ eV}, \quad \Delta\rho = 4.987 \tag{3.46}$$

Case	M(GeV)	$\tan\beta$	$r_{\bar m}$	r_{v_u}	κ_N	$10^4\times\xi_\tau$	$10^{11}\times\left(\frac{n_b^f}{s}\right)_{\max}$	$10^{11}\times\left(\frac{n_b}{s}\right)_{\max}$
(**I**.**1**)	$3\cdot 10^3$	1.939	0.8907	0.9725	1.1457	0.7215	8.53	8.6
(**I**.**2**)	10^4	1.838	0.8414	0.955	1.153	0.6266	8.53	8.59
(**I**.**3**)	10^5	1.904	0.7662	0.9258	1.111	0.5793	8.53	8.59
(**I**.**4**)	10^6	1.986	0.7078	0.9006	1.0742	0.5374	8.54	8.6
(**I**.**5**)	10^7	2.075	0.6628	0.879	1.0442	0.4956	8.55	8.61
(**II**.**1**)	$6\cdot 10^3$	1.928	0.8727	0.9688	1.121	0.7031	8.53	8.6
(**II**.**2**)	10^4	1.84	0.8527	0.9617	1.1322	0.6393	8.54	8.6
(**II**.**3**)	10^5	1.869	0.7784	0.933	1.1013	0.5753	8.54	8.6
(**II**.**4**)	10^6	1.949	0.721	0.9083	1.0672	0.5337	8.54	8.6
(**II**.**5**)	10^7	2.036	0.6766	0.887	1.0393	0.4923	8.54	8.6

Table 16: A$'$ Neutrino Texture, NH. Baryon asymmetry for various values of M and for corresponding minimal (allowed) values of $\tan\beta$. With the choice given in Eqs. (3.46), (3.47) and bfv's of s_{ij}^2. For all cases $r_{\nu 3}\simeq 1$.

and bfv's of all measured oscillation parameters with help of (2.52) and (3.44) for neutrino masses and phases we are getting:

$$(m_1, m_2, m_3) \simeq (0.005719,\ 0.01037,\ 0.05077)\ \text{eV},$$

$$(\delta, \rho_1, \rho_2) \simeq (2.9639,\ 2.911,\ -2.076)\,. \tag{3.47}$$

As far as the baryon asymmetry is concerned, using (3.43) in (2.49) for the CP phase ϕ and expressing couplings $a_{1,3}, b_{2,3}$ in terms of a_2 we get

$$\phi = \text{Arg}\left(\frac{\mathcal{A}_{12}^2\mathcal{A}_{33}}{\mathcal{A}_{13}^2\mathcal{A}_{22}}\right),$$

$$a_1 = 2\left|\frac{\mathcal{A}_{12}}{\mathcal{A}_{22}}\right| a_2,\quad a_3 = \frac{1}{r_{\nu 3}}\left|\frac{\mathcal{A}_{12}\mathcal{A}_{33}}{\mathcal{A}_{22}\mathcal{A}_{13}}\right| a_2,\quad b_2 = \frac{|\mathcal{A}_{22}|}{2|\bar m|a_2},\quad b_3 = \left|\frac{\mathcal{A}_{13}\mathcal{A}_{22}}{\mathcal{A}_{12}}\right|\frac{1}{2r_{\nu 3}|\bar m|a_2}\ . \tag{3.48}$$

For the values of (3.46), (3.47) and bfv's of $s_{12,23,13}^2$ we get

$$\phi = -2.9297\ . \tag{3.49}$$

With these, and for given values of M and $\tan\beta$ by varying a_2 we can investigate the baryon asymmetry. Results are given in Tab. 16.

Texture B$_1$

The B$_1$ Yukawa texture can be written as:

$$\text{Texture B}_1:\quad Y_\nu=\begin{pmatrix} a_1e^{i\alpha_1} & b_1e^{i\beta_1}\\ a_2e^{i\alpha_2} & 0\\ a_3e^{i\alpha_3} & b_3e^{i\beta_3}\end{pmatrix}=\begin{pmatrix} e^{ix} & 0 & 0\\ 0 & e^{iy} & 0\\ 0 & 0 & e^{iz}\end{pmatrix}\begin{pmatrix} a_1 & b_1\\ a_2 & 0\\ a_3e^{i\phi} & b_3\end{pmatrix}\begin{pmatrix} e^{i\omega} & 0\\ 0 & e^{i\rho}\end{pmatrix},$$

$$\text{with}\ \ x=\beta_1-\rho,\quad y=\alpha_2-\alpha_1+\beta_1-\rho,\quad z=\beta_3-\rho,\quad \omega=\alpha_1-\beta_1+\rho,\quad \phi=\alpha_3-\beta_3-\alpha_1+\beta_1. \tag{3.50}$$

With the RHN mass matrix of Eq. (2.29), via the see-saw we will get the light neutrino mass matrix:

$$M_\nu^{(\text{B}_1)}(M_Z)=\begin{pmatrix} 2a_1b_1 & a_2b_1 & (a_1b_3+a_3b_1e^{i\phi})r_{\nu3}\\ a_2b_1 & 0 & a_2b_3r_{\nu3}\\ (a_1b_3+a_3b_1e^{i\phi})r_{\nu3} & a_2b_3r_{\nu3} & 2a_3b_3e^{i\phi}r_{\nu3}^2\end{pmatrix}\bar{m}\ , \tag{3.51}$$

This neutrino mass texture (referred as B$_1$ neutrino texture) works only for inverted neutrino mass ordering [1] (with $m_3=0$) and has two predictive relations. In particular, in terms of measured oscillation parameters we can calculate the phases δ and ρ_1. The exact expressions are:

$$\cos\delta=\frac{m_2(1+t_{23}^2t_{12}^2s_{13}^2)-m_1(t_{12}^2+t_{23}^2s_{13}^2)}{2t_{23}t_{12}s_{13}(m_1+m_2)},\quad \rho_1=\pi-\text{Arg}\left[\frac{(1-t_{23}t_{12}s_{13}e^{-i\delta})^2}{(t_{12}+t_{23}s_{13}e^{-i\delta})^2}\right]\ . \tag{3.52}$$

$$\text{with}\qquad m_1=\sqrt{\Delta m_{atm}^2-\Delta m_{sol}^2},\qquad m_2=\sqrt{\Delta m_{atm}^2},\qquad m_3=0. \tag{3.53}$$

Although the first expression in (3.52) excludes the possibility of using the best fit values for all oscillation parameters, it allows for keeping values of s_{23}^2 and s_{13}^2 within 1σ, while confining s_{12}^2 to 2σ. Remarkably, needed baryon asymmetry can be achieved with relatively low values of $\tan\beta$. For example,

$$\text{for IH of the B}_1\text{ neutrino texture},\quad \text{with}: s_{23}^2=0.604\ (1\sigma),\quad s_{12}^2=0.33\ (2\sigma),\quad s_{13}^2=0.023\ (1\sigma)$$

$$\Longrightarrow \delta=\pm0.307,\quad \rho_1=\pi\mp0.2192,\quad \phi=\pm3.129 \tag{3.54}$$

(Δm^2_{sol} and Δm^2_{atm} are taken bfv's.) to generate baryon asymmetry of desired amount $[\left(\frac{n_b}{s}\right)_{max} \simeq 8.59 \times 10^{-11}]$ in case of $M = 3 \cdot 10^3$ GeV and $M_S = 1$ TeV the value $\tan\beta = 6.32$ is required.

$\mathrm{B_1}'$ **Neutrino Texture: Improved Version**

By addition of the d_5 term to (1,3) and (3,1) entries of the $\mathrm{B_1}$ neutrino texture (3.51), the light neutrino mass matrix becomes:

$$M_\nu^{(\mathrm{B_1}')}(M_Z) = \begin{pmatrix} 2a_1b_1 & a_2b_1 & (a_1b_3 + a_3b_1e^{i\phi})r_{\nu 3} + d_5 \\ a_2b_1 & 0 & a_2b_3r_{\nu 3} \\ (a_1b_3 + a_3b_1e^{i\phi})r_{\nu 3} + d_5 & a_2b_3r_{\nu 3} & 2a_3b_3e^{i\phi}r_{\nu 3}^2 \end{pmatrix}\bar{m} \ , \qquad (3.55)$$

which gives all neutrinos massive and opens up a possibility of choosing two variables such as m_3 and $\Delta\rho \equiv \rho_1 - \rho_2$ as independent ones to operate with. We refer to this (3.55) improved version as the $\mathrm{B_1}'$ neutrino texture. From the condition $M_\nu^{(2,2)} = 0$ we have:

$$m_1|U_{21}|^2 = |m_2(U_{22})^2 + m_3(U_{23})^2e^{i\Delta\rho}|, \quad \rho_1 = \pi - \mathrm{Arg}\left[\frac{m_1(U_{21})^2}{m_2(U_{22})^2 + m_3(U_{23})^2e^{i\Delta\rho}}\right], \quad \text{with} \quad \Delta\rho = \rho_1 - \rho_2. \qquad (3.56)$$

Out of the numerous values $\Delta\rho$ and m_3 can take on, we select those that are not in conflict with the observed oscillation data and at the same time together with the minimal allowed value of $\tan\beta$ generate baryon asymmetry of the needed amount. In case of Inverted Hierarchy both of these requirements can be satisfied. In particular:

$$\text{for IH of the } \mathrm{B_1}' \text{ neutrino texture}: \quad m_3 = 0.00250717 \text{ eV} \quad \text{and} \quad \Delta\rho = 3.6599 \qquad (3.57)$$

determine numerical values of the rest of masses, phases and eventually the neutrino double beta decay parameter:

$$(m_1, m_2, m_3) = (0.049714,\ 0.050461,\ 0.00250717) \text{ eV},$$

$$(\delta, \rho_1, \rho_2) = (0.17303,\ 2.9456,\ -0.71436)\,. \qquad (3.58)$$

$$m_{\beta\beta} \simeq 0.019 \text{ eV}. \qquad (3.59)$$

As far as the baryon asymmetry is concerned, using (3.55) in (2.49), we get:

$$\phi = \text{Arg}\left(\frac{\mathcal{A}_{12}^2\mathcal{A}_{33}}{\mathcal{A}_{23}^2\mathcal{A}_{11}}\right),$$

$$a_1 = \frac{1}{2}\left|\frac{\mathcal{A}_{11}}{\mathcal{A}_{12}}\right| a_2,\quad a_3 = \frac{1}{2r_{\nu 3}}\left|\frac{\mathcal{A}_{33}}{\mathcal{A}_{23}}\right| a_2,\quad b_1 = \frac{|\mathcal{A}_{12}|}{|\bar{m}|a_2},\quad b_3 = \frac{|\mathcal{A}_{23}|}{r_{\nu 3}|\bar{m}|a_2}\ . \tag{3.60}$$

Using all these, we can calculate the baryon asymmetry. The results are given in Tab. 17. The goal of attaining needed baryon asymmetry with the minimal allowed value of $\tan\beta$ and without coming in contradiction with the experimental data can be achieved in case of Normal Hierarchy as well by selecting:

$$\text{For NH of the } \mathrm{B_1}' \text{ neutrino texture}: \quad m_3 = 0.0741678 \text{ eV} \quad \text{and} \quad \Delta\rho = 3.2526 \tag{3.61}$$

Case	M(GeV)	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^4\times\xi_\tau$	$10^{11}\times\left(\frac{n_b^f}{s}\right)_{\max}$	$10^{11}\times\left(\frac{n_b}{s}\right)_{\max}$
(I.1)	$3\cdot 10^3$	2.1	0.8928	0.9731	1.118	0.8134	8.57	8.62
(I.2)	10^4	2.135	0.8499	0.9574	1.0986	0.7826	8.55	8.6
(I.3)	10^5	2.332	0.7856	0.9316	1.0545	0.7924	8.56	8.61
(I.4)	10^6	2.559	0.7385	0.9103	1.0209	0.8066	8.56	8.6
(I.5)	10^7	2.822	0.7048	0.8926	0.9959	0.8242	8.54	8.59
(II.1)	$6\cdot 10^3$	2.118	0.875	0.9695	1.0933	0.8109	8.55	8.6
(II.2)	10^4	2.119	0.858	0.9631	1.0876	0.7896	8.56	8.6
(II.3)	10^5	2.302	0.7948	0.9378	1.0481	0.7932	8.56	8.6
(II.4)	10^6	2.524	0.7484	0.9168	1.017	0.8067	8.55	8.59
(II.5)	10^7	2.786	0.715	0.8994	0.9936	0.826	8.55	8.59

Table 17: $\mathrm{B_1}'$ Neutrino Texture, IH. Baryon asymmetry for various values of M and for corresponding minimal (allowed) values of $\tan\beta$. With the choice given in Eqs. (3.57), (3.58) and bfv's of s_{ij}^2. With $\phi = -2.9846$ and for all cases $r_{\nu 3} \simeq 1$.

give:

$$(m_1, m_2, m_3) = (0.05437,\ 0.0550533,\ 0.0741678)\ \text{eV},$$

$$(\delta, \rho_1, \rho_2) = (0.0034537,\ 0.25965,\ -2.9929)\,. \tag{3.62}$$

$$\phi = 2.2568, \qquad m_{\beta\beta} \simeq 0.051\ \text{eV}. \tag{3.63}$$

Case	M(GeV)	$\tan\beta$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^4\times\xi_\tau$	$10^{11}\times\left(\frac{n_b}{s}\right)_{\max}$
(I.1)	$3\cdot 10^3$	12.612	0.9047	0.9764	1.0026	23.596	8.6
(I.2)	10^4	12.081	0.8733	0.9639	0.9929	20.327	8.6
(I.3)	10^5	12.355	0.8229	0.9425	0.9772	18.774	8.6
(I.4)	10^6	12.696	0.7829	0.9236	0.9652	17.364	8.6
(I.5)	10^7	13.066	0.7515	0.9071	0.9566	15.947	8.6
(II.1)	$6\cdot 10^3$	12.608	0.8858	0.9725	0.994	23.269	8.6
(II.2)	10^4	12.158	0.8735	0.9675	0.9904	21.059	8.6
(II.3)	10^5	12.249	0.8253	0.9467	0.9757	18.883	8.6
(II.4)	10^6	12.582	0.787	0.9284	0.9645	17.46	8.6
(II.5)	10^7	12.943	0.7567	0.9122	0.9565	16.029	8.6

Table 18: B_1' Neutrino Texture, NH. Baryon asymmetry for various values of M and for corresponding minimal (allowed) values of $\tan\beta$. With the choice given in Eqs. (3.61), (3.62) and bfv's of s_{ij}^2. With $\phi = 2.2568$ and for all cases $r_{\nu 3}\simeq 1$ and $\frac{\tilde{n}_b}{s}\simeq 0$.

