

THE DISTRIBUTION OF A, F AND G TYPE STARS PERPENDICULAR  
TO THE GALACTIC PLANE AND IN THE DIRECTION TO THE  
GALACTIC CENTRE

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The offered investigation continues our work «On the space distribution of F type stars» published in the Bulletin of Abastumani Observatory<sup>1</sup>.

The method of the numerical solution of Schwarzschild's integral equation has been improved and more material utilized, the distribution of stars of three different spectral classes being investigated.

We intended primarily to study the space distribution of stars of all the spectral classes according to A. Schwassmann's catalogue<sup>2</sup>. However, taking into account the difficulty of drawing the average curve  $\log Nm$  from ( $m$ ) on the basis of different areas (f. ex. for the types B, M, K) we decided to examine only three spectral types.

The distribution of A, F and G type stars in the direction perpendicular to the galactic plane as well as in the plane of symmetry was studied.

For this purpose we used Schwassmann's catalogue containing data for 19 Kapteyn areas of the northern hemisphere. The density distribution for F type stars was obtained from Malmquist's catalogue<sup>3</sup> of color-indices.

In Malmquist's catalogue are given color-indices of A, F and G type stars situated near the galactic pole.

The selection and counts of F type stars were made on the basis of assumption that their color-index lies between  $+0.24$  and  $+0.38$ . The region of the sky taken by Malmquist being very close to the galactic pole, the density distribution of F type stars derived from his catalogue is represented for this direction only by the curve on Fig. 3. Using Malmquist's catalogue for the study of distribution it was not necessary to assume that the layers are plane-parallel, while this assumption is indispensable when we use Schwassmann's catalogue.

In the present work as well as in the precedent one, it was assumed that in the vicinity of the Sun the equidensity layers are plane-parallel and at the same time parallel to the galactic plane. We shall see below that owing to this plane-parallelism of layers the curves  $Nm$  expressing the number of

stars per square degree brighter than the apparent magnitude ( $m$ ) are connected in such a way that the curves  $Nm$  drawn for different galactic latitudes and longitudes after being multiplied by a corresponding coefficient and shifted along the abscissa by a definite amount must coincide with that drawn for the direction to the galactic pole. Moreover, in the case when the distribution of stars in the plane of symmetry is being investigated, the different directions in longitude must coincide with the direction to the galactic centre (in this case the lines of similar density are assumed to be approximately—parallel straight lines perpendicular to the direction towards the galactic centre).

Making use of this circumstance we can derive the curves  $\log Nm$  for the directions to the pole and to the galactic centre from observations in other directions and obtain the distribution of stars in the directions ( $\zeta$ ) and ( $c$ )\*. This method gives more accurate results than those which would have been obtained if the curve  $\log Nm$  were drawn only on the basis of counts made for directions near the galactic pole or in the galactic plane in the direction to the centre. If the curve  $\log Nm$  as the function of  $m$ , drawn for a given galactic latitude, multiplied by a corresponding factor and shifted along the abscissa does not coincide with the curve  $\log Nm$  drawn for the galactic pole, we may suppose that this fact is due to the absorption, the amount of which can be deduced from the discrepancy between the curves.

To draw the density distribution curves for A, F and G type stars we made use of Schwarzschild's integral equation

$$A_b(m) = \omega \int_0^{\infty} D(r) \varphi(r) r^2 dr. \quad (1)$$

We again introduce  $\zeta = r \sin b$ , where  $b$  is the galactic latitude and  $r$ —the distance from the star or—if the distribution of stars in the direction to the galactic centre ( $c$ ) is being studied— $c = r \cos(l - 325^\circ)$ , where  $l$  is the galactic longitude\*\*.

After substitution and transformation of the equation (1) we obtain (2) and (3), which connect the curves  $\log Nm$  for different galactic latitudes and longitudes:

$$\frac{N_{\zeta}}{2} (m' - 5 \log \operatorname{cosec} b) = \sin^2 b N_b(m'), \quad (2)$$

$$N_{325^\circ} [m' - 5 \log \sec(l - 325^\circ)] = \cos^2(l - 325^\circ) N_l(m'). \quad (3)$$

\* The direction to the galactic centre we designate by ( $c$ ), and that to the pole of galaxy—by ( $\zeta$ ).

\*\* For the longitude of the galactic centre we adopted  $l = 325^\circ$ .

It was indicated above that Schwassmann's catalogue was made use of. Twelve Kapteyn areas enumerated below were chosen as the object of the study: 4, 5, 6, 8, 9, 11, 12, 13, 14, 15, 16, 19. Nine of those: 4, 5, 6, 11, 12, 13, 14, 15, 16, located in different galactic latitudes (from  $b = +27^\circ$  to  $b = +57^\circ$ ) serve for the deduction of distribution in the direction  $\zeta$ .

The distribution of stars in the direction to the galactic centre  $l = 325^\circ$  was derived from the star counts in three areas: 8, 9 and 10, located near the galactic plane:  $b = +3^\circ, -2^\circ, -1^\circ$ .

In Schwassmann's catalogue the spectral types of stars down to 14th magnitude are given. However, only the stars down to  $13^m.25$  were taken into account, because to that limit the above catalogue may be regarded as sufficiently complete.

In tables I, II, and III the values  $\log [\sin^2 b N_b(m')]$  are given for the distribution of A, F and G type stars in high galactic latitudes while tables Ia, IIa, IIIa contain the values  $\log [\cos^2(l - 325^\circ) N_l(m')]$  for the distribution of stars of the same classes in the galactic plane.

TABLE I 366000  
A0—A9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$3 \log \sin b$
14	$57^\circ$	$81^\circ$	—	—	9.77	9.77	0.25	0.37	0.60	0.77	0.77	0.77	0.81	9.77
13	$53^\circ$	$111^\circ$	0.19	0.31	0.31	0.31	0.41	0.41	0.49	0.49	0.55	0.55	0.55	9.71
15	$48^\circ$	$62^\circ$	—	—	9.61	9.61	9.91	0.21	0.21	0.21	0.31	0.51	0.51	9.61
5	$42^\circ$	$91^\circ$	—	—	—	0.18	0.18	0.32	0.48	0.59	0.63	0.71	0.76	9.48
12	$41^\circ$	$123^\circ$	9.75	9.93	9.93	0.05	0.05	0.29	0.45	0.60	0.68	0.71	0.71	9.45
6	$36^\circ$	$75^\circ$	9.61	9.91	0.09	0.21	0.42	0.54	0.54	0.57	0.61	0.65	0.67	9.31
16	$33^\circ$	$56^\circ$	9.51	9.81	0.16	0.25	0.36	0.41	0.41	0.44	0.47	0.47	0.47	9.21
4	$32^\circ$	$107^\circ$	9.17	9.65	9.77	0.07	0.17	0.37	0.59	0.66	0.76	0.81	0.81	9.17
11	$27^\circ$	$123^\circ$	9.45	9.57	9.97	0.12	0.17	0.29	0.39	0.51	0.54	0.57	0.58	8.97

TABLE II 366000  
F0—F9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$3 \log \sin b$
14	$57^\circ$	$81^\circ$	0.37	0.61	0.77	0.95	1.15	1.31	1.38	1.42	1.55	1.70	1.67	9.77
13	$53^\circ$	$111^\circ$	9.71	0.49	0.94	1.16	1.31	1.40	1.54	1.63	1.84	1.79	1.57	9.71
15	$48^\circ$	$62^\circ$	0.09	0.39	0.72	0.91	1.09	1.31	1.40	1.51	1.65	1.73	1.78	9.61
5	$42^\circ$	$91^\circ$	9.96	0.48	0.59	0.82	1.05	1.24	1.38	1.50	1.57	1.64	1.71	9.48
12	$41^\circ$	$123^\circ$	9.93	0.40	0.60	0.71	0.30	1.04	1.18	1.23	1.35	1.45	1.48	9.45
6	$36^\circ$	$75^\circ$	9.31	0.15	0.57	0.65	0.85	0.96	1.03	1.14	1.24	1.35	1.40	9.31
16	$33^\circ$	$56^\circ$	9.69	9.99	0.25	0.53	0.70	0.93	1.10	1.23	1.37	1.44	1.44	9.21
4	$32^\circ$	$107^\circ$	9.47	9.65	9.87	0.45	0.72	0.89	1.00	1.11	1.24	1.35	1.36	9.17
11	$27^\circ$	$123^\circ$	9.27	9.87	0.20	0.37	0.53	0.76	0.91	1.16	1.31	1.45	1.48	8.97

