Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Advances in Space Research xxx (2007) xxx–xxx
ADVANCES IN SPACE RESEARCH (a COSPAR publication)
www.elsevier.com/locate/asr

Investigation of the N–S asymmetry of the differential rotation of Hα filaments and large-scale magnetic elements

M.Sh. Gigolashvili *, D.R. Japaridze, T.G. Mdzinarishvili, B.B. Chargeishvili, V.J. Kukhianidze

E. Kharadze Georgian National Astrophysical Observatory, Georgia, A. Kazbegi Ave 2a, Tbilisi 0162, Georgia

Received 15 October 2006; received in revised form 2 March 2007; accepted 12 March 2007

Abstract

Studying solar rotation, its temporal variation and altitude dependence in the solar atmosphere is timely and important since it is closely related to the mechanism of solar activity. The aim of the present work is to investigate solar differential rotation using rotation rates of Hα filaments in 1957–1993 and those of compact elements of large-scale magnetic fields in 1966–1985. We verify the earlier finding that the differential rotation of both of these magnetic structures is north–south (N–S) asymmetric during the respective time intervals. We find that the N–S asymmetry of Hα filaments changes its sign during the 22-year cycle. The average rotation rate of compact magnetic elements with negative polarity depicts a 22-year oscillation both in the northern and southern hemisphere. Our statistical analysis did not reveal significant N–S asymmetry in the differential rotation of compact elements with positive polarity. Our results on the N–S asymmetry of Hα filaments and compact elements are somewhat controversial since they show opposite simultaneous N–S asymmetries. This may be due to their different heights in the solar atmosphere where radial differential rotation may cause changes in asymmetry.

Keywords: Sun: filaments; Magnetic patterns; Differential rotation

1. Introduction

Solar differential rotation that is already known for many decades still remains a rather unexplained phenomenon. The measured rotation velocities differ from each other not only for different solar formations, which can be explained by the action of different factors in the different layers of solar atmosphere, but they differ even for the same objects. The discrepancy of the obtained results is caused not only by complexity of the phenomenon as such, but also by the rather small velocity differences (from fractions of percent up to several percents), which requires sophisticated methods of statistical processing of the observed data. In spite of studies of various observational data and theoretical investigations for many years the solar differential rotation is not yet fully understood (Howard, 1976; Schroter, 1985; Gilman and DeLuca, 1986; Stix, 1989; Howard et al., 1999).

The temporal variation of characteristics of processes occurring in the solar atmosphere is closely connected with the long-term large-scale manifestation of solar activity. The interaction between solar rotation, convection and magnetic fields is indisputably the reason of such activity. Thus the study of differential rotation and its temporal variation and altitude dependence in the solar atmosphere is an important part of solar physics.

Many authors have reported on the existence of N–S asymmetry in the solar differential rotation (Gilman and Howard, 1984; Antonucci et al., 1990; Hoeksema and Scherrer, 1987; Howard et al., 1984; Balthasar et al., 1986; Javaraiah, 2003; Gigolashvili et al., 2003, 2005a,b; Brajsa et al., 1997, 2000; Duchev, 2001; Georgieva and Kirov, 2003; Georgieva et al., 2003; Mursula and Hiltula,

* Corresponding author.
E-mail address: marinagig@yahoo.com (M.Sh. Gigolashvili).

0273-1177/$30 © 2007 Published by Elsevier Ltd on behalf of COSPAR.
doi:10.1016/j.asr.2007.03.103

2004). However, the obtained results are occasionally different and sometimes even controversial.

In this paper solar differential rotation is investigated statistically by means of rotation rates of Hα filaments in 1957–1993 (Gigolashvili et al., 2003, 2005a,b) and those of compact elements of large-scale magnetic fields in 1966–1985. The possible connection of asymmetry with solar cyclicity is considered. It is proposed that the mechanism of solar activity is responsible for the N–S asymmetry of solar differential rotation.