The baryon asymmetries for cases corresponding to this NH scenario are given in Tab. 18.

Texture B_2

This texture is interesting because, due to specific form of Y_ν, the radiative corrections through the λ_τ coupling do not generate cosmological CP asymmetry. Thus λ_μ may be important, which we investigate below. Thus, this model (and its slight modification discussed below) serves as a good demonstration of the role of ξ_μ correction in emergence of needed Baryon asymmetry.

The B_2 Yukawa texture can be written as:

$$\text{Texture }\; B_2: \quad Y_\nu = \begin{pmatrix} a_1 e^{i\alpha_1} & b_1 e^{i\beta_1} \\ a_2 e^{i\alpha_2} & b_2 e^{i\beta_2} \\ a_3 e^{i\alpha_3} & 0 \end{pmatrix} = \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 e^{i\phi} \\ a_3 & 0 \end{pmatrix} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{i\rho} \end{pmatrix},$$

$$\text{with} \quad x=\beta_1-\rho, \quad y=\alpha_2-\alpha_1+\beta_1-\rho, \quad z=\alpha_3-\alpha_1+\beta_1-\rho,$$

$$\omega=\alpha_1-\beta_1+\rho, \quad \phi=\alpha_1-\beta_1-\alpha_2+\beta_2. \tag{3.64}$$

Via the see-saw we will get the light neutrino mass matrix:

$$M_\nu^{(\mathrm{B}_2)}(M_Z) = \begin{pmatrix} 2a_1b_1 & a_1b_2e^{i\phi}+a_2b_1 & a_3b_1r_{\nu 3} \\ a_1b_2e^{i\phi}+a_2b_1 & 2a_2b_2e^{i\phi} & a_3b_2e^{i\phi}r_{\nu 3} \\ a_3b_1r_{\nu 3} & a_3b_2e^{i\phi}r_{\nu 3} & 0 \end{pmatrix} \bar{m} \ . \tag{3.65}$$

This neutrino mass texture (referred as B_2 neutrino texture) works only for inverted neutrino mass ordering [1] (with $m_3 = 0$) and has two predictive relations. In particular, in terms of measured oscillation parameters we can calculate the phases δ and ρ_1. The exact expressions are:

$$\cos\delta = \frac{m_1 t_{12}^2 t_{23}^2 - m_2(t_{23}^2 + t_{12}^2 s_{13}^2)}{2(m_1+m_2)t_{12}t_{23}s_{13}} \ , \qquad \rho_1 = \pi - \mathrm{Arg}\left(\frac{t_{12}t_{23} - s_{13}e^{i\delta}}{t_{23} + t_{12}s_{13}e^{i\delta}}\right)^2 ,$$

$$\text{with} \qquad m_1 = \sqrt{\Delta m_{atm}^2 - \Delta m_{sol}^2}, \qquad m_2 = \sqrt{\Delta m_{atm}^2}, \qquad m_3 = 0. \tag{3.66}$$

From these relations one can easily check that model works only if at least two of the oscillation parameters $\sin^2\theta_{ij}$ are off by several σ's. Taking bfv's of the oscillation parameters would give the absolute values of the r.h.s. of expression for $\cos\delta$ larger than one. Besides this difficulty, proper value of the baryon asymmetry (generated with help of 1-loop correction of λ_μ) requires even more deviation from the bfv's of the oscillation parameters. The root of the problem is that the value of the phase ϕ is fixed so that the parameter $\sin\phi$ (governing cosmological CP asymmetry) turns out to be too suppressed. For instance, with $s_{12}^2 = 0.333$, $s_{23}^2 = 0.388$, $s_{13}^2 = 0.0241$ and bfv's of Δm_{atm}^2, Δm_{sol}^2, for $M = 3\cdot 10^3$ GeV, with $\tan\beta \simeq 68$ and $M_S = 1$ TeV we obtain needed baryon asymmetry $[\left(\frac{n_b}{s}\right)_{max} \simeq 8.56\times 10^{-11}]$, however for this case the values of $\sin^2\theta_{ij}$ are deviated from the bfv's by $(2-3)\sigma$.

$\mathrm{B}_2{}'$ Neutrino Texture: Improved Version

In order to avoid difficulties with B_2 neutrino texture we add d_5 term to the $(1,2)$ and $(2,1)$ elements of the light neutrino mass matrix. After this, the M_ν will have the form:

$$M_\nu^{(\mathrm{B}_2{}')}(M_Z) = \begin{pmatrix} 2a_1b_1 & a_1b_2e^{i\phi}+a_2b_1+d_5 & a_3b_1r_{\nu 3} \\ a_1b_2e^{i\phi}+a_2b_1+d_5 & 2a_2b_2e^{i\phi} & a_3b_2e^{i\phi}r_{\nu 3} \\ a_3b_1r_{\nu 3} & a_3b_2e^{i\phi}r_{\nu 3} & 0 \end{pmatrix} \bar{m} \ . \tag{3.67}$$

With this modification, all masses are non-zero, and therefore two additional parameters $m_3 \neq 0$ and ρ_2 enter. We refer to this (3.67) improved version as the $\mathrm{B}_2{}'$ neutrino texture. Thus our relations will involve two more independent quantities. For convenience we take m_3 and $\Delta\rho = \rho_1 - \rho_2$ as such. From the condition $M_\nu^{(3,3)} = 0$ we have:

$$m_1 \left|U_{31}\right|^2 = \left|m_2 (U_{32})^2 + m_3 (U_{33})^2 e^{i\Delta\rho}\right| , \quad \rho_1 = \pi - \text{Arg}\left[\frac{m_2\left(U_{31}\right)^2}{m_2 (U_{32})^2 + m_3 (U_{33})^2 e^{i\Delta\rho}}\right] \quad \text{with} \quad \Delta\rho = \rho_1 - \rho_2 . \tag{3.68}$$

From these relations the phases δ and ρ_1 can be calculated in terms of m_3 and $\Delta\rho$.

As it turns out, in this improved version the IH case works well for both neutrino sector and the baryon asymmetry. So, we will start with discussing the IH case. For measured oscillation parameters we take the best fit values given in (3.21) and select pairs $(m_3, \Delta\rho)$ in such a way as to get needed baryon asymmetry. One such choice is:

$$m_3 = 0.01406 \text{ eV}, \quad \Delta\rho = 3.5257 \ , \tag{3.69}$$

which with help of (2.53) and (3.68) determine neutrino masses and phases as:

$$(m_1, m_2, m_3) = (0.0516, \ 0.052323, \ 0.01406) \text{ eV},$$
$$(\delta, \rho_1, \rho_2) = (2.8528, \ 3.1385, \ -0.38724) . \tag{3.70}$$

These for the observable $\nu 02\beta$-decay give $m_{\beta\beta} \simeq 0.0193$ eV.

As far as the baryon asymmetry is concerned, using (3.67) in (2.49) for the CP phase ϕ and expressing couplings $a_{2,3}, b_{1,2}$ in terms of a_1 we get

$$\phi = \text{Arg}\left(\frac{\mathcal{A}_{23}^2 \mathcal{A}_{11}}{\mathcal{A}_{13}^2 \mathcal{A}_{22}}\right) ,$$
$$a_2 = \left|\frac{\mathcal{A}_{22}\mathcal{A}_{13}}{\mathcal{A}_{11}\mathcal{A}_{23}}\right| a_1, \quad a_3 = \frac{2}{r_{\nu 3}} \left|\frac{\mathcal{A}_{13}}{\mathcal{A}_{11}}\right| a_1, \quad b_1 = \frac{|\mathcal{A}_{11}|}{2|\bar{m}|a_1}, \quad b_2 = \left|\frac{\mathcal{A}_{23}\mathcal{A}_{11}}{\mathcal{A}_{13}}\right| \frac{1}{2|\bar{m}|a_1} . \tag{3.71}$$

For the values of (3.69), (3.70) and bfv's for the θ_{ij} angles we get

$$\phi = 2.2301 \ . \tag{3.72}$$

With these, and for given values of M and $\tan\beta$ by varying a_1 we can investigate the baryon asymmetry. Results are given in Tab. 19.

As far as the NH case is concerned, the neutrino sector can work well by certain selection of $(m_3, \Delta\rho)$. However, in order to generate needed baryon asymmetry we need to take values of $\sin^2\theta_{ij}$

Case	M(GeV)	$\tan\beta$	$r_{\nu 3}$	$r_{\bar{m}}$	r_{v_u}	κ_N	$10^4\times\xi_\mu$	$10^{11}\times\left(\frac{n_b^f}{s}\right)_{\max}$	$10^{11}\times\left(\frac{n_b}{s}\right)_{\max}$
(**I.1**)	$3\cdot 10^3$	69.256	0.9965	0.9048	0.9763	1.047	4.18	8.55	8.6
(**I.2**)	10^4	67.557	0.9929	0.8728	0.9638	1.0327	3.589	8.55	8.6
(**I.3**)	10^5	67.376	0.9854	0.8196	0.9415	1.0176	3.34	8.55	8.6
(**I.4**)	10^6	67.359	0.9771	0.7749	0.9213	1.006	3.122	8.55	8.6
(**I.5**)	10^7	67.376	0.9681	0.7373	0.9027	0.997	2.903	8.56	8.6
(**II.1**)	$6\cdot 10^3$	70.391	0.9964	0.8858	0.9725	1.0311	4.093	8.55	8.6
(**II.2**)	10^4	69.003	0.9949	0.8735	0.9675	1.0243	3.691	8.55	8.6
(**II.3**)	10^5	68.322	0.9873	0.8234	0.9462	1.0094	3.33	8.55	8.6
(**II.4**)	10^6	68.321	0.979	0.7813	0.9267	0.9988	3.108	8.55	8.6
(**II.5**)	10^7	68.373	0.9699	0.7459	0.909	0.9907	2.889	8.56	8.61

Table 19: ${\rm B_2}'$ Neutrino Texture, IH neutrinos. Baryon asymmetry for various values of M and for corresponding minimal (allowed) values of $\tan\beta$. For the values of (3.69), (3.70) and bfv's of θ_{ij} mixing angles.

deviated from the bfv's by the $(2-3)\sigma$. For example, with $(s_{12}^2, s_{23}^2, s_{13}^2) = (0.27, 0.629, 0.022)$ and $(m_3, \Delta\rho) = (0.060651\ \text{eV}, 3.12)$ we get

$$\text{for NH of the } {\rm B_2}' \text{ neutrino texture :} \qquad (m_1, m_2, m_3) = (0.033671, 0.034764, 0.060651)\ \text{eV},$$

$$(\delta, \rho_1, \rho_2) = (-0.013,\ -0.12393,\ 3.0393) \implies \phi = -2.7538, \quad m_{\beta\beta} \simeq 0.032\ \text{eV}. \tag{3.73}$$

These for $\tan\beta = 68.1$ and $M = 10^6$ GeV, $M_S = 1$ TeV give the baryon asymmetry $\left(\frac{n_b}{s}\right)_{\max} \simeq 8.59 \cdot 10^{-11}$.

Note that the ${\rm B_2}'$ neutrino texture coincides with the texture P_7 of Ref. [20] if all entries in (3.67) are taken to be real. As was shown in [20] the real neutrino mass texture with $M_\nu^{(3,3)} = 0$ will work for both NH and IH neutrinos (see Tab. 6 of Ref. [20]). Advantage of complex $d = 5$ entry [like in texture (3.67)] is that it gives good possibility for generation of the baryon asymmetry with the λ_μ's radiative correction playing the decisive role. For the first time similar possibility has been considered in [17, 18].

Concluding, note also that the A$'$ and ${\rm B_1}'$ neutrino textures are generalizations of the textures P_5 and P_6 (respectively), considered in [20]. The latter two had no complex phases, while A$'$ and ${\rm B_1}'$ scenarios besides good neutrino fits give possibility for the generation of the baryon asymmetry.