TABLE III 366000  
Go—G9

No.	<i>b</i>	<i>l</i>	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$3 \log \sin$
14	57°	81°	9.77	0.37	0.67	0.97	1.30	1.54	1.72	1.97	2.22	2.37	2.46	9.77
13	53°	111°	0.01	0.58	0.91	1.16	1.32	1.58	1.75	1.98	2.20	2.32	2.37	9.71
15	48°	62°	—	9.91	0.45	0.81	1.17	1.50	1.74	1.94	2.11	2.26	2.33	9.61
5	42°	91°	9.48	0.08	0.66	0.97	1.22	1.43	1.64	1.86	2.06	2.21	2.34	9.48
12	41°	123°	0.05	0.49	0.73	0.91	1.15	1.39	1.62	1.82	2.04	2.21	2.28	9.45
6	36°	75°	9.91	0.26	0.59	0.81	1.02	1.33	1.56	1.77	1.98	2.13	2.19	9.31
16	33°	56°	9.99	0.21	0.57	0.85	1.07	1.26	1.46	1.63	1.83	1.93	1.93	9.21
4	32°	107°	0.01	0.25	0.47	0.79	1.00	1.24	1.46	1.69	1.82	1.92	1.92	9.17
11	27°	123°	9.27	9.87	0.17	0.50	0.80	1.02	1.19	1.42	1.64	1.78	1.81	8.97

TABLE Ia 366000  
A0—A9

No.	<i>b</i>	<i>l</i>	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$3 \log \cos$ ( <i>l</i> —325°)
9	+3°	106°	0.37	0.93	1.25	1.48	1.69	1.85	2.03	2.21	2.34	2.40	2.40	9.67
8	—2°	92°	—	0.29	0.54	0.97	1.21	1.54	1.85	2.02	2.06	2.07	—	9.34
19	—1°	81°	9.93	0.42	0.71	0.95	1.21	1.43	1.57	1.71	1.81	1.84	—	8.93

TABLE IIa 366000  
F0—F9

No.	<i>b</i>	<i>l</i>	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$3 \log \cos$ ( <i>l</i> —325°)
9	+3°	106°	9.97	0.27	0.51	0.75	1.20	1.47	1.65	1.80	2.02	2.20	2.22	9.67
8	—2°	92°	0.24	0.52	0.64	0.86	1.11	1.33	1.59	1.80	1.89	1.89	—	9.34
19	—1°	81°	9.23	9.63	0.19	0.49	0.70	0.89	1.04	1.31	1.50	1.54	—	8.93

TABLE IIIa 366000  
Go—G9

No.	<i>b</i>	<i>l</i>	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$3 \log \cos$ ( <i>l</i> —325°)
9	+3°	106°	0.45	0.78	0.93	1.10	1.35	1.55	1.79	2.03	2.27	2.47	2.50	9.67
8	—2°	92°	0.04	0.38	0.66	0.92	1.15	1.45	1.71	1.94	1.99	1.99	—	9.34
19	—1°	81°	8.93	9.63	9.71	9.93	0.68	1.00	1.16	1.37	1.48	1.50	—	8.93

On the basis of the above formulae (2) and (3) shifting the curves  $\log [\sin^3 b N_b(m)]$  and  $\log [\cos^3 (l-325^\circ) N_l(m)]$  to the left along the axis (*m*) by  $\xi \log \operatorname{cosec} b$  and  $\xi \log \sec (l-325^\circ)$ , respectively, we must obtain the curves  $N_{\pi}(m)$  and  $N_{325^\circ}(m)$  as those of the tables IV, V, VI IVa, Va, VIa

for the galactic pole and for the direction to the galactic centre.

In tables IV, V, VI every two lines correspond to one area; the first line contains the values of *m*' diminished by  $\xi \log \operatorname{cosec} b$ , the second one—the values of  $\log \sin^3 b N_b(m)$ .

The tables IVa, Va, VIa, analogous to tables IV, V, VI, refer, however, to the second case, viz., the direction to the galactic centre, and contain the values of  $\log [\cos^3 (l-325^\circ) N_l(m)]$ ; therefore, while tabulating them we made use of tables Ia, IIa, IIIa, referring to the same case.

On the basis of separate tables IV, V, VI, IVa, Va, VIa the mean average curves  $\log N_{\pi}(m)$  and  $\log N_{325^\circ}(m)$  were drawn. All the six curves are shown on Fig. 1 and Fig. 2.

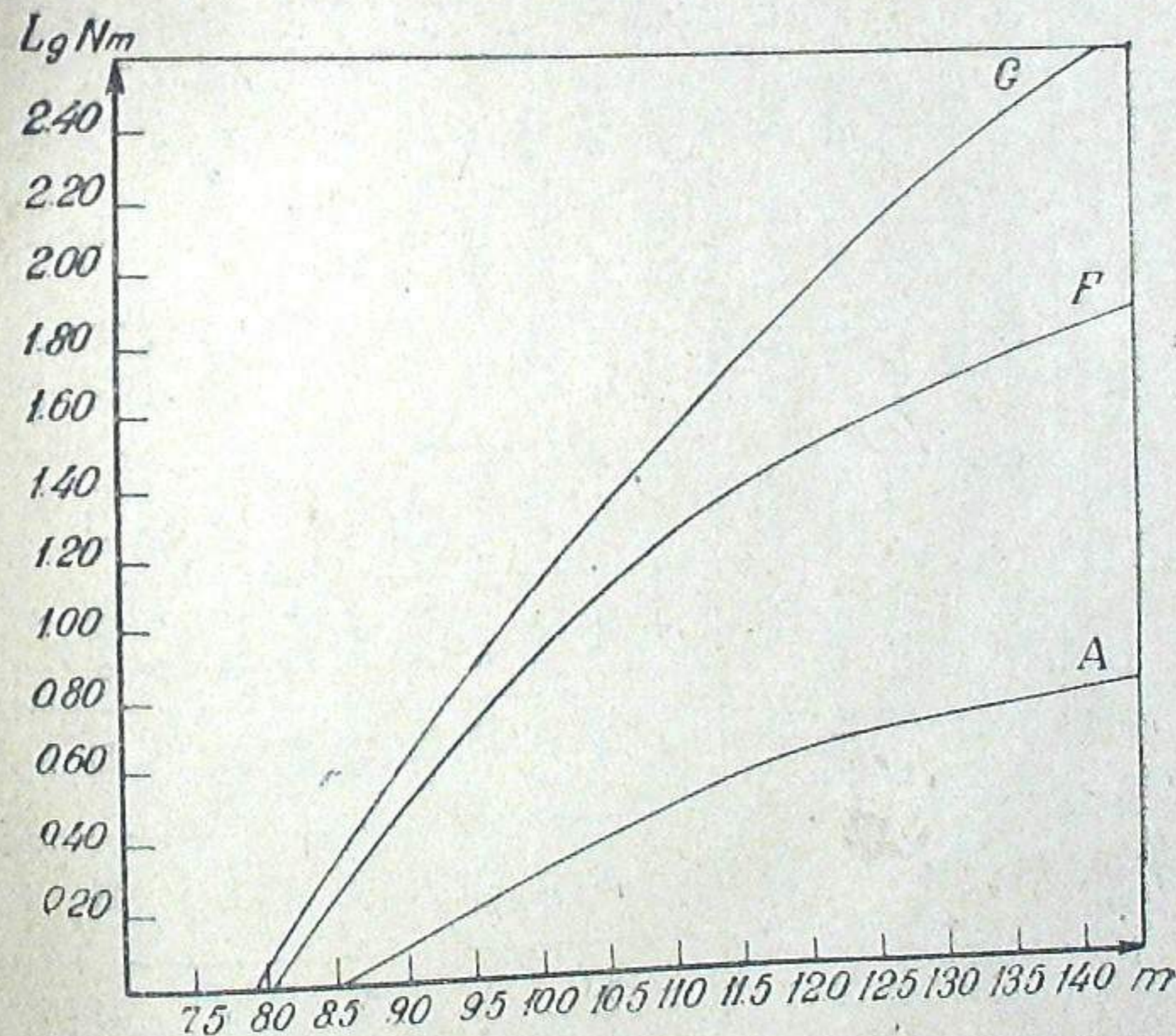


Fig. 1 5b.