2. Observational data

The properties of solar differential rotation are investigated by using the large-scale compact magnetic patterns of McIntosh atlas (1991) and the Hα filaments registered in the photoheliograms of the Abastumani Astrophysical Observatory. Using McIntosh atlas 1675 measurements (990 for solar cycle 20 and 685 for cycle 21) have been carried out for 335 large-scale magnetic compact elements (198 for cycle 20 and 137 for cycle 21).

Annually averaged angular velocities of 716 quiescent Hα filaments are determined from the Hα photoheliograms of the Abastumani Astrophysical Observatory film collection in 1957–1993. The Abastumani Observatory data cover the time span of 37 years (solar cycles 19–22). We have obtained approximately 14,000 measurements of 716 quiescent filaments. The sidereal rotation velocities ω have been computed for each pair of subsequent days in the lifetime of each filament.

3. Method of treatment

Our approach to study the N–S asymmetry is based on the statistical methodology that was worked out in Brownlee's monograph (1965). It is relatively well known that the solar differential rotation can be estimated from the velocities on the solar disk using a standard approximation

$$\omega(\phi) = A + B\sin^2\phi + C\sin^4\phi, \quad (1)$$

where $\omega(\phi)$ is the angular velocity in degrees per day, and ϕ is the heliographic latitude. A, B and C are solar rotation parameters, i.e., A is the equatorial angular velocity and B, C are the latitude gradient coefficients of the rotation rate. Function (1) defines the differential rotation profile. We can restrict ourselves to only the first two terms of the expansion (1), and set $C = 0$.

To avoid cross-talk in estimating coefficients A and B that may result from the non-orthogonality of the fit functions of these coefficients, we use the Gegenbauer polynomials as a set of functions, which leads to the following expansion (Javaraiah and Komm, 1999):

$$\omega(\phi) = \bar{A} + \bar{B}(5\sin^2\phi - 1). \quad (2)$$

Now the coefficients $\bar{A}$ and $\bar{B}$ are free of the above mentioned cross-talk; $\bar{A}$ is the "mean" rotation coefficient and $\bar{B}$ is the latitude gradient coefficient of differential rotation. Coefficients $\bar{A}$ and $\bar{B}$ are related to A and B as follows:

$$\bar{A} = A + (1/5)B, \quad (3)$$
$$\bar{B} = (1/5)B. \quad (4)$$

Let us denote $\omega(\phi) = \eta$ and $5\sin^2\phi - 1 = x$. Then instead of Eq. (2) we have separately for the northern and southern hemispheres

$$\eta_N = \bar{A}_N + \bar{B}_N x, \quad (5)$$
$$\eta_S = \bar{A}_S + \bar{B}_S x. \quad (6)$$

We can determine $x = 5\sin^2\phi - 1$ from observations with high accuracy while instead of η, we can observe Y which is influenced by observational errors. Y is an estimate of $\eta = \omega(\phi)$, derived by averaging data of angular velocity $\omega(\phi)$. Thus, Y contains observational errors as well as random errors caused by the procedure of averaging. We suppose that random estimates of $\eta = \omega(\phi) = \omega(\phi(x))$ are distributed normally around η. In general, this is a standard problem of estimating parameters of linear regression $\eta = a + bx$ where, instead of η, we have data of Y influenced by random errors.

Let us suppose that we have separate sets of observational data for the northern and southern hemispheres for a fixed period t:

$$(x_{N_1}, Y_{N_1}), (x_{N_2}, Y_{N_2}), \ldots, (x_{N_{n_N}}, Y_{N_{n_N}}), \quad (7)$$
$$(x_{S_1}, Y_{S_1}), (x_{S_2}, Y_{S_2}), \ldots, (x_{S_{n_S}}, Y_{S_{n_S}}). \quad (8)$$