3.5 Discussion and Outlook

We have investigated the resonant leptogenesis within the extension of the MSSM by two right handed neutrino superfields with quasi-degenerate masses $\lesssim 10^7$ GeV. It was shown that in this regime the cosmological CP asymmetry arises at one loop level due to charged lepton Yukawa couplings. In particular, needed corrections may come from either of the λ_τ and λ_μ couplings. Which one is relevant from these two couplings depends on the structure of the 3×2 Dirac type Yukawa matrix Y_ν. Aiming to make close connection with the neutrino sector, we first examined all viable neutrino models (considered earlier in Ref. [20]) based on two texture zero Y_ν's augmented by single $\Delta L = 2$, d = 5 operators. This setup is predictive and allows to relate leptonic CP violating phase δ with the cosmological CP violation. In one of such scenarios the role of the λ_μ coupling in CP asymmetry generated at quantum level has been demonstrated. We have also revised the models of Ref. [1] and considered their improved versions by including proper $\Delta L = 2$, d = 5 operators. This allowed to have good fit with the neutrino data and generate needed amount of the baryon asymmetry.

Without specifying their origin, in our considerations we have extensively applied the $\Delta L = 2$, d = 5 operators, of the form given in Eq. (2.25). The d = 5 operator coupling [see Eq. (2.25)] in our case has been directly introduced in the neutrino mass matrices. Here we give one example of possible generation of d = 5 operators we are exploiting within our setup. Besides being of a quantum gravity origin, such d = 5 couplings can be generated from a different sector via renormalizable interactions. For instance, introducing the pair of MSSM singlet states $\mathcal{N}$, $\overline{\mathcal{N}}$ and the superpotential couplings

$$\lambda^{(i)} l_i \mathcal{N} h_u + \bar{\lambda}^{(j)} l_j \overline{\mathcal{N}} h_u - M_* \mathcal{N}\overline{\mathcal{N}} \ ,$$

it is easy to verify that integration of the heavy $\mathcal{N}$, $\overline{\mathcal{N}}$ multiplets leads to the operator in Eq. (2.25) with

$$\tilde{d}_5 e^{ix_5} = 2\lambda^{(i)}\bar{\lambda}^{(j)} \ .$$

Important ingredient here is to maintain forms of the resulting mass matrices and do not mix the states $\mathcal{N}$, $\overline{\mathcal{N}}$ with RHN's $N_{1,2}$. This can be achieved by some (possible flavor) symmetries (which we do not pursue here). Perhaps a safer way to generate those $\Delta L = 2$ effective couplings would be to proceed in a spirit of type II [52], or type III [53] see-saw mechanisms, or exploit alternative possibilities [54, 55] through the introduction of appropriate extra states. Details of such scenarios should be pursued elsewhere.

Throughout our studies we have studied texture zero coupling matrices, but did not attempt to explain and justify considered structures by symmetries. Our approach, being rather phenomenological, was to consider such textures which give predictive and/or consistent scenarios allowing for transparent demonstrations of the suggested mechanism of the loop induced cosmological CP violation. It is desirable to have explanation of texture zeros at more fundamental level, and exploiting flavor symmetries seems to be a good framework.

Since the supersymmetry is a well motivated construction, we have performed our investigations within its framework. However, it would be interesting to examine the considered models also within the non-SUSY setup. For the latter, the scenarios with low $\tan\beta$ look encouraging to start with.

Finally, it would be challenging to embed considered models in Grand Unification (GUT) such as $SU(5)$ and $SO(10)$ GUTs. Due to the high GUT symmetries, additional relations and constraints would emerge making models more predictive.

4 Conclusions

Within the MSSM augmented with two quasi-degenerate right-handed neutrinos all possible two and one texture zero 3×2 Yukawa matrices together with minimal $\mathrm{d} = 5$ operator couplings have been analyzed and their contribution to the light neutrino mass matrices has been thoroughly investigated. All viable neutrino mass matrices have been studied and predictive relations have been derived. Cosmological CP violation has been related to the leptonic CP violating δ phase. Realizations of resonant leptogenesis have been investigated and their consistency with experimental data has been demonstrated.

5 რეზიუმე (Resume in Georgian)

სუსტი, ძლიერი და ელექტრომაგნიტური ურთიერთქმედებების აღმწერი სტანდარტული მოდელი (სმ), მიუხედავად თავისი უაღრესად დიდი წარმატებისა, ვერ ხსნის არანულოვანი მასის მქონე ნეიტრინოების არსებობას და არ იძლევა კოსმოლოგიური CP ასიმეტრიის დამაკმაყოფილებელ ახსნას, როგორც ამ უკანასკნელის წარმოშობის, ისე რიცხვითი მნიშვნელობის თვალსაზრისით. აღნიშნული სირთულეები უკვე რამდენიმე ათეული წლის განმავლობაში წარმოადგენენ ინტენსიური თეორიული და ექსპერიმენტული კვლევების სფეროს. დღეისათვის ექსპერიმენტული და დაკვირვების შედეგები ყველა კითხვაზე პასუხს არ გვცემენ, მაგრამ სიზუსტისა და სიმდიდრის გამო აუცილებლად საჭიროებენ თეორიული მოდელის ფარგლებში ახსნას. სმ-ის ერთერთ ყველაზე საინტერესო გაფართოებას წარმოადგენს მინიმალური სუპერსიმეტრიული სტანდარტული მოდელი (მსსმ), რომლის ორი კვაზი-გადაგვარებული (ხე-მიახლოებაში ზუსტად გადაგვარებული) მარჯვენა ნეიტრინოთი (მნ) განზოგადებით, მივიღეთ და წარმატებით გამოვიყენეთ გარკვეული თანაფარდობები ექსპერიმენტთან თავსებადი ნეიტრინოს მასური მატრიცების მისაღებად და ლეპტონურ CP დამრღვევ δ ფაზასა და კოსმოლოგიურ CP ასიმეტრიას შორის კავშირის დასადგენად. საამისოდ, მსუბუქი ნეიტრინოებისთვის განვიხილეთ ერთი და ორი ტექსტურული ნულის მქონე ყველა შესაძლო იუკავას მატრიცა და შემოვიყვანეთ $\Delta L = 2$ ლეპტონური რიცხვის დამრღვევი, ხუთის ტოლი განზომილების ($\mathrm{d} = 5$) ოპერატორი. ამ ოპერატორს შეაქვს წვლილი მსუბუქი ნეიტრინოს მასურ მატრიცაში. მსსმ-ის ასეთი მოდიფიკაციით ჩვენ:

1) მოვახდინეთ ექსპერიმენტთან თავსებადი მსუბუქი ნეიტრინოს მასური მატრიცების კლასიფიკაცია და მათი შესწავლით გამოვიყვანეთ წინასწარმეტყველების უნარის მქონე ანალიზური გამოსახულებები, რომელთა გამოყენებით მივიღეთ სრული ინფორმაცია მსუბუქი ნეიტრინოს მასების, ლეპტონური CP დამრღვევი δ ფაზისა და მაიორანას ფაზების შესახებ. ასევე, მოვახერხეთ CP დამრღვევი δ ფაზის დაკავშირება თერმული ლეპტოგენეზისის CP ფაზასთან (A. Achelashvili and Z. Tavartkiladze, Int. J. Mod. Phys. A **31**, no. 13, 1650077 (2016).).

2) განვიხილეთ რეზონანსული ლეპტოგენეზისის საკითხი. კვანტური შესწორებების დეტალური შესწავლით, ვაჩვენეთ ლეპტონური ასიმეტრიის წარმოშობის შესაძლებლობა 1-მარყუჟოვან დონეზე და დავადგინეთ, რომ ამ პროცესში გადამწყვეტ როლს თამაშობს ტაუ ლეპტონის იუკავას ბმა, თუმცა ზოგ შემთხვევაში, არსებითი მნიშვნელობა აქვს მიუ ლეპტონის იუკავას ბმას. წინასწარმეტყველების უნარის მქონე კონკრეტულ ნეიტრინოს მოდელზე მოვახდინეთ CP დამრღვევი δ ფაზისა და კოსმოლოგიური CP ასიმეტრიის დაკავშირების შესაძლებლობის დემონსტრირება და ვაჩვენეთ, რომ ბარიონული ასიმეტრიის სასურველი მნიშვნელობა მიიღება რეზონანსული ლეპტოგენეზისის გამოყენებით. წარმოვადგინეთ ჩვენი მოდელის რენორმალიზაციური ულტრაიისფერი სისრულის ერთი მაგალითი და დავამტკიცეთ ყველა მიღებული შედეგის მდგრადობა. ანალიზის მაქსიმალური სიზუსტისთვის ვსარგებლობდით რენორმალიზაციური ჯგუფის განტოლებებზე დამყარებული მკაცრი მათემატიკური მეთოდებით. მარჯვენა სნეიტრინოების—მნ-ის სკალარული პარტნიორების დაშლების გავლენა შევაფასეთ დეტალური კვლევის შედეგად (A. Achelashvili and Z. Tavartkiladze, Phys. Rev. D **96**, no. 1, 015015 (2017).).

3) დავამტკიცეთ, კოსმოლოგიური CP ასიმეტრიის რადიაციული შესწორებებით გაჩენის შესაძლებლობის შესწავლით, რომ ნეიტრინოს გარკვეული ტექსტურებისთვის მხოლოდ λ_μ-ს გათვალისწინება იძლევა კოსმოლოგიურ CP დარღვევას ერთ-მარყუჟოვან მიახლოებაში, თუმცა შემთხვევების უმრავლესობაში გადამწყვეტია λ_τ-ს წვლილი. ექსპერიმენტთან თავსებადი ყველა მასური მატრიცის განხილვისას, მხედველობაში მივიღეთ როგორც λ_τ, ისე λ_μ ბმით განპირობებული რადიაციული შესწორებები, განვიხილეთ მსუბუქი ნეიტრინოსთვის ორი ტექსტურული ნულის მქონე ყველა 3×2 იუკავას მატრიცა და ჩავატარეთ ლეპტოგენეზისის დეტალური ანალიზი. ანალოგიური მიდგომით შევისწავლეთ ერთი ტექსტურული ნულის მქონე დირაკისეული იუკავას მატრიცები და ვაჩვენეთ მიღებული შედეგების თავსებადობა ექსპერიმენტულ მონაცემებთან (A. Achelashvili and Z. Tavartkiladze, Nucl. Phys. B **929**, 21 (2018). A. Achelashvili and Z. Tavartkiladze, AIP Conf.Proc. 1900 (2017) no.1, 020012.).

4) მივიღეთ, ორი (კვაზი) გადაგვარებული მნ-ის სცენარის ფარგლებში, ზოგადი თანაფარდობები CP ასიმეტრიისთვის (შესაბამისი შესწორებებით). მიუხედავად იმისა, რომ ჩვენს ნაშრომში მარყუჟოვანი მიახლოებით განპირობებული CP დარღვევა გამოყენებულ იქნა ტექსტურული ნულების მქონე სპეციფიკურ მოდელებში, მიღებული შედეგების გამოყენება შესაძლებელია ორი (კვაზი) გადაგვარებული მნ-ის შემცველ ნებისმიერ მოდელში.

6 Resume

The Standard Model of weak, strong and electromagnetic interactions (SM), despite its enormous success is incapable of accommodating non-zero masses of neutrinos, neither does it provide satisfactory explanation of the cosmological CP asymmetry in terms of both, its origin and numerical value. These issues, however, have been the subject of intense theoretical and experimental research for several decades. Nowadays, experimental and observational data, though not answering all the questions, but nevertheless rich and precise enough to address various aspects of the theory, is in dire need of accounting for within relevant models. One of the most promising extensions of SM is the Minimal Supersymmetric Standard Model (MSSM), which we augment by two quasi-degenerate (strictly degenerate at tree level) right-handed neutrinos (RHN), thus paving the way for certain predictive relations which have been successfully used in sorting out viable light neutrino mass matrices, as well as establishing the connection of leptonic CP violating phase δ with the cosmological CP asymmetry. Towards this end, we considered all possible two and one texture zero 3×2 light neutrino Yukawa matrices and introduced one $\Delta L = 2$ lepton number violating dimension five (d = 5) operator contributing to the light neutrino mass matrix. Tweaking MSSM this way, we:

1) Classified all experimentally viable light neutrino mass matrices, leading to several predictions and analytically derived predictive relations, thus obtaining all numerical information regarding light neutrino masses, leptonic CP violating phase δ and Majorana phases in each case. We also related the CP violating δ phase to the CP phase of the thermal leptogenesis (A. Achelashvili and Z. Tavartkiladze, Int. J. Mod. Phys. A **31**, no. 13, 1650077 (2016).).

2) Addressed the issue of resonant leptogenesis. Investigating the quantum corrections in details, we showed that the lepton asymmetry is induced at 1-loop level and decisive role is played by the tau lepton Yukawa coupling, although in some cases the mu lepton Yukawa coupling is of crucial importance. On a concrete and predictive neutrino model, which enables to predict the CP vio-

lating δ phase and relate it to the cosmological CP asymmetry, we demonstrated that the needed amount of the baryon asymmetry could be generated via the resonant leptogenesis. We presented one example of renormalizable ultra violet completion of our model and proved the robustness of all obtained results. To make our study as thorough as possible, we extensively used rigorous methods based on RG equations. Impact of the decays of the right-handed sneutrinos—the scalar partners of the RHNs-was estimated through the most detailed investigation (A. Achelashvili and Z. Tavartkiladze, Phys. Rev. D **96**, no. 1, 015015 (2017).).