TABLE IV G660000  
A0-A9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$5 \log \cos ec b$
14	57°	81°	—	—	9.62	10.12	10.62	11.12	11.62	12.12	12.62	13.12	13.62	—0.38
13	53°	111°	8.51	9.01	9.51	10.01	10.51	11.01	11.51	12.01	12.51	13.01	13.51	—0.49
15	48°	62°	—	—	9.61	9.61	9.91	10.21	10.51	11.01	11.51	12.01	12.51	—0.64
5	42°	91°	—	—	—	9.63	10.13	10.63	11.13	11.63	12.13	12.63	13.13	—0.87
12	41°	123°	8.05	8.55	9.05	9.55	10.05	10.55	11.05	11.55	12.05	12.55	13.05	—0.95
6	36°	75°	7.84	8.34	8.84	9.34	9.84	10.34	10.84	11.34	11.84	12.34	12.84	—1.16
16	33°	56°	9.51	9.81	0.16	0.25	0.36	0.41	0.41	0.44	0.47	0.47	0.47	—1.32
4	32°	107°	7.68	8.12	8.62	9.12	9.62	10.12	10.62	11.12	11.62	12.12	12.62	—1.38
11	27°	123°	9.45	9.57	9.97	0.12	0.17	0.29	0.39	0.51	0.54	0.57	0.58	—1.71

TABLE V G660000  
F0-F9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$5 \log \cos ec b$
14	57°	81°	8.62	9.12	9.62	10.12	10.62	11.12	11.62	12.12	12.62	13.12	13.62	—0.38
13	53°	111°	8.51	9.01	9.51	10.01	10.51	11.01	11.51	12.01	12.51	13.01	13.51	—0.49
15	48°	62°	0.09	0.39	0.72	0.91	1.09	1.31	1.40	1.51	1.65	1.73	1.78	—0.64
5	42°	91°	8.13	8.63	9.13	9.63	10.13	10.63	11.13	11.63	12.13	12.63	13.13	—0.87
12	41°	123°	9.96	0.48	0.59	0.82	1.05	1.24	1.38	1.50	1.57	1.64	1.71	—0.95
6	36°	75°	7.84	8.34	8.84	9.34	9.84	10.34	10.84	11.34	11.84	12.34	12.84	—1.16
16	33°	56°	9.31	0.15	0.57	0.65	0.85	0.96	1.03	1.14	1.24	1.35	1.40	—1.32
4	32°	107°	7.68	8.18	8.68	9.18	9.68	10.18	10.68	11.18	11.68	12.18	12.68	—1.38
11	27°	123°	9.69	9.99	0.25	0.53	0.70	0.93	1.10	1.23	1.37	1.44	1.44	—1.71

TABLE VI G66000  
Go—G9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$5 \log \cos ec b$
14	57°	81°	8.62	9.12	9.62	10.12	10.62	11.12	11.62	12.12	12.62	13.12	13.62	-0.38
13	53°	111°	0.01	0.58	0.91	1.16	1.32	1.58	1.75	1.98	2.20	2.32	2.37	-0.49
15	48°	62°	—	8.86	9.36	9.86	10.36	10.86	11.36	11.86	12.36	12.86	13.36	-0.64
5	42°	91°	8.13	8.63	9.13	9.63	10.13	10.63	11.13	11.63	12.13	12.63	13.13	-0.87
12	41°	123°	0.05	0.49	0.73	0.91	1.15	1.39	1.62	1.82	2.04	2.21	2.28	-0.95
6	36°	75°	7.84	8.34	8.84	9.34	9.84	10.34	10.84	11.34	11.84	12.34	12.84	-1.16
16	33°	56°	9.99	0.21	0.57	0.85	1.07	1.26	1.46	1.63	1.83	1.93	1.93	-1.32
4	32°	107°	7.62	8.12	8.62	9.12	9.62	10.12	10.62	11.12	11.62	12.12	12.62	-1.38
11	27°	123°	9.27	9.87	0.17	0.50	0.80	1.02	1.19	1.42	1.64	1.78	1.81	-1.71

TABLE IVa G66000  
Ao—A7

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$5 \log \sec (l-325^\circ)$
9	+3°	106°	8.45	8.95	9.45	9.95	10.45	10.95	11.45	11.95	12.45	12.95	13.45	9.45
8	-2°	92°	0.37	0.93	1.25	1.48	1.69	1.85	2.03	2.21	2.34	2.40	2.40	8.90
19	-1°	81°	7.21	7.71	8.21	8.71	9.21	9.71	10.21	10.71	11.21	11.71	12.21	8.21

TABLE Va G66000  
Fo—F9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$5 \log \sec (l-325^\circ)$
9	+3°	106°	8.45	8.95	9.45	9.95	10.45	10.95	11.45	11.95	12.45	12.95	13.45	9.45
8	-2°	92°	0.24	0.52	0.64	0.86	1.11	1.33	1.59	1.80	1.89	1.89	1.89	8.90
19	-1°	81°	7.21	7.71	8.21	8.71	9.21	9.71	10.21	10.71	11.21	11.71	12.21	8.21

TABLE VIa G66000  
Go—G9

No.	$b$	$l$	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	$5 \log \sec (l-325^\circ)$
9	+3°	106°	8.45	8.95	9.45	9.95	10.45	10.95	11.45	11.95	12.45	12.95	13.45	9.45
8	-2°	92°	0.04	0.38	0.66	0.92	1.15	1.45	1.71	1.94	1.99	1.99	1.99	8.90
19	-1°	81°	7.21	7.71	8.21	8.71	9.21	9.71	10.21	10.71	11.21	11.71	12.21	8.21