Suppose that the observed values $Y_{N_1}, Y_{N_2}, \ldots Y_{N_{n_N}}$ and $Y_{S_1}, Y_{S_2}, \ldots, Y_{S_{n_S}}$ are independent estimates of normally distributed random values with parameters η_{N_i}, $\sigma^2_N (i = 1, 2, \ldots, n_N)$ and η_{S_j}, $\sigma^2_S (j = 1, 2, \ldots, n_S)$; and η_N and η_S are linear functions of x:

$$\eta_N = \alpha_N + \beta_N(x_N - \bar{x}_N), \quad (9)$$
$$\eta_S = \alpha_S + \beta_S(x_S - \bar{x}_S), \quad (10)$$

where $\bar{x}_N = 1/n_N \sum_{i=1}^{n_N} x_{N_i}$ and $\bar{x}_S = 1/n_S \sum_{j=1}^{n_S} x_{S_j}$.

Estimates of regression lines Eqs. (9) and (10) are given by

$$\hat{Y}_N = a_N + b_N(x_N - \bar{x}_N), \quad (11)$$
$$\hat{Y}_S = a_S + b_S(x_S - \bar{x}_S), \quad (12)$$

where $\hat{Y}_N$, a_N, b_N and $\hat{Y}_S$, a_S, b_S are obtained from the observational data (7) and (8) by using the standard least squares method. Coefficients α_N, β_N and α_S, β_S are related to $\bar{A}_N$, $\bar{B}_N$ and $\bar{A}_S$, $\bar{B}_S$ as follows:

$$\bar{A}_N = \alpha_N - \beta_N\bar{x}_N, \quad \bar{B}_N = \beta_N, \quad (13)$$
$$\bar{A}_S = \alpha_S - \beta_S\bar{x}_S, \quad \bar{B}_S = \beta_S. \quad (14)$$

The question arises whether the two estimated regression lines (11) and (12) correspond to the same or to two different regression lines for a fixed period of observational time. This problem is solved as follows:

Please cite this article in press as: Gigolashvili, M.S. et al., Investigation of the N–S asymmetry of the differential rotation ..., J. Adv. Space Res. (2007), doi:10.1016/j.asr.2007.03.103

(1) From Eqs. (7) and (8), we calculate a_N, b_N, a_S, b_S, s_N^2 and s_S^2, where s_N^2 and s_S^2 are the estimates of the variance σ_N^2 and σ_S^2:

$$s_N^2 = \frac{1}{n_N - 2}\sum_{i=1}^{n_N}(Y_{N_i} - \hat{Y}_{N_i})^2, \tag{15}$$

$$s_S^2 = \frac{1}{n_S - 2}\sum_{j=1}^{n_S}(Y_{S_j} - \hat{Y}_{S_j})^2. \tag{16}$$

Then we test the null hypotheses $\sigma_S^2 = \sigma_N^2 = \sigma^2$ by application of the F-criterion. Let us denote $s_{max} = \max(s_N, s_S)$, $s_{min} = \min(s_N, s_S)$, then $(s_{max}^2/s_{min}^2) \sim F(n_N - 2, n_S - 2)$, where $n_N - 2$ and $n_S - 2$ are degrees of freedom of the F-distribution. If the F-criterion rejects the null hypothesis, we conclude that the two regression lines are different. If the F-criterion accepts the null hypothesis, we construct the joint estimate s^2 of σ^2

$$s^2 = \frac{(n_N - 2)s_N^2 + (n_S - 2)s_S^2}{n_N + n_S - 4} \tag{17}$$

and then test the hypothesis $\beta_N = \beta_S$.

(2) We test the hypothesis $\beta_N = \beta_S$ by determining the statistics T_b as follows:

$$T_b = \frac{b_N - b_S}{s\left\{\left[\sum_{i=1}^{n_N}(x_{N_i} - \bar{x}_N)^2\right]^{-1} + \left[\sum_{j=1}^{n_S}(x_{S_j} - \bar{x}_S)^2\right]^{-1}\right\}^{\frac{1}{2}}} \approx t(n_N + n_S - 4), \tag{18}$$

where s is given in Eq. (17), and t is Student's distribution with $n_N + n_S - 4$ degrees of freedom. If the null hypothesis is rejected, we conclude that the lines of regression Eqs. (11) and (12) are different because they have different slopes. On the other hand, if the null hypothesis is accepted, we construct the joint estimate of the slope and then test another hypothesis: whether the regression lines coincide or are parallel with each other or not.