3) Proved, having studied the rise of cosmological CP asymmetry by radiative corrections through the charged lepton Yukawa couplings, that in specific neutrino textures only inclusion of the λ_μ generates cosmological CP violation at 1-loop level. In most cases, however, decisive role is played by the λ_τ coupling. In each case of experimentally favored light neutrino mass matrices we took into account radiative corrections induced by both, λ_μ and λ_τ couplings, considered all two texture zero 3×2 Dirac Yukawa matrices of neutrinos and performed detailed analysis of leptogenesis. We applied the same approach to one texture zero Dirac Yukawa matrices as well and showed compatibility of obtained results with current experimental data (A. Achelashvili and Z. Tavartkiladze, Nucl. Phys. B **929**, 21 (2018). A. Achelashvili and Z. Tavartkiladze, AIP Conf.Proc. 1900 (2017) no.1, 020012.).

4) Obtained, within the scenarios with two (quasi) degenerate RHNs, the general expressions for CP asymmetry (with corresponding corrections). Although in our work obtained results of the loop induced cosmological CP violation have been used for specific texture zero models (see Refs. [16–18]), the application can be extended to any model with two (quasi) degenerate right handed neutrinos.

A Renormalization Group Studies

A.1 Running of Y_ν, Y_e and M_N Matrices

RG equations for the charged lepton and neutrino Dirac Yukawa matrices, appearing in the superpotential of Eq. (2.1), at 1-loop order have the forms [56, 57]:

$$16\pi^2 \frac{d}{dt} Y_e = 3Y_e Y_e^\dagger Y_e + Y_\nu Y_\nu^\dagger Y_e + Y_e \left[\mathrm{tr} \left(3Y_d^\dagger Y_d + Y_e^\dagger Y_e \right) - c_e^a g_a^2 \right] , \qquad c_e^a = (\frac{9}{5}, 3, 0), \tag{A.1}$$

$$16\pi^2 \frac{d}{dt} Y_\nu = Y_e Y_e^\dagger Y_\nu + 3Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu \left[\mathrm{tr} \left(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu \right) - c_\nu^a g_a^2 \right] , \qquad c_\nu^a = (\frac{3}{5}, 3, 0). \tag{A.2}$$

$g_a = (g_1, g_2, g_3)$ denote gauge couplings of $U(1)_Y, SU(2)_w$ and $SU(3)_c$ gauge groups respectively. Their 1-loop RG have forms $16\pi^2 \frac{d}{dt} g_a = b_a g_a^3$, with $b_a = (\frac{33}{5}, 1, -3)$, where the hypercharge of $U(1)_Y$ is taken in $SU(5)$ normalization.

The RG for the RHN mass matrix at 2-loop level has the form [57]:

$$16\pi^2 \frac{d}{dt} M_N = 2M_N Y_\nu^\dagger Y_\nu - \frac{1}{8\pi^2} M_N \left[Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu + Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu \,\mathrm{tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) \right]$$

$$+ \frac{1}{8\pi^2} M_N Y_\nu^\dagger Y_\nu \left(\frac{3}{5} g_1^2 + 3g_2^2 \right) + (\mathrm{transpose}) \ , \qquad (A.3)$$

Let's start with renormalization of the Y_ν's matrix elements. Ignoring in Eq. (A.2) the $\mathcal{O}(Y_\nu^3)$ order entries (which are very small because within our studies $|(Y_\nu)_{ij}| \lesssim 10^{-4}$), and from charged fermion Yukawas keeping λ_τ, λ_μ, λ_t and λ_b, we will have:

$$16\pi^2 \frac{d}{dt} \ln(Y_\nu)_{ij} \simeq \delta_{i3}\lambda_\tau^2 + \delta_{i2}\lambda_\mu^2 + 3\lambda_t^2 - c_\nu^a g_a^2 \ . \qquad (A.4)$$

This gives the solution

$$(Y_\nu)_{ij}(\mu) = (Y_{\nu G})_{ij} (\eta_\tau(\mu))^{\delta_{i3}} (\eta_\mu(\mu))^{\delta_{i2}} \eta_t^3(\mu) \eta_{g\nu}(\mu), \qquad (A.5)$$

where $Y_{\nu G}$ denotes Yukawa matrix at scale M_G and the scale dependent RG factors are given by:

$$\eta_{t,b,\tau,\mu}(\mu) = \exp\left(-\frac{1}{16\pi^2} \int_t^{t_G} \lambda_{t,b,\tau,\mu}^2(t')dt' \right) , \quad \eta_a(\mu) = \exp\left(\frac{1}{16\pi^2} \int_t^{t_G} g_a^2(t')dt' \right)$$

$$\eta_{g\nu}(\mu) = \exp\left(\frac{1}{16\pi^2} \int_t^{t_G} c_\nu^a g_a^2(t')dt' \right) = \eta_1^{3/5}(\mu)\eta_2^3(\mu), \quad \text{with} \quad t = \ln\mu \ , \ t' = \ln\mu' \ , \ t_G = \ln M_G. \qquad (A.6)$$

From these, for the combination $Y_\nu^\dagger Y_\nu$ at scale $\mu = M$ we get expression given in Eq. (3.14).

On the other hand, for the RHN mass splitting and for the phase mismatch [depending on $\xi_{\tau,\mu}$ defined in Eq. (3.15)], the integrals/factors of Eqs. (3.11), (3.12), (3.13) and (3.14) will be relevant.

A.2 Relating $M_\nu(M_Z)$ and $M_\nu(M)$

Details of derivations, of the results presented in this subsection, are given in Appendix A.2 of Ref. [16]. At scale M, after decoupling of the RHN states, the neutrino mass matrix is generated and has the form:

$$M_\nu^{ij}(M) = -\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \frac{v_u^2(M)}{Me^{-i(\omega+\rho)}} \ , \qquad (A.7)$$

where '×' stand for entries depending on Yukawa couplings. After renormalization, keeping λ_τ, λ_t, λ_b and g_a in the RGs, the neutrino mass matrix at scale M_Z has the form:

$$M_\nu^{ij}(M_Z) = \begin{pmatrix} \times & \times & (\times)\cdot r_{\nu 3} \\ \times & \times & (\times)\cdot r_{\nu 3} \\ (\times)\cdot r_{\nu 3} & (\times)\cdot r_{\nu 3} & (\times)\cdot r_{\nu 3}^2 \end{pmatrix} \bar{m} \ , \tag{A.8}$$

with $\bar{m}$ given in Eq. (3.23) and × in Eq. (A.8) denotes entries determined at scale M and corresponding to those in (A.7), and RG factors $r_{\nu 3}$, $r_{\bar{m}}$ are given respectively in Eqs. (A.17), (A.18) of Ref. [16]:

$$r_{\nu 3} = \left(\frac{\eta_\tau(t_Z)}{\eta_\tau(t_{M_S})}\right)^{1/2} \left(\frac{\eta_\tau(t_{M_S})}{\eta_\tau(t_M)}\right) \ , \tag{A.9}$$

$$r_{\bar{m}} = \eta_\lambda^4 \left(\frac{\eta_t(t_{m_t})}{\eta_t(t_M)}\right)^{12} \left(\frac{\eta_b(t_Z)}{\eta_b(t_{M_S})}\right)^{12} \left(\frac{\eta_\tau(t_Z)}{\eta_\tau(t_{M_S})}\right)^4 \left(\frac{\eta_2(t_Z)}{\eta_2(t_M)}\right)^{\frac{15}{2}} \left(\frac{\eta_1^{3/5}(t_Z)\eta_1^{2/5}(t_{M_S})}{\eta_1(t_M)}\right)^{\frac{3}{2}} \ , \tag{A.10}$$

where

$$\eta_\lambda = \exp\left(-\frac{1}{16\pi^2}\int_{t_{m_h}}^{t_{M_S}} \lambda(t)dt\right) \ , \tag{A.11}$$

and remaining η-factors are defined in Eq. (A.6).

We will also need the RG factor relating the VEV $v_u(M)$ to the $v(M_Z)$. Thus we define:

$$r_{v_u} = \frac{v_u(M)}{v(M_Z)s_\beta} \ . \tag{A.12}$$

Analytic expression for r_{v_u} derived from appropriate RGs is given by Eq. (A.20) of Ref. [16]:

$$r_{v_u} = \frac{v_u(M)}{v(M_Z)s_\beta} = \left(\frac{\eta_t(t_{m_t})}{\eta_t(t_M)}\right)^3 \left(\frac{\eta_b(t_Z)}{\eta_b(t_{M_S})}\right)^3 \left(\frac{\eta_\tau(t_Z)}{\eta_\tau(t_{M_S})}\right) \left(\frac{\eta_2^3(t_Z)\eta_2^{-2}(t_{M_S})}{\eta_2(t_M)}\right)^{\frac{3}{4}} \left(\frac{\eta_1^3(t_Z)\eta_1^{-2}(t_{M_S})}{\eta_1(t_M)}\right)^{\frac{3}{20}} \ . \tag{A.13}$$

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A.3 Calculation Procedure and Used Schemes

To find the RG factors, appearing in the baryon asymmetry and in the neutrino mass matrix renormalization, we numerically solve renormalization group equations from the scale M_Z up to the $M_G \simeq 2 \cdot 10^{16}$ GeV scale. For simplicity, for all SUSY particle masses we take common mass scale M_S. Thus, in the energy interval $M_Z \leq \mu < M_S$, the Standard Model RGs for $\overline{\rm MS}$ coupling constants are used. However, in the interval $M_S \leq \mu \leq M_G$, since we are dealing with the SUSY, the RGs for the $\overline{\rm DR}$ couplings are applied. Below we give boundary and matching conditions for the gauge couplings $g_{1,2,3}$, for Yukawa constants $\lambda_{t,b,\tau,\mu}$ and for the Higgs self-coupling λ.

Gauge couplings $\alpha_a = \frac{g_a^2}{4\pi}$

We choose our inputs for the $\overline{\rm MS}$ gauge couplings at scale M_Z as follows:

$$\alpha_1^{-1}(M_Z) = \frac{3}{5}c_w^2\alpha_{em}^{-1}(M_Z) + \frac{3}{5}c_w^2\frac{8}{9\pi}\ln\frac{m_t}{M_Z} \ , \qquad \alpha_2^{-1}(M_Z) = s_w^2\alpha_{em}^{-1}(M_Z) + s_w^2\frac{8}{9\pi}\ln\frac{m_t}{M_Z} \ ,$$

$$\alpha_3^{-1}(M_Z) = \alpha_s^{-1}(M_Z) + \frac{1}{3\pi}\ln\frac{m_t}{M_Z} \ , \tag{A.14}$$

where logarithmic terms $\ln\frac{m_t}{M_Z}$ are due to the top quark threshold correction [58, 59]. Taking $\alpha_s(M_Z) = 0.1185$, $\alpha_{em}^{-1}(M_Z) = 127.934$ and $s_w^2 = 0.2313$, from (A.14) we obtain:

$$\alpha_1^{-1}(M_Z) = 59.0057 + \frac{8c_w^2}{15\pi}\ln\frac{m_t}{M_Z} \ , \quad \alpha_2^{-1}(M_Z) = 29.5911 + \frac{8s_w^2}{9\pi}\ln\frac{m_t}{M_Z} \ ,$$

$$\alpha_3^{-1}(M_Z) = 8.4388 + \frac{1}{3\pi}\ln\frac{m_t}{M_Z} \ . \tag{A.15}$$

With these inputs we run $g_{1,2,3}$ via the 2-loop RGs from M_Z up to the scale M_S.

At scale $\mu = M_S$ we use the matching conditions between $\overline{\rm DR} - \overline{\rm MS}$ gauge couplings [60, 61]:

$$\text{at } \mu = M_S: \quad \frac{1}{\alpha_1^{\overline{\rm DR}}} = \frac{1}{\alpha_1^{\overline{\rm MS}}} \ , \qquad \frac{1}{\alpha_2^{\overline{\rm DR}}} = \frac{1}{\alpha_2^{\overline{\rm MS}}} - \frac{1}{6\pi} \ , \qquad \frac{1}{\alpha_3^{\overline{\rm DR}}} = \frac{1}{\alpha_3^{\overline{\rm MS}}} - \frac{1}{4\pi} \ . \tag{A.16}$$

Above the scale M_S we apply 2-loop SUSY RG equations in $\overline{\rm DR}$ scheme [56].