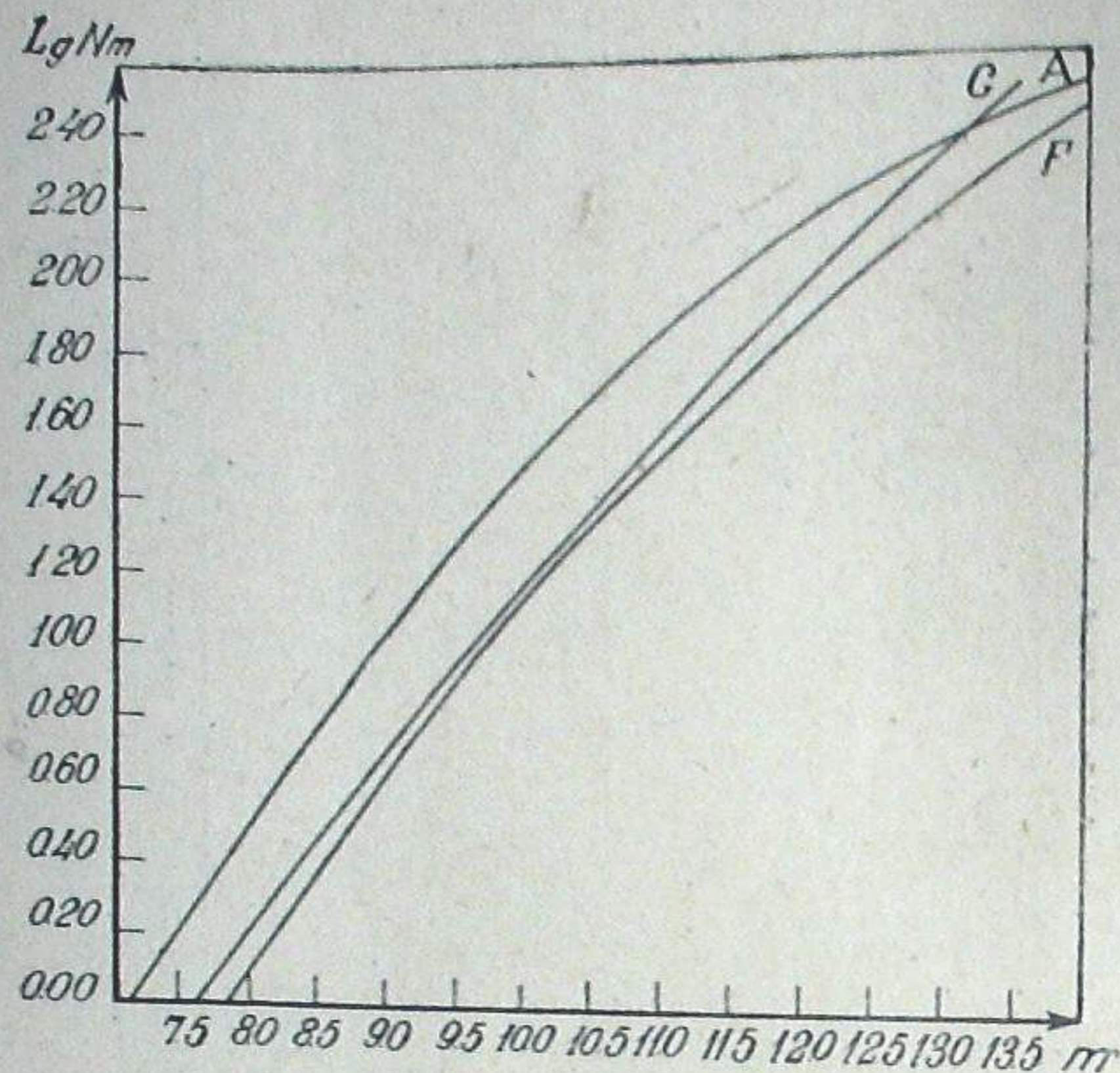


Fig. 2 5b.

It is to see from the tables that the average curves were drawn on the basis of counts in nine areas in the first case and in three areas in the second one.

On the basis of each table one average mean curve was drawn for A, F and G type stars. The distance between the curves drawn for different areas does not exceed 0.2 stellar magnitudes.

The small influence of cosmic absorption may, probably, be explained by the fact that the stars selected are at a relatively short distance from us. Moreover, the areas, which served for the investigation of distribution in  $\tau$ , are located in nearly adjacent latitudes.

In our precedent work Schwarzschild's equation took the following form:

$$\begin{aligned} A_{\tau} \left( \frac{m'}{2} \right) &= \omega \int_0^{\infty} D(\tau) \varphi(m' - 5 \log \tau) \tau^2 d\tau \\ A_{325^{\circ}} \left( \frac{m'}{2} \right) &= \omega \int_0^{\infty} D(c) \varphi(m' - 5 \log c) c^2 dc \end{aligned} \quad (4)$$

To solve these equations numerically by Bok's method we introduce in (4)  $5 \log \tau - 5 = n$ , hence,  $\tau = 10^{0.2n-1}$ ,  $d\tau = 10^{0.2n+1} 0.2 \log_e 10 dn$ . We obtain

$$A_{\tau} \left( \frac{m}{2} \right) = \omega \int_{-\infty}^{+\infty} D(\tau) \varphi(m-n) 10^{0.6n+3} 0.2 \log_e 10 dn$$

or, denoting

$$10^{0.6n+3} 0.2 \log_e 10 = k(n) \quad \text{and} \quad D(\tau) k(n) = d(n)$$

$$A_{\tau} \left( \frac{m}{2} \right) = \omega \int_{-\infty}^{+\infty} d(n) \varphi(m-n) dn. \quad (5)$$

This equation may be utilized also for the distribution of stars in the direction towards the galactic centre.

On the basis of (5) we write a series of equations for a series of values  $A_{\tau} \left( \frac{m}{2} \right)$  and  $A_{325^{\circ}} \left( \frac{m}{2} \right)$  which are obtained numerically taking the derivatives from the curves of Fig. 1 and Fig. 2 in points  $m_1, m_2, \dots, m_i$ . To solve these equations as a system of linear equations only the knowledge of the luminosity function  $\varphi(M)$  is required.

To derive the luminosity function of A and F type stars we made use of the catalogue: «The Spectroscopic Absolute Magnitudes and Parallaxes of 4179 Stars» by W. S. Adams, A. H. Joy, M. L. Humason and Ada Margaret Brayton<sup>1</sup>. Counting the stars A and F over the whole catalogue and arranging them after absolute magnitudes we obtain the possibility to draw, according to the below indicated method, certain conclusions about the luminosity function  $\varphi(M)$ . The results of counts of stars of various absolute magnitudes, according to the named catalogue for the stars A and F are given in Table VII in columns 3 and 5.

The luminosity function for G type stars was taken from the work by Van Rhijn and Schwassmann<sup>2</sup>. For G type stars the luminosity function has two maxima corresponding to giants and dwarfs; this circumstance as was shown by the experience of calculation entails certain difficulties for the solution of such numerical equations as (6), (7) and (8).

We shall see below that in calculating the two maxima are reduced to one. Table VII shows that the number of F type stars augments with diminishing brightness from  $1^m 25$ ,  $1^m 75$  to  $3^m 25$  dropping sharply for  $3^m 75$ ,  $4^m 25$  and  $4^m 75$ . However, it is to see from the table that F type stars have two maxima: the first maximum is due to giants, the second one—to dwarfs. As the great majority of F type stars belongs to dwarfs and the number of

TABLE VII 366020

M	$\varphi(m)$	A	A <sub>c</sub>	F	F <sub>c</sub>	G
-3.0-2.6	-2.75			2		0.001·10 <sup>6</sup>
-2.5-2.1	-2.25			2		0.001·10 <sup>6</sup>
-2.0-1.6	-1.75	1		7		0.001·10 <sup>6</sup>
-1.5-1.1	-1.25			5		0.002·10 <sup>6</sup>
-1.0-0.6	-0.75			7		0.004·10 <sup>6</sup>
-0.5-0.1	-0.25	3		9		0.011·10 <sup>6</sup>
+0.0+0.4	+0.25	4		1		0.03·10 <sup>6</sup>
+0.5+0.9	+0.75	24		1		0.08·10 <sup>6</sup>
+1.0+1.4	+1.25	52	14	8		0.17·10 <sup>6</sup>
+1.5+1.9	+1.75	145	52	24	8	0.26·10 <sup>6</sup>
+2.0+2.4	+2.25	154	154	75	63	0.30·10 <sup>6</sup>
+2.5+2.9	+2.75	18	52	162	187	0.22·10 <sup>6</sup>
+3.0+3.4	+3.25	4	14	354	354	0.24·10 <sup>6</sup>
+3.5+3.9	+3.75			214	187	1.15·10 <sup>6</sup>
+4.0+4.4	+4.25			52	63	5.50·10 <sup>6</sup>
+4.5+4.9	+4.75	3		3	8	9.35·10 <sup>6</sup>
+5.0+5.4	+5.25	3				12.60·10 <sup>6</sup>
+5.5+5.9	+5.75					14.80·10 <sup>6</sup>
+6.0+6.4	+6.25					14.10·10 <sup>6</sup>
+6.5+6.9	+6.75					11.00·10 <sup>6</sup>
+7.0+7.4	+7.25					7.60·10 <sup>6</sup>
+7.5+7.9	+7.75					4.79·10 <sup>6</sup>
+8.0+8.4	+8.25					
+8.5+8.9	+8.75					

giants among them is relatively small, we neglected the maximum due to giants and retained the one due to dwarfs taking into consideration that among faint stars of Bergedorf's catalogue there are few giants of the F type.