(3) If the null hypothesis $\beta_1 = \beta_2$ is accepted, the estimate of β, that is b, is constructed as follows:

$$b = \frac{\sum_{i=1}^{n_N} Y_{N_i}(x_{N_i} - \bar{x}_N) + \sum_{j=1}^{n_S} Y_{S_j}(x_{S_j} - \bar{x}_S)}{\sum_{i=1}^{n_N}(x_{N_i} - \bar{x}_N)^2 + \sum_{j=1}^{n_S}(x_{S_j} - \bar{x}_S)^2}. \tag{19}$$

Then we test the hypothesis $\alpha_1 = \alpha_2$. For this purpose we use the statistics T_a, constructed as follows:

$$T_a = \frac{a_N - a_S - b(\bar{x}_N - \bar{x}_S)}{s\left[\frac{1}{n_N} + \frac{1}{n_S} + \frac{(\bar{x}_N - \bar{x}_S)^2}{\sum_{i=1}^{n_N}(x_{N_i} - \bar{x}_N)^2 + \sum_{j=1}^{n_S}(x_{S_j} - \bar{x}_S)^2}\right]^{\frac{1}{2}}} \approx t(n_N + n_S - 3), \tag{20}$$

where b is given in Eq. (19), s^2 is the joint estimate of σ^2, and t is Student's distribution with $n_N + n_S - 3$ degrees of freedom. This is the final step in our analysis.

4. Results

The results from the above-described statistical analysis are summarized in Table 1 for Hα filaments, in Table 2 for large-scale magnetic elements with negative polarity and in Table 3 for large-scale magnetic elements with positive polarity. It should be noted that T_a statistics changes its sign depending on solar cycle and the corresponding Student's criterion for Hα filaments and for large-scale magnetic elements with negative polarity indicates that these differences are significant with the 90% confidence level.

We also studied the time variation of T_a statistics near maxima and minima of solar activity. For Hα filaments the values of T_a statistics were calculated for the minimum periods 1964–1966, 1973–1978 and 1984–1988, as well as for maximum periods 1957–1963, 1967–1972, 1979–1983 and 1989–1993. The calculated values of T_a statistics are depicted in Fig. 1 with bars indicating the 90% confidence level.

For large-scale magnetic elements with negative polarity and with positive polarity the average values of T_a statistics were calculated separately for the minimum periods 1964–1966 and 1973–1978, as well as for maximum periods 1967–1972, 1979–1983. These values together with the 90% confidence level bars are depicted correspondingly in Figs. 2 and 3.

5. Discussion

To avoid cross-talk between the coefficients in practice, it is not only sufficient to select an orthogonal set of polynomials like Gegenbauer or Legendre polynomials, it must also be possible to determine the coefficients independently, and that depends on the latitudinal distribution of the available data. Our data consists of annually averaged angular velocities that are determined in successive 5° wide latitude belts.

Let as assume that coefficient B is the same for both hemispheres and $C = 0$ in Eq. (1). In this case Eq. (2) in the range around 26.5 degrees reduces to

$$\omega(\phi) = \bar{A}. \tag{21}$$

One of the sets of our data of Hα filaments contains the range $25° \pm 2.5°$ and we assume that for this region Eq. (21) is applicable. Q4

Let us consider the histogram of latitudinal distribution of the available Hα data (see Fig. 4). As it is seen from Fig. 4 our data cover a wide range of latitudes around the critical 26.5 range quite well. We calculated coefficients $\bar{A}$ from Eq. (21) and compared them with ones calculated from Eq. (2) for each solar cycle as well as for the whole period 1957–1993. They were found to be the same within 90% confidence level. That means that cross-talk between the coefficients $\bar{A}$ and $\bar{B}$ in our analysis can be neglected.