Yukawa Couplings and λ

At the scale M_S all SUSY states decouple and we are left with the Standard Model with one Higgs doublet. Thus, Yukawa couplings we are considering and the self-coupling are determined as:

$$\lambda_t(m_t) = \frac{m_t(m_t)}{v(m_t)} \ , \quad \lambda_b(M_Z) = \frac{2.89\text{GeV}}{v(M_Z)} \ , \quad \lambda_\tau(M_Z) = \frac{1.746\text{GeV}}{v(M_Z)} \ , \ \lambda_\mu(M_Z) = \frac{0.1027\text{GeV}}{v(M_Z)},$$

$$\lambda(m_h) = \frac{1}{4}\left(\frac{m_h}{v(m_h)}\right)^2, \qquad \text{with} \quad v(M_Z) = 174.1 \text{ GeV} \ , \quad m_h = 125.15 \text{ GeV} \ , \tag{A.17}$$

where $m_t(m_t)$ is the top quark running mass related to the pole mass as:

$$m_t(m_t) = p_t M_t^{pole} \ . \tag{A.18}$$

The factor p_t is $p_t \simeq 1/1.0603$ [62], while the recent measured value of the top's pole mass is [63]:

$$M_t^{pole} = (173.34 \pm 0.76)\ \text{GeV}. \tag{A.19}$$

We take the values of (A.17) as boundary conditions for solving 2-loop RG equations [59, 64] for $\lambda_{t,b,\tau,\mu}$ and λ from the M_Z scale up to the scale M_S.

Above the M_S scale, we have MSSM states including two doublets h_u and h_d, which couple with up type quarks and down type quarks/charged leptons respectively. Thus, Yukawa couplings we are considering at M_S are $\approx \lambda_t(M_S)/s_\beta, \lambda_b(M_S)/c_\beta$ and $\lambda_{\tau,\mu}(M_S)/c_\beta$, with $s_\beta \equiv \sin\beta, c_\beta \equiv \cos\beta$. Above the scale M_S we apply 2-loop SUSY RG equations in $\overline{\text{DR}}$ scheme [56]. Thus, at $\mu = M_S$ we use the matching conditions between $\overline{\text{DR}} - \overline{\text{MS}}$ couplings:

$$\text{at } \mu = M_S: \qquad \lambda_t^{\overline{\text{DR}}} \simeq \frac{\lambda_t^{\overline{\text{MS}}}}{s_\beta}\left[1 + \frac{1}{16\pi^2}\left(\frac{g_1^2}{120} + \frac{3g_2^2}{8} - \frac{4g_3^2}{3}\right)\right],$$

$$\lambda_b^{\overline{\text{DR}}} \simeq \frac{\lambda_b^{\overline{\text{MS}}}}{c_\beta}\left[1+\frac{1}{16\pi^2}\left(\frac{13g_1^2}{120} + \frac{3g_2^2}{8} - \frac{4g_3^2}{3}\right)\right], \quad \lambda_{\tau,\mu}^{\overline{\text{DR}}} \simeq \frac{\lambda_{\tau,\mu}^{\overline{\text{MS}}}}{c_\beta}\left[1+\frac{1}{16\pi^2}\left(-\frac{9g_1^2}{40} + \frac{3g_2^2}{8}\right)\right], \tag{A.20}$$

where expressions in brackets of r.h.s. of the relations are due to the $\overline{\text{DR}} - \overline{\text{MS}}$ conversions [61]. With Eq. (A.20)'s matchings we run corresponding couplings from the scale M_S up to the M_G scale. Throughout this work, above the mass scale M_S without using the superscript $\overline{\text{DR}}$ we assume the couplings determined in this scheme.

B Baryon Asymmetry from RHS Decays

In this appendix we give details of the contribution to the net baryon asymmetry from the right handed sneutrinos (RHS) - the scalar partners of the RHNs. Estimation of this contribution for specific textures was given in [1], while more detailed investigation was given in [16] (from the lepton couplings taking into account only λ_τ and A_τ in the proper RGs). Since we have seen that for some cases for the cosmological CP asymmetry decisive is the RG correction via the λ_μ Yukawa coupling, here we extend its calculation by taking into account also effects from λ_μ and A_μ into the asymmetry generated by the RHS decays.

We will consider soft SUSY breaking scalar potential

$$V_{SB}^{\nu} = \tilde{l}^T A_\nu \tilde{N} h_u - \frac{1}{2}\tilde{N}^T B_N \tilde{N} + \text{h.c.} + \tilde{l}^\dagger m_{\tilde{l}}^2 \tilde{l} + \tilde{N}^\dagger m_{\tilde{N}}^2 \tilde{N}\ , \tag{B.1}$$

which will be relevant for deriving RHS masses and their couplings to the components of the l and h_u superfields. Using general expressions of Ref. [56] we write down 1-loop RGs for A_ν and B_N, which have the forms:

$$16\pi^2\frac{d}{dt}A_\nu = Y_eY_e^\dagger A_\nu + 2\hat{A}_eY_e^\dagger Y_\nu + 5Y_\nu Y_\nu^\dagger A_\nu + A_\nu\left[\mathrm{tr}(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) + 4Y_\nu^\dagger Y_\nu - c_\nu^a g_a^2\right]$$

$$+ 2Y_\nu\left[\mathrm{tr}(3Y_u^\dagger \hat{A}_u + Y_\nu^\dagger A_\nu) + c_\nu^a g_a^2 M_{\tilde{V}_a}\right] , \qquad (B.2)$$

$$16\pi^2\frac{d}{dt}B_N = 2B_NY_\nu^\dagger Y_\nu + 2Y_\nu^TY_\nu^* B_N + 4M_NY_\nu^\dagger A_\nu + 4A_\nu^TY_\nu^* M_N \ . \qquad (B.3)$$

We parameterize the matrices B_N and A_ν as:

$$B_N = (M_N)_{12}m_B\begin{pmatrix} \delta_{BN}^{(1)} & 1 \\ 1 & \delta_{BN}^{(2)} \end{pmatrix} , \qquad A_\nu = m_A a_\nu \ , \qquad (B.4)$$

where entries $(M_N)_{12}, m_B$, $\delta_{BN}^{(1,2)}$ and elements of the matrix a_ν run (their RGs can be derived from the RG equations given above), while m_A is a constant. The matrix $\hat{A}_e$ (similar to the structure of Y_e Yukawa matrix) is

$$\hat{A}_e = \mathrm{Diag}\,(A_e, A_\mu, A_\tau)\,. \qquad (B.5)$$

Assuming proportionality / alignment of the soft SUSY breaking terms and corresponding superpotential couplings, we will use the following boundary conditions:

$$\text{at } \mu = M_G: \qquad a_\nu = Y_\nu \ , \qquad \delta_{BN}^{(1)} = \delta_{BN}^{(2)} = 0, \quad \hat{A}_e = m_A\mathrm{Diag}\,(\lambda_e, \lambda_\mu, \lambda_\tau)$$

$$\hat{A}_u = m_AY_{uG} \ , \qquad \hat{A}_d = m_AY_{dG} \ . \qquad (B.6)$$

Using (B.3) for B_N's entries in (B.4) we have:

$$16\pi^2\frac{d}{dt}\delta_{BN}^{(1)} \simeq 4(Y_\nu^\dagger Y_\nu)_{21} + 8\frac{m_A}{m_B}(Y_\nu^\dagger a_\nu)_{21} \ , \qquad 16\pi^2\frac{d}{dt}\delta_{BN}^{(2)} \simeq 4(Y_\nu^\dagger Y_\nu)_{12} + 8\frac{m_A}{m_B}(Y_\nu^\dagger a_\nu)_{12}. \qquad (B.7)$$

For the elements of a_ν we have

$$16\pi^2\frac{d}{dt}\left(\frac{(a_\nu)_{ij}}{(Y_\nu)_{ij}}\right) \simeq 2\frac{1}{m_A}(\delta_{i3}\lambda_\tau A_\tau + \delta_{i2}\lambda_\mu A_\mu) + \frac{2}{m_A}(3\lambda_tA_t + c_\nu^a g_a^2 M_{\tilde{V}_a}) \ , \qquad (B.8)$$

which show the violation of the alignment between a_ν and Y_ν due to RG effects. At r.h.s. of (B.8) we kept $\lambda_{\mu,\tau,t}, A_{\mu,\tau,t}$, gauge couplings and gaugino masses. From this we derive

$$a_\nu \simeq \begin{pmatrix} 1+\epsilon_0 & 0 & 0 \\ 0 & 1+\epsilon_0+\epsilon_\mu & 0 \\ 0 & 0 & 1+\epsilon_0+\epsilon_\tau \end{pmatrix} Y_\nu$$

$$\text{with} \quad \epsilon_0 = -\frac{1}{8\pi^2 m_A}\int_t^{t_G} dt(3\lambda_t A_t + c_\nu^a g_a^2 M_{\tilde{V}_a}) \ , \qquad \epsilon_{\mu,\tau} = -\frac{1}{8\pi^2 m_A}\int_t^{t_G} dt \lambda_{\mu,\tau} A_{\mu,\tau} \ . \tag{B.9}$$

Keeping in mind that the powers of the Y_ν couplings can be ignored due to their smallness, the m_B can be treated as a constant, and from (B.9), (B.7), (B.4) we obtain:

$$\text{at } \mu = M: \quad B_N = m_B M \begin{pmatrix} -\alpha\delta_N(1+\bar{\epsilon}_1) & 1 \\ 1 & -\alpha\delta_N^*(1+\bar{\epsilon}_2) \end{pmatrix}, \quad \alpha = 1 + 2\frac{m_A}{m_B} \tag{B.10}$$

and

$$\bar{\epsilon}_1 = \frac{1}{4\pi^2\alpha\delta_N}\int_{t_M}^{t_G} dt \left(Y_\nu^\dagger (\frac{\alpha}{16\pi^2} Y_e Y_e^\dagger + 2\frac{m_A}{m_B}\hat{\epsilon})Y_\nu\right)_{21}, \quad \bar{\epsilon}_2 = \frac{1}{4\pi^2\alpha\delta_N^*}\int_{t_M}^{t_G} dt \left(Y_\nu^\dagger (\frac{\alpha^*}{16\pi^2} Y_e Y_e^\dagger + 2\frac{m_A^*}{m_B^*}\hat{\epsilon}^*)Y_\nu\right)_{21}^* ,$$

$$\text{with} \quad \hat{\epsilon} = \text{Diag}\left(\epsilon_0, \ \epsilon_0 + \epsilon_\mu, \ \epsilon_0 + \epsilon_\tau\right). \tag{B.11}$$

The form of B_N given in Eq. (B.10) will be used to construct the RHS mass matrix. Before doing this, using Eq. (A.5) and ignoring the coupling λ_e (as it turns out from the lepton Yukawa couplings all relevant effects are due to $\lambda_{\mu,\tau}$), for $\bar{\epsilon}_{1,2}$ at scale $\mu = M$ we can get expressions:

$$\bar{\epsilon}_1(M) = \frac{1}{4\pi^2\alpha\delta_N}(Y_\nu^\dagger \hat{K} Y_\nu)_{21}\Big|_{\mu=M}, \qquad \bar{\epsilon}_2(M) = \frac{1}{4\pi^2\alpha\delta_N^*}(Y_\nu^T \hat{K} Y_\nu^*)_{21}\Big|_{\mu=M}$$

$$\text{with} \quad \hat{K} = \frac{1}{\eta_t^6 \eta_{g\nu}^2}\text{Diag}\left[2\frac{m_A}{m_B}I_0 \ , \ \frac{1}{\eta_\mu^2}\left(2\frac{m_A}{m_B}I_1^{(\mu)} + \frac{\alpha}{16\pi^2}I_2^{(\mu)}\right) \ , \ \frac{1}{\eta_\tau^2}\left(2\frac{m_A}{m_B}I_1^{(\tau)} + \frac{\alpha}{16\pi^2}I_2^{(\tau)}\right)\right],$$

$$I_0 = \int_{t_M}^{t_G} dt \eta_t^6 \eta_{g\nu}^2 \epsilon_0 \ , \quad I_1^{(\mu,\tau)} = \int_{t_M}^{t_G} dt \eta_t^6 \eta_{g\nu}^2 (\epsilon_0 + \epsilon_{\mu,\tau})\eta_{\mu,\tau}^2 \ , \quad I_2^{(\mu,\tau)} = \int_{t_M}^{t_G} dt \eta_t^6 \eta_{g\nu}^2 \lambda_{\mu,\tau}^2 \eta_{\mu,\tau}^2. \tag{B.12}$$

Keeping the B_N-term in (B.1) and including the mass^2 term $\tilde{N}^\dagger M_N^\dagger M_N \tilde{N}$ coming from the superpotential, the quadratic (with respect to $\tilde{N}$'s) potential will be:

$$V_{\tilde{N}}^{(2)} = \tilde{N}^\dagger M_N^\dagger M_N \tilde{N} - \left(\frac{1}{2}\tilde{N}^T B_N \tilde{N} + \text{h.c.}\right) \ . \tag{B.13}$$

With the transformation of the N superfields $N = U_N N'$ (according to Eq. (3.4), the U_N diagonalizes the fermionic RHN mass matrix), we obtain:

$$V_{\tilde{N}}^{(2)} = \tilde{N}'^\dagger (M_N^{Diag})^2 \tilde{N}' - \left(\frac{1}{2}\tilde{N}'^T U_N^T B_N U_N \tilde{N}' + \text{h.c.}\right). \tag{B.14}$$