The stars of the type A do not show two maxima as they are not sharply divided into giants and dwarfs.

The number of A type stars augments with diminishing brightness beginning from the absolute magnitude 0<sup>m</sup>.75 down to 2<sup>m</sup>.25, a more rapid increase making itself noticeable from 1<sup>m</sup>.25 to 2<sup>m</sup>.25, and drops abruptly for 2<sup>m</sup>.75, 3<sup>m</sup>.25.

Taking into account all that was said about A and F type stars we can draw some conclusions about the luminosity curve. In fact, if we assume that the curve  $\varphi(M)$  is symmetrical with regard to the maximum ( $m_0$ ), the effect of selection can be eliminated easily enough if we adopt for the relative value of  $\varphi(m_0+k)$  the geometrical mean from values of Table VII for ( $m+k$ ) and ( $m-k$ ). Moreover, the assumption is made that beyond the limits

of the intervals 1<sup>m</sup>.25-3<sup>m</sup>.25 and 1<sup>m</sup>.25-4<sup>m</sup>.75 for A and F type stars, respectively,  $\varphi(M) = 0$ . This assumption is, apparently, fulfilled accurately enough for our purposes and it greatly simplifies the computations. Thus, we obtain for the function  $\varphi(M)$  values listed in 4th and 6th columns for A and F type stars, respectively.

Taking into account all the above we come to the conclusion that the great majority of the stars has an absolute brightness comprised in the intervals +1<sup>m</sup>.25-+3<sup>m</sup>.25, +1<sup>m</sup>.75-+4<sup>m</sup>.75 and +0<sup>m</sup>.25-+7<sup>m</sup>.75 for A, F and G types respectively.

In the equation (5)  $M = m - n$ ; therefore, we can write

$$\text{for } A + 1.25 < m - n < + 3.25 \text{ or } m - 1.25 > n > m - 3.25,$$

$$\text{,, } F + 1.75 < m - n < + 4.75 \text{ ,, } m - 1.75 > n > m - 4.75,$$

$$\text{,, } G + 0.25 < m - n < + 7.75 \text{ ,, } m - 0.25 > n > m - 7.75.$$

Using these limits we write the equation (5) for the stars of these types

$$A_{\frac{\pi}{2}, c}(m) = \omega \int_{m-3.25}^{m-1.25} d(n) \varphi(m-n) dn,$$

$$A_{\frac{\pi}{2}, c}(m) = \omega \int_{m-4.75}^{m-1.75} d(n) \varphi(m-n) dn,$$

$$A_{\frac{\pi}{2}, c}(m) = \omega \int_{m-7.75}^{m-0.25} d(n) \varphi(m-n) dn.$$

For the numerical solution we obtain equations (6), (7) and (8), where  $\omega = 1$ .

$$A_{\frac{\pi}{2}, c}(m) = \frac{1}{2} d(m-1.25) \varphi(1.25) + d(m-1.75) \varphi(1.75) + d(m-2.25) \varphi(2.25) + \\ + d(m-2.75) \varphi(2.75) + \frac{1}{2} d(m-3.25) \varphi(3.25). \quad (6)$$

$$A_{\frac{\pi}{2}, c}(m) = \frac{1}{2} d(m-1.75) \varphi(1.75) + d(m-2.25) \varphi(2.25) + d(m-2.75) \varphi(2.75) + \\ + d(m-3.25) \varphi(3.25) + d(m-3.75) \varphi(3.75) + d(m-4.25) \varphi(4.25) + \\ + \frac{1}{2} d(m-4.75) \varphi(4.75). \quad (7)$$

$$A_{\frac{\pi}{2}, c}(m) = \frac{1}{2} d(m-0.25)\varphi(0.25) + d(m-0.75)\varphi(0.75) + d(m-1.25)\varphi(1.25) + d(m-1.75)\varphi(1.75) + d(m-2.25)\varphi(2.25) + d(m-2.75)\varphi(2.75) + d(m-3.25)\varphi(3.25) + d(m-3.75)\varphi(3.75) + d(m-4.25)\varphi(4.25) + d(m-4.75)\varphi(4.75) + d(m-5.25)\varphi(5.25) + d(m-5.75)\varphi(5.75) + d(m-6.25)\varphi(6.25) + d(m-6.75)\varphi(6.75) + d(m-7.25)\varphi(7.25) + \frac{1}{2} d(m-7.75)\varphi(7.75). \tag{8}$$

If we obtain for A, F and G stars a series of values  $A_{\frac{\pi}{2}}$  and  $A_{325^\circ}$  corresponding to a series of values of  $m_i$ , we shall have a series of equations of the same kind as (6), (7) and (8).

To obtain the values  $A_{\frac{\pi}{2}}$ ,  $A_{325^\circ}$  we take the derivatives from the curves plotted on Fig. 1 and Fig. 2.

In Table VIII the values of  $m_i$  are listed in uneven columns.

TABLE VIII 666020

$m$	$A_{\frac{\pi}{2}}$	$m$	$A_c$	$m$	$A_{\frac{\pi}{2}}$	$m$	$A_c$	$m$	$A_{\frac{\pi}{2}}$	$m$	$A_c$
8.75	5.20	7.50	2.5	8.25	1.6	8.25	2.3	8.25	2.3	8.00	2.0
9.25	8.00	8.00	4.2	8.75	2.6	8.75	4.4	8.75	4.2	8.50	3.4
9.75	8.60	8.50	7.4	9.25	4.2	9.25	6.7	9.25	6.8	9.00	5.6
10.25	10.80	9.00	12.0	9.75	6.1	9.75	10.5	9.75	11.5	9.50	9.8
10.75	10.60	9.50	19.0	10.25	8.6	10.25	17.4	10.25	16.2	10.00	15.2
11.25	12.80	10.00	31.6	10.75	9.6	10.75	25.6	10.75	27.8	10.50	31.2
11.75	15.40	10.50	41.0	11.25	11.0	11.25	35.6	11.25	42.6	11.00	35.6
12.25	8.80	11.00	53.6	11.75	12.0	11.75	47.0	11.75	66.4	11.50	61.4
12.75	9.80	11.50	65.8	12.25	15.0	12.25	68.0	12.25	93.6	12.00	97.6
13.25	5.00	12.00	92.0	12.75	16.6	12.75	98.0	12.75	124.0	12.50	136.0
13.75	1.40	12.50	100.0	13.25	18.2	13.25	120.0	13.25	150.0	13.00	204.0
14.25	1.24	13.00	108.0	13.75	24.6	13.75	134.0	13.75	160.0	13.50	244.0
		13.50	106.0	14.25	26.2	14.25	142.0	14.25	190.0	14.00	316.0
		14.00	128.0	14.75	27.0	14.75	160.0	14.75	250.0	14.50	438.0
								15.25	296.0	15.00	450.0
								15.75	320.0	15.50	620.0
								16.25	450.0	16.00	640.0
								16.75	550.0	16.50	660.0
								17.25	570.0	17.00	670.0
								17.75	620.0	17.50	680.0

Note: Columns 1, 2, 5, 6, 9, 10 correspond to Fig. 1 and columns 3, 4, 7, 8, 11, 12 — to Fig. 2.

The derivatives  $A_{\frac{\pi}{2}}(m)$ ,  $A_{325^\circ}(m)$  from the curves plotted on Fig. 1 and Fig. 2 in points corresponding to  $(m_i)$  are given in even columns.