In our opinion, the successive changes in the sign of T_a statistics mean that for Hα filaments and magnetic elements with negative sign there exists an oscillation in the

Please cite this article in press as: Gigolashvili, M.S. et al., Investigation of the N–S asymmetry of the differential rotation ..., J. Adv. Space Res. (2007), doi:10.1016/j.asr.2007.03.103

Table 1
Results of statistical analysis for Hα filaments

Solar cycle No.	Years	s^2_{max}/s^2_{min}	T_b	T_a	n_N	n_S	F	t
19	1957–1963	1.053	−1.577	−2.168	63	51	1.56	1.659
20	1964–1975	1.002	−0.709	1.736	97	83	1.43	1.654
21	1976–1985	1.060	0.054	−2.506	84	88	1.44	1.654
22	1986–1993	1.481	−0.057	2.249	63	72	1.50	1.657
19–22	1957–1993	1.021	−0.805	−0.810	307	294	1.21	1.650

Table 2
Results of statistical analysis for large-scale magnetic elements with negative polarity

Solar cycle No.	Years	s^2_{max}/s^2_{min}	T_b	T_a	n_N	n_S	F	t
20	1966–1975	1.068	−0.563	−2.779	57	60	1.53	1.65
21	1976–1985	1.179	−0.100	3.913	58	54	1.55	1.65
20–21	1966–1985	1.168	−0.361	0.747	115	114	1.40	1.65

Table 3
Results of statistical analysis for large-scale magnetic elements with positive polarity

Solar cycle No.	Years	s^2_{max}/s^2_{min}	T_b	T_a	n_N	n_S	F	t
20	1966–1975	1.143	0.432	−0.156	55	59	1.54	1.65
21	1976–1985	1.061	0.470	0.103	58	53	1.55	1.65
20–21	1966–1985	0.546	0.487	−0.065	113	112	1.40	1.65

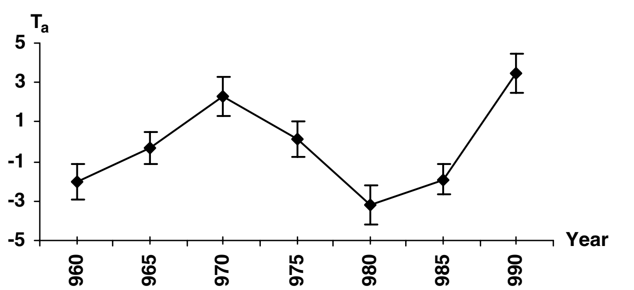

Fig. 1. Time variation of T_a statistics of Hα filaments during maxima and minima of solar cycles 19–22.

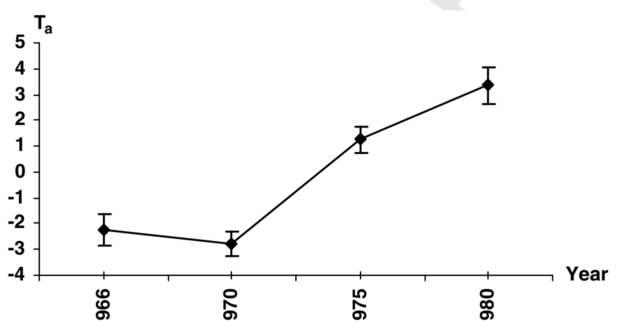

Fig. 2. Time variation of T_a statistics of compact magnetic large-scale elements with negative polarity during maxima and minima of solar cycles 20–21.

"mean" rotation velocity for both northern and southern hemispheres of the solar magnetic cycle of approximately 22 years. The N–S asymmetry also exists in the differential rotation of large-scale magnetic elements with positive polarity, but in this case the confidence level is very low.