With phase redefinition

$$\tilde{N}' = \tilde{P}_1 \tilde{N}'' \ , \qquad \tilde{P}_1 = \text{Diag}\left(e^{-i\tilde{\omega}_1/2}, e^{-i\tilde{\omega}_2/2}\right) \ , \qquad \text{with} \quad \tilde{\omega}_{1,2} = \text{Arg}[m_B(1 \mp \tilde{\alpha}|\delta_N|)] \tag{B.15}$$

and by going to the real scalar components

$$\tilde{N}''_1 = \frac{1}{\sqrt{2}}(\tilde{N}_1^R + i\tilde{N}_1^I) \ , \qquad \tilde{N}''_2 = \frac{1}{\sqrt{2}}(\tilde{N}_2^R + i\tilde{N}_2^I) \ , \tag{B.16}$$

and using (B.10), we will have:

$$-\left(\frac{1}{2}\tilde{N}'^T U_N^T B_N U_N \tilde{N}' + \text{h.c.}\right) = -\frac{|Mm_B|}{2}\,|1-\tilde{\alpha}|\delta_N||\left((\tilde{N}_1^R)^2 - (\tilde{N}_1^I)^2\right)$$

$$-\frac{|Mm_B|}{2}\,|1+\tilde{\alpha}|\delta_N||\left((\tilde{N}_2^R)^2 - (\tilde{N}_2^I)^2\right) - |M|\text{Re}(m_B\delta_\epsilon)\left(\tilde{N}_1^R\tilde{N}_2^R - \tilde{N}_1^I\tilde{N}_2^I\right) + |M|\text{Im}(m_B\delta_\epsilon)\left(\tilde{N}_1^I\tilde{N}_2^R + \tilde{N}_1^R\tilde{N}_2^I\right)$$

$$\text{with} \quad \tilde{\alpha} = \alpha(1 + \frac{\bar{\epsilon}_1 + \bar{\epsilon}_2}{2}), \quad \delta_\epsilon = i\alpha|\delta_N|\frac{\bar{\epsilon}_1 - \bar{\epsilon}_2}{2}e^{-i(\tilde{\omega}_1+\tilde{\omega}_2)/2} \ . \tag{B.17}$$

From (B.14) and (B.17) we obtain the mass^2 terms:

$$V_{\tilde{N}}^{(2)} = \frac{1}{2}\tilde{n}^{0T} M_{\tilde{n}}^2 \tilde{n}^0 \ , \quad \text{with} \quad \tilde{n}^{0T} = \left(\tilde{N}_1^R, \tilde{N}_1^I, \tilde{N}_2^R, \tilde{N}_2^I\right) \tag{B.18}$$

and

$$M_{\tilde{n}}^2 = \begin{pmatrix} (\tilde{M}_1^0)^2 & 0 & -|M|\text{Re}(m_B\delta_\epsilon) & |M|\text{Im}(m_B\delta_\epsilon) \\ 0 & (\tilde{M}_2^0)^2 & |M|\text{Im}(m_B\delta_\epsilon) & |M|\text{Re}(m_B\delta_\epsilon) \\ -|M|\text{Re}(m_B\delta_\epsilon) & |M|\text{Im}(m_B\delta_\epsilon) & (\tilde{M}_3^0)^2 & 0 \\ |M|\text{Im}(m_B\delta_\epsilon) & |M|\text{Re}(m_B\delta_\epsilon) & 0 & (\tilde{M}_4^0)^2 \end{pmatrix} \tag{B.19}$$

where

$$(\tilde{M}_1^0)^2 = |M|^2(1-|\delta_N|)^2 - |m_BM|\,|1-\tilde{\alpha}|\delta_N||\,, \quad (\tilde{M}_2^0)^2 = |M|^2(1-|\delta_N|)^2 + |m_BM|\,|1-\tilde{\alpha}|\delta_N|| \ ,$$

$$(\tilde{M}_3^0)^2 = |M|^2(1+|\delta_N|)^2 - |m_BM|\,|1+\tilde{\alpha}|\delta_N||\,, \quad (\tilde{M}_4^0)^2 = |M|^2(1+|\delta_N|)^2 + |m_BM|\,|1+\tilde{\alpha}|\delta_N|| \tag{B.20}$$

The coupling of $\tilde{n}^0$ states with the fermions emerges from the F-term of the superpotential $l^TY_\nu N h_u$. Following the transformations, indicated above, we will have:

$$(l^TY_\nu N h_u)_F \to \tilde{h}_u l^T Y_\nu \tilde{N} = e^{-i\tilde{\omega}_2/2}\tilde{h}_u l^T Y_\nu U_N \left(\rho_u e^{i(\tilde{\omega}_2-\tilde{\omega}_1)/2}, \rho_d\right)\tilde{n}^0 \ ,$$

$$\text{with} \quad \rho_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix} \ , \quad \rho_d = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & i \end{pmatrix} \ . \tag{B.21}$$

Diagonalizing the matrix (B.19) by the transformation

$$V_{\tilde{n}}^T M_{\tilde{n}}^2 V_{\tilde{n}} = (M_{\tilde{n}}^{Diag})^2, \quad \tilde{n}^0 = V_{\tilde{n}}\tilde{n}, \tag{B.22}$$

the fermion coupling with the scalar $\tilde{n}$ mass eigenstates will be

$$\tilde{h}_u l^T Y_F \tilde{n} \qquad \text{with} \quad Y_F = Y_\nu \tilde{V}^0 V_{\tilde{n}} \ , \quad \tilde{V}^0 = U_N \left(\rho_u e^{-i\tilde{\omega}_1/2}, \rho_d e^{-i\tilde{\omega}_2/2}\right) \ . \tag{B.23}$$

The coupling with the slepton $\tilde{l}$ is derived from the interaction term $h_u \tilde{l}^T \left(Y_\nu M_N^* \tilde{N}^* - A_\nu \tilde{N}\right)$. Going from $\tilde{N}$ to the $\tilde{n}$ states, one obtains:

$$h_u \tilde{l}^T Y_B \tilde{n} \quad \text{with} \quad Y_B = (Y_\nu M_N^* \tilde{V}^{0*} - A_\nu \tilde{V}^0) V_{\tilde{n}} \ . \tag{B.24}$$

For given values of M, m_B and m_A, with help of Eqs. (B.19), (B.23) and (B.24), we will have coupling matrices Y_F, Y_B and all other quantities needed for calculation of the baryon asymmetry created via the decays of the $\tilde{n}_{1,2,3,4}$ states.

B.1 Calculating $\frac{n_b}{s}$ - Asymmetry Via $\tilde{n}$ Decays

Due to the SUSY breaking terms, the masses of RHS's differ from their fermionic partners' masses. For each mass-eigenstate RHS's $\tilde{n}_{i=1,2,3,4}$ we have one of the masses $\tilde{M}_{i=1,2,3,4}$ respectively. With the SUSY M_S scale $\frac{M_S}{M} \lesssim 1/3$, the states $\tilde{n}_i$ remain nearly degenerate and for the resonant $\tilde{n}$-decays the resummed effective amplitude technique [10] will be applied. Effective amplitudes for the real $\tilde{n}_i$ decay, say into the lepton l_α ($\alpha = 1, 2, 3$) and antilepton $\bar{l}_\alpha$ respectively are given by [10]

$$\hat{S}_{\alpha i} = S_{\alpha i} - \sum_j S_{\alpha j} \frac{\Pi_{ji}(\tilde{M}_i)(1 - \delta_{ij})}{\tilde{M}_i^2 - \tilde{M}_j^2 + \Pi_{jj}(\tilde{M}_i)} \ , \quad \hat{\bar{S}}_{\alpha i} = S_{\alpha i}^* - \sum_j S_{\alpha j}^* \frac{\Pi_{ji}(\tilde{M}_i)(1 - \delta_{ij})}{\tilde{M}_i^2 - \tilde{M}_j^2 + \Pi_{jj}(\tilde{M}_i)} \ , \tag{B.25}$$

where $S_{\alpha i}$ is a tree level amplitude and Π_{ij} is a two point Green function's (polarization operator of $\tilde{n}_i - \tilde{n}_j$) absorptive part. The CP asymmetry is then given by

$$\epsilon_i^{sc} = \frac{\sum_\alpha \left(|\hat{S}_{\alpha i}|^2 - |\hat{\bar{S}}_{\alpha i}|^2\right)}{\sum_\alpha \left(|\hat{S}_{\alpha i}|^2 + |\hat{\bar{S}}_{\alpha i}|^2\right)} \ . \tag{B.26}$$

With Y_F and Y_B given by Eqs. (B.23) and (B.24) we can calculate polarization diagram's (with external legs $\tilde{n}_i$ and $\tilde{n}_j$) absorptive part Π_{ij}. These at 1-loop level are given by:

$$\Pi_{ij}(p) = \frac{ip^2}{8\pi}\left(1 - \frac{M_S^2}{p^2}\right)^2 \left(Y_F^\dagger Y_F + Y_F^T Y_F^*\right)_{ij} + \frac{i}{8\pi}\left(s_\beta^2 + c_\beta^2 (1 - \frac{M_S^2}{p^2})\right)\left(Y_B^\dagger Y_B + Y_B^T Y_B^*\right)_{ij} \ , \tag{B.27}$$

where p denotes external momentum in the diagram and upon evaluation of (B.26), for Π one should use (B.27) with $p = \tilde{M}_i$. In (B.27), taking into account the SUSY masses M_S of all non SM states, we are using the refined expression for the Π_{ij}.

In an unbroken SUSY limit, neglecting finite temperature effects ($T \to 0$), the $\tilde{N}$ decay does not produce lepton asymmetry due to the following reason. The decays of $\tilde{N}$ in the fermion and scalar channels are respectivelly $\tilde{N} \to l\tilde{h}_u$ and $\tilde{N} \to \tilde{l}^* h_u^*$. Since the rates of these processes are the same due to SUSY (at $T = 0$), the lepton asymmetries created from these decays cancel each other. With $T \neq 0$, the cancellation does not take place and one has

$$\tilde{\epsilon}_i = \epsilon_i(\tilde{n}_i \to l\tilde{h}_u)\Delta_{BF} \ , \tag{B.28}$$

with a temperature dependent factor Δ_{BF} given in [65].[12] Therefore, we just need to compute $\epsilon_i(\tilde{n}_i \to l\tilde{h}_u)$, which is the asymmetry created by $\tilde{n}_i$ decays in two fermions. Thus, in (B.25) we take $S_{\alpha i} = (Y_F)_{\alpha i}$ and calculate $\epsilon_i(\tilde{n}_i \to l\tilde{h}_u)$ with (B.26). The baryon asymmetry created from the lepton asymmetry due to $\tilde{n}$ decays is given by:

$$\frac{\tilde{n}_b}{s} \simeq -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \frac{\tilde{\epsilon}_i}{\Delta_{BF}}\eta_i = -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \epsilon_i(\tilde{n}_i \to l\tilde{h}_u)\eta_i \ , \tag{B.29}$$

where an effective number of degrees of freedom (including two RHN superfields) $g_* = 228.75$ was used. η_i are efficiency factors which depend on $\tilde{m}_i \simeq \frac{(v\sin\beta)^2}{M}2(Y_F^\dagger Y_F)_{ii}$, and account for temperature effects once integration of the Boltzmann equations is performed [65].

Calculating the contribution $\frac{\Delta n_b}{s} = \frac{\tilde{n}_b}{s}$ to the baryon asymmetry from the RHS decays, we have examined various values of pairs (m_A, m_B) in the range of 100 GeV - few TeV. As it turned out, the ratio $\frac{\tilde{n}_b}{n_b^f}$ is always suppressed($< 3.4 \cdot 10^{-2}$). The results for each neutrino scenario, we have considered in this work, for one specific choice of (m_A, m_B), are given in Table 20 (see its caption for more information). The ranges for $\frac{\tilde{n}_b}{s}$ are due to the fact that for each scenario we have considered different values of $\tan\beta, M$ and M_S. Upon the calculations, with obtained values of $\tilde{m}_i$, according to Ref. [65] we picked up the corresponding values of η_i and used them in (B.29). While giving the results of the net baryon asymmetry, for each case (see sections 3.3 and 3.4), we have included corresponding contributions from $\frac{\tilde{n}_b}{s}$ as well. As we see from the results of Tab. 20, the $\frac{\tilde{n}_b}{s}$ is suppressed/subleading for all cases. We have also witnessed (by varying the phases of $m_{A,B}$) that the complexities of m_A and m_B practically do not change the results. This happens because the m_A in the Y_B coupling matrix appears in front of the Y_ν [see Eq. (B.24)], which is strongly suppressed. Irrelevance of the m_B's phase can be seen from the structure of (B.19). Suppression of $\frac{\tilde{n}_b}{s}$ will always

[12] This expression is valid with alignment $A_\nu = m_A Y_\nu$, which we are assuming to be true at the GUT scale and thus Eq. (B.28) can be well applicable to our estimates.