Thus, there are 4 columns containing the data relative to A type stars; the first two columns refer to the distribution of stars in the direction to the galactic pole; the third and the fourth column refer to the distribution in the direction to the galactic centre. In the first and third columns the values of  $m_i$  are given; in the second and fourth columns the derivatives from those points are listed.

In Table VIII the columns are divided by horizontal lines. The values of derivatives above this line are derived from the curves of Fig. 1 and Fig. 2, which are actually based on star counts. Under the lines the derivatives (i. e. the values  $A_{\frac{\pi}{2}}$ ,  $A_{325^\circ}$ ) obtained by extrapolation are given.

The equations (6), (7) and (8) were solved numerically in a somewhat different way than in the foregoing work. Successive approximations were used without changing generally Bok's method.

In the first approximation of numerical equations the value  $d(n)$ , which in the equation is multiplied by the maximum value  $\varphi_{max}(M)$ , is adopted as the unknown quantity.

For the maximum value  $\varphi_{max}(M)$  we take  $\Sigma\varphi(M)$ , i. e. the sum of all the  $\varphi(M)$  for the given type.

The maximum value  $\varphi_{max}(M)$  corresponds in equations (6) and (7) to the middle member because the luminosity function for A and F types was drawn symmetrically. For the type G the luminosity function  $\varphi(M)$  has two maxima, as was indicated above; however, according to our method of solution of equation (8) we reduced them to a single maximum value  $\varphi_{max}(M)$  in the following way.

It is known that the real brightness of the star is defined by the formula  $I = 10^{-0.4M}$ . On the other hand, for a star of a constant apparent brightness we have  $I = c_1 r^2$ , where  $r$  is the distance; hence we obtain the expression for the volume within which the stars of absolute brightness  $M$  have apparent magnitudes comprised between  $m$  and  $m+dm$ , where  $dm$  is assumed to be constant:

$$V = 4\pi r^2 dr = \frac{4\pi}{5} r^3 dm = \frac{4\pi}{5} \left(\frac{I}{c_1}\right)^{3/2} dm,$$

$$V = c \cdot I^{3/2} = c \cdot 10^{-0.6M}$$

because under the given absolute value  $dm = \frac{5dr}{r}$ .

To obtain the mean value of the volume  $10^{-0.6M}$ , within which the stars of an absolute magnitude  $(M)$  will seem to us as having absolute apparent



brightness between  $(m - \frac{1}{2})$  and  $(m + \frac{1}{2})$ , we must make use of the following formula:

$$\bar{V} = c \cdot 10^{-0.6\bar{M}} = \frac{\int \varphi(M) \cdot c \cdot 10^{-0.6M} dM}{\int \varphi(M) dM} \quad (9)$$

Hereupon we find the absolute magnitude  $\bar{M}$ .

The solution of equation (9) gives for the mean value  $M = 4.25$ . After this, we solve the equation (8) in the first approximation, considering that  $\varphi(4.25)$  has the value equal to the sum of all real  $\varphi(M)$  and  $\varphi(M)$  under  $M = 4.25$  equals 0.

It must be remarked that Van Rhijn and Schwassmann give the luminosity function  $\varphi(M)$  for G from  $M = -4.5$  to  $M = +9.0$ ; however, we shortened this interval taking the value  $\varphi(M)$  from  $M = 0.25$  to  $M = 7.75$ .

It was done with the purpose to diminish the number of equation members for the numerical solution by Bok's method. In spite of this shortening of the interval of  $M$ , there are 16 members in equation (8), much more than in (6) or (7).

Having for A and F the value  $\bar{M}$  under which  $\varphi(M)$  attains the maximum and for the type G the mean value  $M$  calculated according to formula (9) we solve the system of equations (6), (7) and (8) by means of successive approximations.

We considered in each approximation beginning from the second one in each equation as known from the precedent approximation all the terms except the one having for coefficient  $\varphi(M)$ . These values of  $d$  were calculated in the new approximation. But for a definitive solution it is necessary that the number of equations be sufficient; i. e. the extrapolation of  $A_x(m)$  and  $A_{325^\circ}(m)$  is necessary. These extrapolated values are given as was shown above in Table VII below the horizontal line.

To calculate the values  $D(\zeta)$  and  $D(c)$ , we solved the equations (6), (7) and (8) in three approximations, the final results being obtained after the third approximation. Here we must indicate that the densities determined  $D(\zeta)$  and  $D(c)$  are connected with real densities by means of the relation (10). If we denote the obtained densities by  $D'(\zeta)$  and the real ones by  $D(\zeta)$  they will be connected as below:

$$D(\zeta) = \frac{D'(\zeta)}{k} = D'(\zeta) \frac{\int \varphi(M) dM}{\omega} = D'(\zeta) k' \quad (10)$$

where  $\omega$  is the solid angle.

Multiplying  $D'(\zeta)$  by the corresponding coefficient  $k'$ , in this case

$$k' = \frac{\int \varphi(M) dM}{\omega},$$

we obtain the real density.

In the same way all the numerical values of  $D'(\zeta)$  and  $D'(c)$  obtained are reduced to real densities  $D(\zeta)$  and  $D(c)$  in one unit of volume.

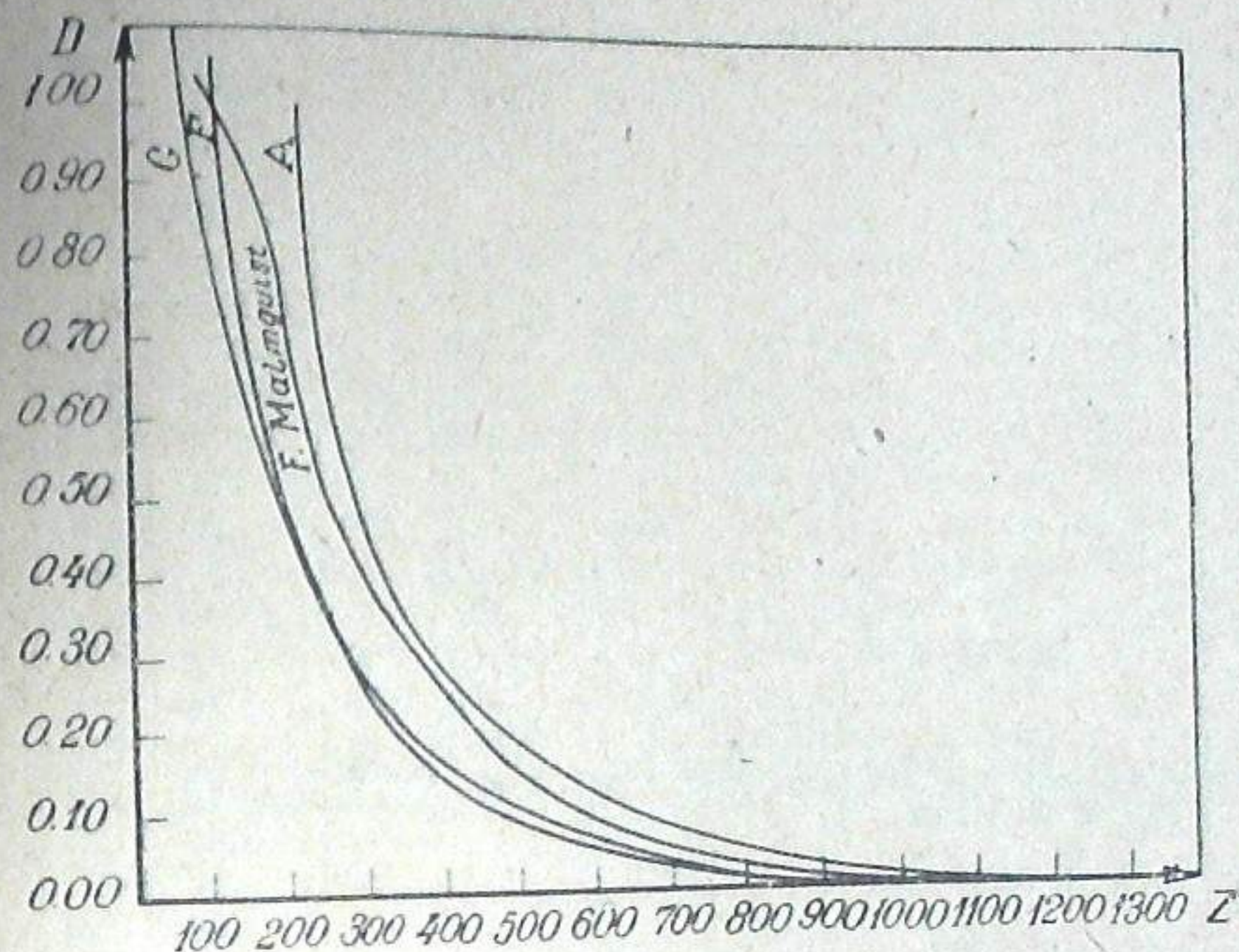


Fig. 3 65b.