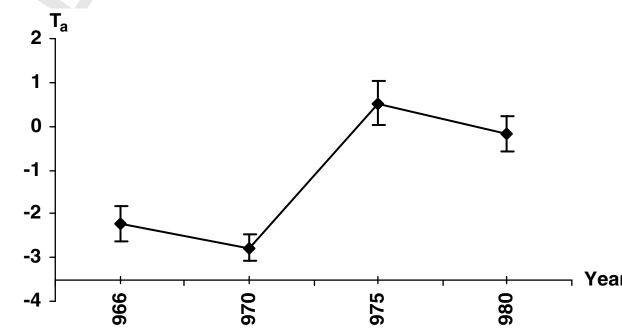

Fig. 3. Time variation of T_a statistics of compact magnetic large-scale elements with positive polarity during maxima and minima of solar cycles 20–21.

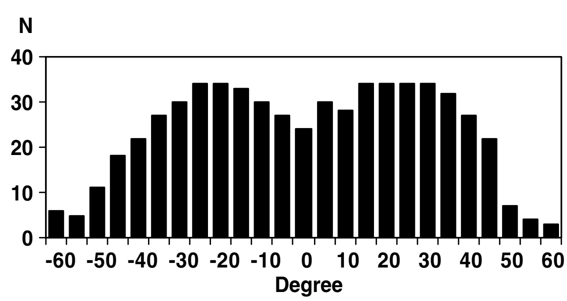

Fig. 4. The histogram of latitudinal distribution of the available Hα data. Latitudes in degrees are measured on abscissa and frequencies of available data of annually averaged angular velocities for Hα filaments are on ordinate.

Based on statistical analysis we could not reveal a significant N–S asymmetry in the differential rotation of large-scale magnetic elements with positive polarity.

Please cite this article in press as: Gigolashvili, M.S. et al., Investigation of the N–S asymmetry of the differential rotation ..., J. Adv. Space Res. (2007), doi:10.1016/j.asr.2007.03.103

Differences in velocity fields between different hemispheres and magnetic polarities confirm our earlier work (Gigolashvili et al., 2003, 2005a). The fact that results for the N–S asymmetry of Hα filaments and large scale magnetic fields are somewhat controversial, showing opposite signs of asymmetry, can be explained by their different heights in the solar atmosphere where, due to radial differential rotation of atmosphere, physical features causing changes of sign of asymmetry as waves propagate to higher layers. Thus the change of sign of asymmetry does not occur simultaneously at all heights but propagates with some period that can be revealed by a detailed study of long series of data corresponding to different atmospheric heights.

6. Conclusion

1. Statistical results on the N–S asymmetry in solar differential rotation of Hα filaments and magnetic elements with negative polarity imply that there exists an oscillation in the "mean" rotation velocity for both northern and southern hemispheres and the solar magnetic cycle (approximately 22 years).
2. T_a statistics changes its sign depending on solar cycle and the corresponding Student's criterion indicates that these differences are significant. We also studied time variation of T_a statistics near solar maxima and minima.
3. Thus, we can conclude that the N–S asymmetry of solar rotation rate changes over the solar cycle: the difference in the velocity reaches the maximum value around activity maxima, and changes its sign from near activity minima.

These conclusions must be verified by the further investigations using the expanded observational data.

References

Antonucci, E., Hoeksema, J.T., Scherrer, P.H. Rotation of the photospheric magnetic fields-A north-south asymmetry. Astrophys. J. 360, 296–304, 1990.

Balthasar, H., Vazquez, M., Wohl, H. Differential rotation of sunspots groups in the period from 1874 through 1976 and changes of the rotation velocity within the solar cycle. Astron. Astrophys. 155, 87–98, 1986.

Brajsa, R., Ruzdjak, V., Vrsnak, B., et al. On the possible changes of the solar differential rotation during the activity cycle determined using microwave low brightness temperature regions and Hα filaments as tracers. Solar Phys. 171, 1–34, 1997.