Neutrino Model	$10^{11}\times\frac{\tilde{n}_b}{s}$
Texture P_1, NH, data of tab. 7	$0.23-0.28$
Texture P_2, NH, data of tab. 8	$0.16-0.23$
Texture P_3, NH, data of tab. 9	~ 0.1
Texture P_3, IH, data of tab. 9	$0.07-0.09$
Texture P_4, NH, data of tab. 10	$0.07-0.08$
Texture A$'$, NH, data of Eqs. (3.46), (3.47)	$0.05-0.07$
Texture $\mathrm{B_1}'$, IH, data of Eqs. (3.57), (3.58)	$0.04-0.049$
Texture $\mathrm{B_1}'$, NH, data of Eqs. (3.61) – (3.63)	$\simeq 0$
Texture $\mathrm{B_2}'$, IH, data of Eqs. (3.69), (3.70)	$0.042-0.05$
Texture $\mathrm{B_2}'$, NH, data of Eq. (3.73)	$\approx 1.4\times 10^{-4}$

Table 20: Values of $\frac{\Delta n_b}{s}=\frac{\tilde{n}_b}{s}$ - contributions to the Baryon asymmetry via decays of the right handed sneutrinos for $(m_A, m_B)=(100i, 500)$ GeV and for various neutrino textures. Asymmetries are calculated with those values of a_i and b_j Yukawas that give $\left(\frac{n_b}{s}\right)_{\max}$. (For the latter see sections 3.3 and 3.4.)

happen for the value of $|m_B|$ in the range of 100 GeV - few TeV, because the mass degeneracy of $\tilde{n}_i$ states is lifted in such a way that resonant enhancement of $\frac{\tilde{n}_b}{s}$ is not realized. (Unlike the case of soft leptogenesis [65] which requires $|m_B| \lesssim 10$ MeV. Without special arrangement, such suppressed values of $|m_B|$ seem unnatural and we have not considered them within our studies.)

C Issues Related to the Baryogenesis

In this appendix we highlight and discuss some key concepts of Baryogenesis. For comprehensive reviews we refer to [66], from which we have greatly benefited.

C.1 Freeze Out: Origin of Species

The early history of the universe can be described to a high degree of accuracy in terms of most of its constituents being in thermal equilibrium. If the thermal equilibrium has been held since the early period of the universe, the present state of the universe would be completely specified by the present temperature. However, thermal equilibrium has been disturbed many times, by various processes, such as: neutrino decoupling, decoupling of the background radiation, primordial nucleosynthesis, inflation, baryogenesis, decoupling of relic WIMPs etc. To find out whether a particle species is coupled to or decoupled from the plasma one needs to compare the interaction rate Γ of the particle with the expansion rate of the universe H:

$$\Gamma \geq \mathrm{H}\ (\text{coupled})\ , \quad \Gamma \leq \mathrm{H}\ (\text{decoupled}) \tag{C.1}$$

where Γ is the interaction rate (per particle) for the reaction(s) that keep the species in equilibrium. If a massive particle species remained in equilibrium until present, its abundance: $\frac{\mathrm{n}}{\mathrm{s}} \sim \left(\frac{\mathrm{m}}{\mathrm{T}}\right)^{3/2} \exp(-\frac{\mathrm{m}}{\mathrm{T}})$ would be absolutely negligible because of the exponential suppression. If the interactions of the species freezes out(i.e. $\Gamma \leq \mathrm{H}$) at a temperature such that $\frac{\mathrm{m}}{\mathrm{T}}$ is not much greater than 1, the species can have a significant relic abundance today.

C.2 Baryon Asymmetry of The Universe

In this section we outline the problem of baryon asymmetry within the Standard Model and demonstrate how it can emerge, although in insufficient amount, on a more sophisticated level, such as an SU(5) GUT. The problem itself is rooted in the observed fact that the Universe seemingly does not contain antimattter in high concentrations. Such cosmological asymmetry between matter(baryons) and antimatter(antibaryons) remains a mistery even at the SU(3)xSU(2)xU(1) level. There is no cosmological model capable of generating just baryons on condition that all relevant baryon producing interactions conserve baryon nunber. Prior to the advent of GUT based models, in all cosmological models asymmetric initial conditions were set in advance which seemed unsatisfactory. The Standard Model based on baryon number conserving interactions does not fix the photon number density(corresponding to temperature of 2.7 K) to the observed nucleon density $\mathrm{n_N}$. Such a ratio is introduced by hand as an initial condition:

$$\frac{\mathrm{n_N}}{\mathrm{n}_\gamma} \simeq 10^{-9} \tag{C.2}$$

When the Universe was not hot enough for baryons(quarks) and antibaryons(antiquarks) to be produced in pairs, the above mentioned condition would lead to the baryon asymmetry:

$$\delta = \frac{\mathrm{n_q} - \mathrm{n_q^c}}{\mathrm{n_q} + \mathrm{n_q^c}} \simeq 10^{-9} \tag{C.3}$$

where $\mathrm{n_q}$ and $\mathrm{n_q^c}$ are quark and antiquark number densities respectively. Naturally, the fundamental question arises: why shoud there be such an asymmetry with precisely such value of δ? It would seem much more natural to assume that initially the Universe was in a symmetric state (irrespectively of initial conditions) and later, because of fundamental interactions of physics ended up with baryon asymmetry. To realize such a scenario it is necessary to postulate a new, baryon number changing interaction in addition to those that are already present in SU(3)xSU(2)xU(1) which would satisfy the following conditions:

1)The new interaction is expected to violate both C and CP invariances.

2)Its existence should be indicative of a period in cosmological expansion of the Universe when the B, C and CP invariance violating processes were in conflict with thermodynamical equilibrium. Obviously, both C and CP symmetries exclude the possibility of non-zero δ defined in (C.3) because corresponding transformations replaces $\mathrm{n_q}$ with $\mathrm{n_q^c}$ and vice versa. The requirement of violated thermal equillibrium may not seem so obvious but can be justified using CPT invariance which forces all particle and antiparticle states to have the same masses and therefore to have the same weights in Boltzmann distribution. Thus, no CPT invariant interaction can lead to non-zero value of δ in thermodynamical equilibrium. The simplest GUT model based on SU(5) gauge group possesses all necessary ingredients guaranteeing non-zero value of δ which, however, is disfavored by current experimental data, rendering the simplest SU(5) model obsolete. Nevertheless, SU(5) model and its shortcomings are still worth of studying. It predicts B, C and CP violating processes involving interactions with X-bosons(and with the Higgs particle as well) after these particle had been pushed out of equilibrium because of cosmological expansion. To demonstrate this possibility, we need to figure out the rates of relevant reactions as functions of energy(or, temperature). The condition of thermodynamical equilibrium requires that reaction rates exceed the rate of cosmological expansion of the Universe (C.1). It turns out that in two-body collisions mediated by X and Y bosons:

$$\mathrm{uu} \to \mathrm{e}^+\mathrm{d}^\mathrm{c}, \quad \mathrm{ud} \to \nu^\mathrm{c}\mathrm{d}^\mathrm{c}, \quad \mathrm{ud} \to \mathrm{e}^+\mathrm{u}^\mathrm{c} \tag{C.4}$$

the required transition from thermodynamical equilibrium to the non-equilibrium state is impossible. However, decays and inverse decays of heavy X-bosons have a threshold and can, therefore, make the above mentioned transitions possible. When $\mathrm{kT} > \mathrm{M_X}$, X-bosons must exist in the dynamical equilibrium and there number must be comparable to the number of ordinary particles(for example, $\mathrm{N_X} \simeq \mathrm{N}_\gamma$). Under these circumstances, X and $\mathrm{X^c}$ bosons decay violating B and CP invariances, producing more quarks than antiquarks. Ordinarily, excess of baryons would eventually vanish because if inverse decay processes, but when the Universe cools down to the temperature for which $\mathrm{kT} < \mathrm{M_X}$ the number of X-bosons(and inverse decays) gets suppressed by the Boltzmann factor $\exp(-\frac{\mathrm{M_X}}{\mathrm{kT}})$, consequently baryon production virtually stops and the baryon excess generated earlier gets 'frozen in'. There are two decay channels involving X-bosons; So, four decay widths should exist accounting for X and $\mathrm{X^c}$ boson decays:

$$\gamma_1 \equiv \Gamma(\mathrm{X} \to \mathrm{l^c q^c}), \quad \text{with} \;\; \mathrm{B_1} = -1/3 \tag{C.5}$$

$$\gamma_2 \equiv \Gamma(\mathrm{X} \to \mathrm{qq}), \quad \text{with} \;\; \mathrm{B_2} = 2/3 \tag{C.6}$$

and

$$\gamma_1^\mathrm{c} \equiv \Gamma(\mathrm{X^c} \to \mathrm{lq}), \quad \text{with} \;\; \mathrm{B}_1' = 1/3 \tag{C.7}$$

$$\gamma_2^\mathrm{c} \equiv \Gamma(\mathrm{X^c} \to \mathrm{q^c q^c}), \quad \text{with} \;\; \mathrm{B}_2' = -2/3 \tag{C.8}$$

CPT invariance causes total decay widths of particles and antiparticles to be the same:

$$\gamma_1 + \gamma_2 = \gamma_1^\mathrm{c} + \gamma_2^\mathrm{c} \tag{C.9}$$

At the same time CPT invariance ensures that $\gamma_1 = \gamma_1^c$ and $\gamma_2 = \gamma_2^c$ only in Born approximation. With C and CP violating interactions, higher order terms can emerge leading to:

$$\gamma_1 - \gamma_2 = \gamma_1^c - \gamma_2^c \neq 0 \tag{C.10}$$

meaning that, although X and X^c bosons had initially been present in the same amount, the departure from thermodynamical equilibrium would have forced them to produce excess of baryons over antibaryons:

$$\delta \sim \gamma_1 B_1 + \gamma_2 B_2 + \gamma_1^c B_1' + \gamma_2^c B_2' = (\gamma_1 - \gamma_1^c)(B_1 - B_2) \tag{C.11}$$

This clearly proves that the origin of non-zero δ is related to B, C and CP violation. The problem is however, that on the other hand, δ must be much smaller than 10^{-9} , since $\gamma_1 - \gamma_1^c$ is necessarily a higher order term [67] and is likely to be further multiplied by a small CP-phase [68]. Another problem associated with the SU(5) model is conservation of B−L. Namely, B−L=0, even if B≠0, and any baryon asymmetry generated will be washed out in subsequent topological transitions(C.4). For successful baryogenesis, generation of non-vanishing B−L is needed. The abovementioned problems marring the SU(5) GUT model can be alleviated in more complex approaches, such as SO(10), where neither B nor B−L is conserved and experimentally observable value of $\gamma_1 - \gamma_1^c$ can be reached. One of the attractive features of SO(10) is the presence of heavy scalar bosons and gauge bosons which generate the d=7 operators. These particles have (B−L)-violating two-body decays, which can generate the observed baryon asymmetry of the universe naturally. This would not be possible in case of (B−L)-preserving decays of GUT scale particles such as the ones in SU(5). The idea to use grand unified theories for implementing Sakharov‘s conditions for baryogenesis was practically abandoned after the realization that the sphalerons, which violate B+L symmetry would erase any baryon asymmetry that obeyed the Δ(B−L)=0 selection rule [49]. This is because the effective interactions generated by sphalerons are in thermal equilibrium in the range: 10^2GeV≤T≤ 10^{12}GeV and violate B+L symmetry. However, if baryon asymmetry was generated by (B−L)-violating decays of GUT scale particles, they would be immune to sphaleron destruction. This mechanism of baryogenesis, which also induces the d=7 B-violating operators, is very efficient and occurs quite generically in SO(10) models(for details and related discussion see Ref. [69]). The d=7 B and (B−L)-violating operators arise in unified SO(10) models, both in the non-supersymmetric and SUSY versions. For comparison, the leading baryon number violating operators in the Standard Model are of dimension 6(d=6), all carrying lepton number L=1 along with B=1. Consequently, this operators preserve B−L. The same operators are present in SU(5) and SO(10) based models, suppressed by two inverse powers of GUT scale masses. For the d=7 operators arising in SO(10), (B−L)=±2. While they are suppressed by one additional power of a heavy mass scale, they can naturally lead to sphaleron-proof baryogenesis. In several instances there also was found that these operators may lead to observable (B−L)-violating nucleon decay [69].