These results are given in Table IX.

In calculating we had to assume that for A type stars the density in the direction to the pole and to the galactic centre is constant as far as 200 and 126 parsec, respectively. The density of F and G type stars in both directions is assumed to be constant up to the distance of 100 and 50 parsec, respectively; this is reflected in Table IX. If we assume the density in the vicinity of the Sun to be equal to a unit we obtain according to Table X the curves plotted on Fig. 3 and Fig. 4 which show that the density decreases relatively with the distance.

To check our results we give the comparison of our data (the distribution of density in the direction  $\zeta$ ) with those by other authors. For this purpose Table IX from the work by Van Rhijn and Schwassmann<sup>6</sup> was taken in which the results of star density determinations obtained by Van Rhijn and Oort are compared. We reduced the densities of this table to our distances by means of the formula given by the authors:

$$\zeta = r \sin 50^\circ.$$

TABLE IX ცხრილი

A0-A9				F0-F9			G0-G9			
Z	$D(z)10^{-4}$	Z	$D(z)10^{-4}$	Z	$D(z)10^{-4}$	$D(z)10^{-4}$	Z	$D(z)10^{-4}$	Z	$D(z)10^{-4}$
200	3.470	112	11.414	100	8.113	12.365	50	131.80	45	169.51
250	3.550	141	8.733	126	7.556	16.570	63	149.04	56	143.06
320	1.378	180	8.356	160	5.368	10.278	72	123.05	71	179.17
400	1.214	225	6.658	200	4.236	7.260	100	137.54	89	141.22
500	0.525	280	4.854	250	4.005	6.920	126	69.92	112	124.66
630	0.295	355	4.790	320	1.774	5.508	160	84.18	141	134.56
790	0.281	450	2.838	400	1.118	3.820	200	60.50	180	69.56
1000	0.030	560	1.935	500	0.480	1.918	250	51.06	225	63.94
1260	0.046	710	1.033	630	0.430	1.555	320	34.50	280	57.73
1600	0.006	892	0.902	790	0.212	1.302	400	19.55	355	39.56
		1120	0.450	1000	0.098	0.812	500	10.58	450	34.04
		1410	0.262	1260	0.076	0.410	630	3.75	560	8.28
		1800	0.115	1600	0.040	0.218	790	2.30	710	5.29
		2250	0.066	2000	0.020	0.122	1000	1.52	892	3.68
							1260	0.90	1120	1.98
							1600	0.48	1410	1.33
							2000	0.34	1800	0.69
							2500	0.21	2250	0.37
							3200	0.12	2800	0.18
							4000	0.05	3550	0.09

The results are presented in Table X.

TABLE X ცხრილი

Z	A			F			G		
	Rh	O	V	Rh	O	V	Rh	O	V
40	1.00	—	1.00	1.00	1.00	1.00	1.00	1.00	1.00
60	—	—	1.00	—	—	1.00	—	—	1.00
80	1.00	0.82	1.00	1.00	—	1.00	—	—	1.00
130	0.53	0.55	1.00	—	—	0.93	1.00	0.92	0.93
150	0.30	1.00	1.00	—	—	0.93	—	—	0.54
230	0.12	0.18	1.00	0.62	0.75	0.75	0.58	0.77	0.51
310	0.08	0.08	0.50	0.32	0.55	0.50	0.38	0.52	0.42
390	0.05	0.04	0.35	0.17	0.37	0.26	0.31	0.36	0.28
580	0.02	0.03	0.11	0.11	0.25	0.14	0.30	0.25	0.15
770	0.01	0.02	0.08	0.03	0.08	0.06	0.14	0.10	0.05
960	0.00	0.00	0.02	0.01	0.04	0.03	0.06	0.04	0.02
1900	0.00	0.00	0.00	—	—	0.02	0.02	0.02	0.01
				0.00	0.00	0.00	0.00	0.00	0.00

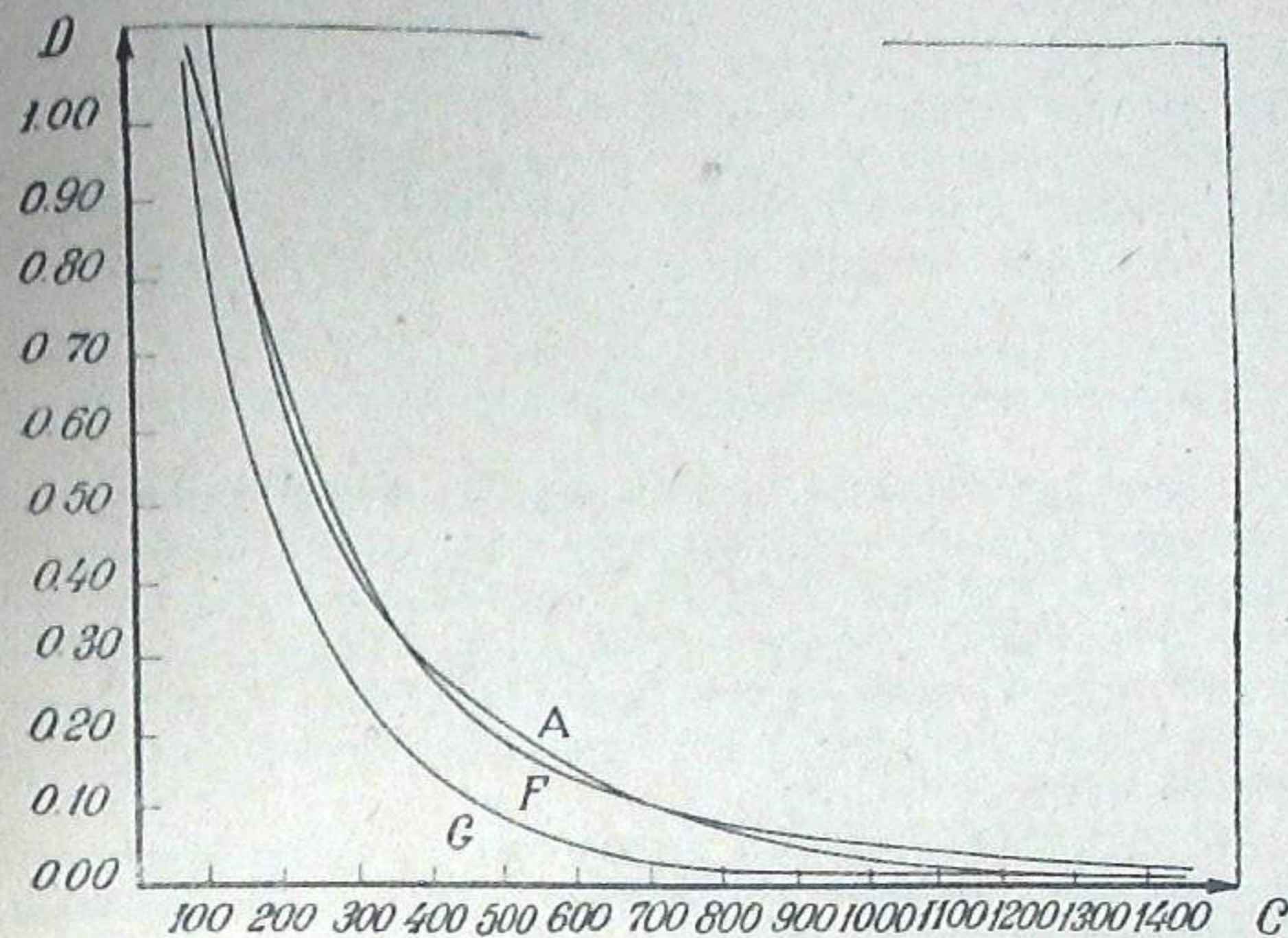


Fig. 4 ნახ.