Brajsa, R., Ruzdjak, V., Vrsnak, B., et al. Statistical weights and selective height corrections in the determination of the solar rotation velocity. Solar Phys. 196, 279–297, 2000.

Brownlee, K.A. Statistical Theory and Methodology in Science and Engineering. John Wiley & Sons, Inc, 1965. Q3

Duchev, P.I. An estimation of the long-term variation of a north-south asymmetry of the long-lived solar filaments. Solar Phys. 199, 211–215, 2001.

Georgieva, K., Kirov, B. Different periodicities in the rotation of the northern and southern solar hemispheres. Proc. Of SOHO 12/ GONG+ 2002 'Local and Global Helioseismology: The Present and Future', Big Bear Lake, California (USA), pp. 275–278, 2003.

Georgieva, K., Kirov, B., Javaraiah, J., et al. Solar rotation and solar wind-magnetosphere coupling. Planetary Space Sci. 53, 197–207, 2003.

Gigolashvili, M., Japaridze, D., Kukhianidze, V. Variations of the solar differential rotation associated with polarity reversal. Solar Phys. 231, 23–28, 2005a.

Gigolashvili, M.Sh., Japaridze, D.R., Mdzinarishvili, T.G., et al. N–S asymmetry in the solar differential rotation during 1957–1993. Solar Phys. 227 (1), 27–38, 2005b.

Gigolashvili, M.Sh., Mdzinarishvili, T.G., Japaridze, D.R., et al. N–S Asymmetry in the differential rotation of the Sun and its variation with the Solar Activity Cycles. New Astronomy 8, 529–536, 2003.

Gilman, P.A., DeLuca, E.E. Dynamo theory for the sun and stars, in: Zeilik, M., Gibson, D.M. (Eds.), Cool Stars, Stellar Systems, and the Sun, 4th Cambridge Workshop, Santa Fe, Springer-Verlag, Berlin, Lecture Notes in Phys. 254, pp. 163–172, 1986.

Gilman, P.A., Howard, R. Variations in solar rotation with the sunspot cycle. Astrophys. J. 283, 385–391, 1984.

Hoeksema, J.T., Scherrer, P.H. Rotation of the coronal magnetic field. Astrophys. J. 318, 428–436, 1987.

Howard, R. A possible variation of the solar rotation with the activity cycle. Astrophys. J. 210, L159–L161, 1976.

Howard, R., Gilman, P.A., Gilman, P.I. Rotation of the Sun measured from Mount Wilson white-light images. Astrophys. J. 283, 373–384, 1984.

Howard, R.F., Gupta, S.S., Sivaraman, K.R. Measurement of Kodaikanal white-light images-II. Rotation comparison and marging with Mount Wilson data. Solar Phys. 186, 25–41, 1999.

Javaraiah, J. Long-term variations in the solar differential rotation. Solar Phys. 212, 23–49, 2003.

Javaraiah, J., Komm, R.W. Short-term periodicities of the sun's 'mean' and differential rotation. Solar Phys. 184, 41–60, 1999.

McIntosh, P.S., Willock, E.C., Thompson, R.J. Atlas of Stackplots derived from Solar Synoptic Charts: Evolution of Large-Scale Magnetic Fields and Coronal Holes from H-alpha Synoptic Charts: 1966-87, National data Center, p. 196, 1991.

Mursula, K., Hiltula, T. Systematically asymmetric heliospheric magnetic field: evidence for a quadrupole mode and non-axisymmetry with polarity flip-flops. Sol. Phys. 224, 133–143, 2004.

Schroter, E.H. The solar differential rotation: present status of observations. Solar Phys. 100, 141–168, 1985.

Stix, M. The Sun's Differential Rotation, in: Klarc, G. (Ed.), Reviews in Modern Astronomy, vol. 2, pp. 248–256, 1989.