C.3 Leptogenesis

In the Standard Model, considering only renormalizable interactions, perturbation theory guarantees conservation of baryon and lepton numbers to all orders. However, certain non-perturbative effects(like sphalerons) may give rise to baryon and lepton number violating reactions. Such reactions are suppressed by a factor $\exp\left(-\frac{8\pi^2}{g^2}\right) \simeq 10^{-162}$, where g is the SU(2) coupling constant. At temperatures above 300 GeV, this exponential suppression disappears due to thermal fluctuations. Nevertheless, net baryon/lepton numbers get produced in insufficient amount because the reactions responsible for baryon/lepton number violation take place in thermal equilibrium and besides, the same reactions are suppressed by some small parameters, smallness of which is dictated by the need to violate conservation of both CP and baryon/lepton numbers. An attempt to generate non-zero baryon number density in the GUT based extensions of the Standard Model, through the decay processes of leptoquarks is bound to fail, because despite having different values for B and L, the decay channels have the same value of B-L. This in turn means that in the Universe with equal numbers of particles and antiparticles of all types, densities of B, L and B-L will inevitably be equal to zero. To address this problem, it is tempting to introduce in the early Universe some heavy particle, decays of which would produce a non-zero density of B-L. Generated this way the non-zero density of B-L number would not vanish in thermal equilibrium and it could become a source of non-zero density of baryon number. Suppose that in thermal equilibrium there are conserved quantum numbers Q_a and each of the particle species 'i' being in equilibrium carries a value q_{ai} for the quantum number Q_a. Chemical potential μ_i of a particle of the 'i' species is conserved for all interactions in thermal equilibrium and therefore can be expressed as a linear combination of conserved quantum numbers:

$$\mu_i = \sum_a q_{ai}\mu_a \tag{C.12}$$

Since these particles are involved in interactions which take place at temperatures above 10^{16} K (or, 800 GeV), they are highly relativistic and the number density of particle species 'i' can be written as:

$$n_i = \frac{g_i}{(2\pi\hbar)^3}\int\frac{d^3p}{e^{(p-\mu_i)/k_BT}\mp 1} = 4\pi g_i\left(\frac{k_BT}{2\pi\hbar}\right)^3\int_0^\infty\frac{x^2dx}{e^{x-\mu_i/k_BT}\mp 1} \tag{C.13}$$

where $x \equiv \frac{p}{k_BT}$ and g_i is the number of helicity (and other sources of multiplicity) states for each particle and the '$-$' sign corresponds to bosons and the '+' sign to fermions. The expression for antiparticle density $\bar{n}_i$ is similar to (C.13) with μ_i replaced with -μ_i. Thus,

$$n_i - \bar{n}_i = 8\pi g_i\left(\frac{k_BT}{2\pi\hbar}\right)^3\sinh\left(\frac{\mu_i}{k_BT}\right)\int_0^\infty\frac{x^2e^xdx}{e^{2x}\mp 2e^x\cosh(\mu_i/k_BT)+1} \tag{C.14}$$

It can be safely assumed that $|\mu_i \ll 1|$ for all particle species and therefore:

$$n_i - \bar{n}_i = 8\pi g_i\left(\frac{k_BT}{2\pi\hbar}\right)^3\left(\frac{\mu_i}{k_BT}\right)\int_0^\infty\frac{x^2e^xdx}{(e^x\mp 1)^2} \tag{C.15}$$

Using (C.12) we re-write the last expression as:

$$n_i - \bar{n}_i = f(T)\tilde{g}_i\mu_i = f(T)\tilde{g}_i\sum_a q_{ai}\mu_a \tag{C.16}$$

This in turn can be used to express density of the conserved quantum number Q_a:

$$n_a = \sum_i q_{ai}(n_i - \bar{n}_i) = f(T)\sum_b M_{ab}\mu_b \tag{C.17}$$

where

$$M_{ab} \equiv \sum_i \tilde{g}_i q_{ai} q_{bi} \tag{C.18}$$

from (C.17):

$$n_i - \bar{n}_i = \sum_{ab}\tilde{g}_i q_{ai} M^{-1}_{ab} n_b \tag{C.19}$$

for any particle species 'i'. Using data provided in the table below

Particle	$\tilde{g}$	B	L	T_3	Y
u_L	3	1/3	0	1/2	-1/6
d_L	3	1/3	0	-1/2	-1/6
u_R	3	1/3	0	0	-2/3
d_R	3	1/3	0	0	1/3
ν_L	1	0	1	1/2	1/2
e_L	1	0	1	-1/2	1/2
e_R	1	0	1	0	1
W^+	4	0	0	1	0
ϕ^+	2	0	0	1/2	-1/2
ϕ^0	2	0	0	-1/2	-1/2
gluons	4	0	0	0	0

Table 21: Particles of the Standard Model, together with the number $\tilde{g}$ of their helicity and color states (with an extra factor 2 for bosons), and the values of their baryon number, lepton number, and gauge quantum numbers. Only one "generation" of quarks and leptons and only one doublet of scalar fields are shown. The subscripts L and R denote the helicity states of quarks u and d and leptons ν and e. Antiparticles are not shown separately, and the photon and Z^0 are not shown because they are their own antiparticles, and so do not contribute to the densities of any quantum numbers. Color quantum numbers are not shown.

we derive a formula for baryon number density in thermal equilibrium.

$$n_B \equiv \sum_i B_i(n_i - \bar{n}_i) = \sum_i \tilde{g}_i B_i \left((B-L)_i M^{-1}_{B-L,B-L} + Y_i M^{-1}_{Y,B-L}\right) n_{B-L} =$$

$$= \left(\frac{4}{3} M^{-1}_{B-L,B-L} - \frac{2}{3} M^{-1}_{Y,B-L}\right) N_g n_{B-L} = \left(\frac{8N_g + 4N_d}{22N_g + 13N_d}\right) n_{B-L} \tag{C.20}$$

where N_g and N_d stand for the number of quark/lepton families and Higgs douplets respectively. Experimentally allowed minimal model involves 3 generations and 1 Higgs doublet, thus giving $n_B = \left(\frac{28}{79}\right) n_{B-L}$.

C.4 Instantons, Sphalerons and the Early Universe

Non-Abelian gauge theories allow for the existence of topologically different vacua which are separated from each other by a barrier, providing the possibility of topological transitions in the early Universe. These transitions lead to anomalous non-conservation of fermion number in the Standard Model. The probability of taking the field from one vacuum to another depends on temperature and on contributions of two competing processes. The first being the sub-barrier tunneling, which is the dominant of the two when the temperature at the time of transition is small compared to the height of the barrier. Corresponding Euclidean solution to the field equations is called an instanton. The second process dominates when the temperature is high enough for thermal fluctuations to take the field over the barrier and to another vacuum without tunneling. The static field configuration corresponding to the maximum of the potential and determining the rate of transitions in this case is called a sphaleron. Before delving deeper into topological transitions it makes sense to consider the SU(2)xU(1) group first. Corresponding Lagrangian

$$L_f = i\bar{\psi}_L\gamma^\mu(\partial_\mu + igA_\mu + ig'Y_L B_\mu)\psi_L + i\bar{\psi}_R\gamma^\mu(\partial_\mu + ig'Y_R B_\mu)\psi_R \tag{C.21}$$

is invariant under both SU(2) transformations:

$$\psi_L \to U\psi_L, \qquad \psi_R \to \psi_R \tag{C.22}$$

and U(1) transformations:

$$\psi_L \to e^{-ig'Y_L\lambda(x)}\psi_L, \qquad \psi_R \to e^{-ig'Y_R\lambda(x)}\psi_R \tag{C.23}$$

provided gauge fields A_μ and B_μ transform as:

$$A_\mu \to \tilde{A}_\mu = UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \quad \text{and} \quad B_\mu \to \tilde{B}_\mu = B_\mu + \partial_\mu\lambda \tag{C.24}$$

respectively. An important role is also played by tensor $F_{\mu\nu}$ defined as:

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + ig(A_\mu A_\nu - A_\nu A_\mu) \tag{C.25}$$

and its dual tensor:

$$\tilde{F}^{\alpha\beta} \equiv \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta} \tag{C.26}$$

To understand the non-trivial structure of the vacuum in non-Abelian theories, it is convenient to start with SU(N) theory without fermionic and bosonic fields. Vacuum implies vanishing of

$F_{\mu\nu}$, but not of the vector potential A_μ. Vanishing of $F_{\mu\nu}$ simply means that vector potential is a gauge-transform of zero, i.e. vector potential with components:

$$A_0 = 0, \qquad A_i = \frac{i}{g}(\partial_i U)U^{-1} \tag{C.27}$$

where U(x) is an arbitrary time-independent unitary matrix, also describes vacuum. All possible U(x) functions form homotopy classes. Two functions belong to the same homotopy class if they can be related by a non-singular continuous transformation. If not, the functions are said to belong to different homotopy classes. Homotopy classes are characterized by the winding number:

$$\nu \equiv -\frac{1}{24\pi^2}\int \mathrm{tr}\left(\epsilon^{ijk}(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right) d^3x \tag{C.28}$$

where ϵ^{ijk} is a totally antisymmetric Levi-Civita symbol. In the electroweak theory at temperatures T> 100 GeV symmetry is restored and the rate of topological transitions is very high. Since the SU(2) gauge fields interact only with the left-handed fermions and have the same strength for each doublet, the left current can be written as:

$$\partial_\mu^{(f)} J_L^\mu = -\frac{g^2}{16\pi^2}\mathrm{tr}(F\tilde{F}) \tag{C.29}$$

where f indicates a fermion doublet and runs from 1 to 12. Values f= $1, 2, 3$ correspond to leptonic doublets while f= $4-12$ number three quark families. For example, the choice of f= 1 immediately selects the first lepton family with the corresponding current: f $= 1$, $\quad \mathrm{J}_\mathrm{L}^\mu = \bar{\mathrm{e}}_\mathrm{L}\gamma^\mu \mathrm{e}_\mathrm{L} + \bar{\nu}_\mathrm{e}\gamma^\mu\nu_\mathrm{e}$. From two vacuum configurations (C.27) with winding numbers ν_0 and ν_1 specified on two different space-like hyperspaces the following relation is true:

$$\int \mathrm{tr}(F\tilde{F})d^4x = \frac{16\pi^2}{g^2}(\nu_1 - \nu_0) \tag{C.30}$$

which means that the field configurations interpolating between two topologically different vacua has a non-vanishing field strength and hence, 'in between' non-zero positive potential energy. From (C.29) and (C.30) it is clear that a topological transition increasing the winding number by $\Delta\nu$ units, decreases the fermion number in each doublet by the same $\Delta\nu$ units leading to non-conservation of the total fermion number. With the color index taken into account, there are nine quark doublets and since the baryon number of each quark is equal to $\frac{1}{3}$, the following relation holds:

$$\Delta L_e = \Delta L_\mu = \Delta L_\tau = \frac{1}{3}\Delta B \tag{C.31}$$

where indices e, μ, τ indicate the leptonic doublets, while ΔB stands for an overall change of the baryon number. Of course, total lepton and baryon numbers change by three units each: $\Delta L = \Delta B = -3$. The energy of disappearing fermions is transferred to the remaining and newly created fermions and antifermions. There are certain interactions in the Electroweak theory which convert left-handed fermions into right-handed ones. This means non-conservation of the total fermion number, hence some linear combination of baryon and lepton numbers $B + aL$ (from (C.20) $a = 28/51$) should vanish at thermal equilibrium. On the other hand from (C.31) it follows that B-L is conserved. Topological transitions in the early Universe can ensure equilibrium only if their rate per fermion exceeds the expansion rate of the Universe. Thus, even if B+aL were generated in the early Universe, it would be washed out by topological transitions for temperatures: 10^{12}GeV > T > 10^{2}GeV. Hence, if B-L=0, no pre-existent baryon number survives.

C.5 Baryogenesis Via Leptogenesis. See-Saw Mechanism

From (C.20) it is obvious that for baryon asymmetry to emerge, non-zero initial value for B-L is necessary. Even if initially $B_i = 0$ and $L_i \neq 0$, topological transitions will subsequently ensure non-zero final baryon number density, given by:

$$B_f = -\frac{a}{1+a} L_i, \quad \text{with} \quad a = \frac{28}{51} \tag{C.32}$$

As to the non-zero initial value for L_i, it can be generated in out-of-equilibrium decays of heavy neutrinos [5]. Heavy neutrinos can be produced after inflation, either in the preheating phase or after thermalization. Subsequently, their concentration freezes out and their out-of-equilibrium decays give rise to lepton asymmetry L_i. Heavy neutrinos can be naturally incorporated in the Standard Model to explain neutrino masses in neutrino oscillations. We start with the Yukawa coupling term responsible for Dirac masses of neutrinos:

$$L_Y^{\nu} = -f_{ij}^{(\nu)} \chi \bar{\nu}_L^i \nu_R^j + h.c. \tag{C.33}$$

where $i = 1, 2, 3$ is the lepton family index. Invariance of (C.33) requires that the right-handed neutrinos be SU(2) singlets, with neither color nor hypercharge. On the other hand, respecting all the gauge symmetries of the theory the Majorana mass term can be introduced as well:

$$L_M^{(\nu)} = -\frac{1}{2} M_{ij} (\bar{\nu}_R^c)^i \nu_R^j \tag{C.34}$$

where 'c' stands for charge conjugation. Once the symmetry is broken, the expectation value χ_0 of the χ field emerges and the Dirac masses of neutrino can be evaluated from the matrix:

$$(M_D)_{ij} = f_{ij}^{(\nu)} \chi_0 \tag{C.35}$$

For the sake of simplicity we consider the case of one generation and write the total mass term as:

$$L^{(\nu)} = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c \tag{C.36}$$

Under the assumption of $m_D \ll M$ diagonalization of (C.36) leads to the mass eigenvalues:

$$m_\nu \simeq -\frac{m_D^2}{M}, \qquad m_N \simeq M \tag{C.37}$$

and their corresponding eigenstates:

$$\nu \simeq \nu_L + \nu_L^c, \qquad N \simeq \nu_R + \nu_R^c \tag{C.38}$$

which describe light and heavy Majorana fermions. Appropriately choosing the value for M one can obtain light neutrino masses within a reasonable range. This method of generating light neutrino masses is called the see-saw mechanism. Confining the theory to just Dirac neutrino mass terms would result in ending up with unnaturally small Yukawa couplings and unbroken L. Lepton number violation stems from having both Dirac and Majorana mass terms in Lagrangian. Just Dirac mass terms are not enough to violate lepton number. However, with heavy Majorana mass, the lepton number is also violated. Heavy majorana neutrinos, being absolutely identical to their antiparticles($N = N^c$), can decay into a lepton-higgs pair $N \to l\phi$ or into the CP-conjugated state $N \to \bar{l}\bar{\phi}$, thus violating the lepton number by two units. It is worth noting, that in case of three generations neutrino mass eigenstates do not coincide with flavor(weak) states. Instead, they are related by lepton mixing matrix. This explains neutrino oscillations and with complex Yuakawa couplings one can have sources of CP violation.

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