It is to see from this table that our results agree quite well with those obtained by Van Rhijn and Oort, if we pass over the fact that the relative distribution of A type stars is a little different.

August, 1937.

Literature: ლიტერატურა:

1. Bull. Abast. Obs. 1, p. 87, 1937.
2. Berg. Spektr.—Durchm. B. I, 1935.
3. Invest. of the Stars in High Gal. Lat. II, 1936.
4. Aph. J. 81, p. 187, 1935.
5. Mitt. Hamb. Sternw. Berg. B. 7, 41 p. 151, 1935.

A, F და G ტიპის ვარსკვლავთა განაწილება გალაქტიკის სიბრტყის მართობულ და სივრცით სიბრტყეში

ა. ვაშაკიძე

(რეზუმე)

ეს წერილი წარმოადგენს გაგრძელებას ავტორის მიერ უკვე გამოქვეყნებული შრომისა „F ტიპის ვარსკვლავთა სივრცეში განაწილების შესახებ“<sup>1</sup>. მთავარი ამოცანა იმაში მდგომარეობს, რომ გამოვკვლიოთ იქნეს A, F და G კლასის ვარსკვლავთა განაწილება როგორც გალაქტიკის სიბრტყის მართობული მიმართულებით (ჩრდილოეთ ნახევარსფეროში) ისე თვით გალაქტიკის სიბრტყეში, გალაქტიკის ცენტრის მიმართულებით.

როგორც პირველ წერილში, ისევე აქ, მიჩნეულია, რომ მზის ახლოს მყოფი ფენები, გალაქტიკის სიბრტყის მიმართ და ურთი-ერთ პარალელურნი არიან.  $N_6(m)$  და  $N_4(m)$  მრუდები, რომელნიც  $m$  ხილულ სიდიდეზედ უფრო კაშკაშა ვარსკვლავთა რიცხვს გამოხატავენ რომელიმე გალაქტიკური განედის და გრძედის კვადრატულ გრადუსზედ, დაკავშირებულნი არიან (2) და (3) ფორმულებით  $N_{\frac{\pi}{2}}(m)$  და  $N_{325^\circ}(m)$  მრუდებთან, რომელნიც იმავეს გამოხატავენ გალაქტიკის  $b=90^\circ$  და  $l=325^\circ$ -სათვის.

(2) და (3) ფორმულა საშუალებას გვაძლევს  $N_{\frac{\pi}{2}}(m)$  და  $N_{325^\circ}(m)$  მრუ-

დებები უფრო მეტი სიზუსტით მივიღოთ, რადგანაც ამ ფორმულებით ნებისმიერი გალაქტიკური განედისათვის და გრძედისათვის აგებული  $N_6(m)$  და  $N_4(m)$  მრუდები ზემოდხსენებულ მრუდებზე მიიყვანება. ამ უკანასკნელთაგან კი ვარსკვლავთა სიმკვრივის განაწილება გამოიყვანება. ამ გზით ასეთ განაწილებას უფრო მეტი სიზუსტით მივიღებთ, ვიდრე იმ შემთხვევაში, თუ მრუდებს მხოლოდ გალაქტიკის პოლუსისა და ცენტრის მიმართულებით დათვლილ ვარსკვლავების საფუძველზე ავაგებდით.

თუ გალაქტიკის სხვადასხვა განედისათვის და გრძედისათვის აგებული  $N_6(m)$  და  $N_4(m)$  მრუდები არ ემთხვევიან გალაქტიკის პოლუსისათვის და ცენტრისათვის აგებულ მრუდებს, მაშინ ამ ნებისმიერ ( $b$  და  $l$ ) მიმართულებით უფრო მეტი ან ნაკლები შთანთქმა არსებობს ვარსკვლავურ სივრცეში, ვიდრე ზემოდხსენებულ მიმართულებით. ეს შთანთქმა კი განისაზღვრება მრუდებს შორის განსხვავების საფუძველზე.

A, F და G ტიპის ვარსკვლავთათვის  $N_{\frac{\pi}{2}}(m)$  და  $N_{325^\circ}(m)$  მრუდების

აგებისა და ვარსკვლავთა სიხშირის განაწილების გამოყვანისათვის Schwarzschild-ის ინტეგრალური განტოლება (1) იყო აღებული. ეს განტოლება ჩვენ J. Bok-ის მეთოდით ამოვხსენით.

ჩვენი გამოკვლევის ობიექტად უმთავრესად Schwassmann<sup>2</sup>-ის კატალოგში მოცემული ჩრდილოეთ ნახევარსფეროს Kapteyn-ის თორმეტი არე იყო აღებული.

I, II, III, Ia, IIa და IIIa ცხრილებში მოცემულია A, F და G ტიპის ვარსკვლავთათვის  $\sin^2 b N_6(m)$  და  $\cos^2(l-325^\circ)N_4(m)$ -ის მნიშვნელობანი. IV, V, VI, IVa, Va და VIa ცხრილებში თვითეული არესათვის მოცემულია ორ-ორი მწკრივი, რომელთაგანაც პირველი ეკუთვნის (2) და (3) გამოსახვათა მარცხენა მხარეს, ხოლო მეორე—მარჯვენას.

ამ ცხრილების საფუძველზე ზემოდ ხსენებულ კლასების ვარსკვლავთათვის ავაგეთ საშუალო  $N_{\frac{\pi}{2}}(m)$  და  $N_{325^\circ}(m)$  მრუდები შთანთქმის მხედველობაში

მიუღებლივ. ამ მრუდების გაწარმოებით ვღებულობთ (6), (7) და (8) განტოლებებში შემავალ  $A_{\frac{\pi}{2}}(m)$  და  $A_4(m)$  მნიშვნელობებს. განტოლებათა საბოლოო

გადაწყვეტისათვის საჭიროა მივიღოთ A, F და G კლასის ვარსკვლავთა აბსო-

ლუტური სიკაშკაშის მრუდები  $\varphi(M)$ . უკანასკნელნი A და F ტიპის ვარსკვლავთათვის გამოვიყვანეთ სპეციალური კატალოგიდან<sup>1</sup>, ხოლო G ტიპის ვარსკვლავთათვის კი ვისარგებლეთ Van Rhijn და Schwassmann<sup>2</sup>-ის შრომით. ამ ტიპის ვარსკვლავთა აბსოლუტური სიკაშკაშის მრუდები მოცემულია ცხრ. VII-ის სახით.

ცხრ. IX-ში მოცემულია ვარსკვლავთა სიმკვრივის კლება მოცულობის ერთეულში მანძილის მიხედვით გალაქტიკის პოლუსის და მისი ცენტრის მიმართულებით. ამ ცხრილის საფუძველზე აგებულია სათანადო მრუდები (ნახ. 3 და 4).

შემოწმების მიზნით ჩვენი შედეგები Van Rhijn-ის და Oort-ის მიერ სხვა გზით მიღებულ მონაცემებს შევადარეთ (ცხრ. X). როგორც შედარებამ გვიჩვენა, F და G ტიპის ვარსკვლავთათვის ჩვენი შედეგები ზემოდდასახელებულ ავტორთა შედეგებს უახლოვდება.

აგვისტო, 1937.