Gravitational radiation generated by cosmological phase transition magnetic fields

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We study gravitational waves generated by the cosmological magnetic fields induced via bubble collisions during the electroweak (EW) and QCD phase transitions. The magnetic field generation mechanisms considered here are based on the use of the fundamental EW minimal supersymmetric and QCD Lagrangians. The gravitational waves spectrum is computed using a magnetohydrodynamic turbulence model. We find that the gravitational wave spectrum amplitude generated by the EW phase transition peaks at a frequency of approximately 1–2 mHz, and is of the order of 10^{-20}–10^{-21}; thus this signal is possibly detectable by the Laser Interferometer Space Antenna (LISA). The gravitational waves generated during the QCD phase transition, however, are outside the LISA sensitivity bands.

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I. INTRODUCTION

There are various mechanisms that might generate gravitational waves in the early Universe. For reviews see [1–3]. A well-known one is the generation of gravitational waves during the cosmological electroweak (EW) or QCD phase transitions [4–7]. These mechanisms include bubble wall motions and collisions if the phase transition is first order [8], as well as cosmological magnetic fields and hydrodynamical or magnetohydrodynamic (MHD) turbulence [9–15].

A brief overview of our present work is as follows:

(i) We use basic EW and QCD Lagrangians to derive the magnetic fields created in the plasma during the EW phase transitions and the QCD phase transitions.
(ii) In the framework of the standard MHD theory the magnetic field produces turbulence in the plasma. The value of the Alfvén velocity, v_A, is a key parameter when considering the generation of gravitational waves.
(iii) From the parameters found via our theory of the EW and QCD phase transitions, we use the formalism of the gravitational waves generation by MHD turbulence to estimate the peak frequency and amplitude of the gravitational waves produced during these cosmological phase transitions.

The direct detection of the relic gravitational waves will open the new prospects to understand the physical processes in the early Universe [1]. The main objective of the present paper is to study if the gravitational waves produced by the magnetic fields created during cosmological phase transitions could be detected by the current and/or nearest future missions. To be observable the gravitational waves signal must satisfy two conditions: it must be within the observation frequency bands, and its amplitude should exceed substantially the instrumental noise [for the stochastic backgrounds signal to noise ratio (SNR) must be taken to be 5] [1]. Our present study is to determine if the gravitational waves produced either by the EW or the QCD phase transitions might be detectable by the Laser Interferometer Space Antenna (LISA), whose sensitivity reaches maximum at frequencies 1–100 mHz [3].

In order to produce a detectable gravitational wave signal the cosmological phase transition must be first order, so that bubbles of the new vacuum nucleate within the false vacuum [8] at a critical temperature. Otherwise, there is a crossover transition. It has been shown that with the standard EW model there is no first order phase transitions [16], and there is no explanation of baryogenesis. However, there has been a great deal of activity in the minimal supersymmetric (MSSM) extension of the standard EW model [17]. In this case a MSSM having a top squark with a mass similar to the Higgs mass leads to the first order EW phase transition. This EW MSSM theory is consistent with baryogenesis [18]. On the other hand, recent lattice QCD calculations have shown [19] that the QCD phase transition is a first order cosmological phase transition, with bubble nucleation and collisions.

The EW phase transition is particularly interesting for exploring possible cosmological magnetic fields since the electromagnetic field along with the $W^{\pm}$ and Z fields are the gauge fields of the standard model. For the QCD phase transitions the electromagnetic field is included in the Lagrangian through coupling to quarks. In both cases the magnetic field with large enough energy density can have different cosmological signatures. In particular, the big bang nucleosynthesis (BBN) bound on the magnetic field energy density is $\rho_B \leq 0.1\rho_{\rm rad}$ [20], giving $\Omega_B = \rho_B/\rho_{\rm cr} \leq 2.4 \times 10^{-6} h_0^{-2}$, where Ω_B is the energy density parameter at the present time, $\rho_{\rm cr}$ is the present critical energy density (i.e. the total energy density for the flat Universe), and h_0 is the current Hubble parameter in the units of 100 km/ sec /Mpc. This corresponds to a limit on

the effective magnetic field comoving amplitude $B^{\rm eff} = \sqrt{8\pi\rho_B} \leq 8 \times 10^{-7}$ G.[1] Similar limits can be obtained through the available data of Laser Interferometer Gravitational Observatory (LIGO), if it is assumed that the primordial magnetic field generates relic gravitational waves via its anisotropic stress [21]. Stronger limits on the primordial magnetic field are provided by the cosmic microwave background (CMB) anisotropy data, $B^{\rm eff} \leq 5 \times 10^{-9}$ G; see Ref. [22] for a review and references therein.

Several studies have been performed to estimate the gravitational wave signal from the first order EW phase transition in MSSM and next to minimal supersymmetric standard model (nMSSM); see Refs. [23]. Using a model similar to the tunneling from a false to a true vacuum [8] it was argued that gravitational waves generated by the EW phase transition are possibly detectable [7]. However, based on a similar model, it was concluded that the magnitude of the gravitational waves generated in a MSSM model of the EW phase transition have a lower amplitude than that required to be detected by LISA, while in a nMSSM model the signal is larger [24].

In the present work we readdress the gravitational waves generated by the primordial magnetic fields created during the cosmological phase transitions. There are several models of phase transitions resulting in magnetic field production [25]. For reviews and references, see Refs. [26,27]. Our calculations for the EW phase transition are based on previous studies of the magnetic field generation by nucleation [28,29] and collisions [30,31] of the EW bubbles. In these studies the basic MSSM EW Lagrangian is used. Similarly, to generate magnetic fields during the QCD bubble collisions [32,33] the basic QCD Lagrangian is used [34]. For the QCD phase transitions the bubble walls are composed mainly of the gluonic field. After the collision of two bubbles an interior gluonic wall is formed, resulting in a magnetic wall production (due to coupling of quarks within nucleons to the gluonic wall causing alignment of nucleon magnetic dipole moments).

The magnetic fields generated by the cosmological phase transitions lead to MHD turbulence. In the present work we make use of the basic MHD formulation [35] and determine the main parameter for MHD turbulence—the Alfvén velocity, v_A, for the magnetic fields produced both by the EW and QCD phase transitions. The MHD turbulence model described in details in Refs. [13,14] is then used to calculate the gravitational waves produced by MHD turbulence present during EW or QCD phase transitions. In contrast to the previous works [4,13,36], we do not parameterize the gravitational wave signal in terms of certain phase transitions parameters, but use our solutions based on the fundamental EW and QCD Lagrangians. On the other hand, our results are model dependent in the sense that they use the value of the bubble wall velocity $v_b = 1/2$, found in Ref. [28]. However, the value of $v_b = 1/2$ was derived using the fundamental EW MSSM theory for bubble nucleation rather than a model (see below).

The outline of our paper is as follows. In Sec. II we review the main MHD equations, the origin of turbulence, and discuss the main assumptions used in our present work. In Sec. III we discuss the EW and QCD equations used for deriving the magnetic fields produced by the two cosmological phase transitions, and give our results. In Sec. IV we study the gravitational waves generated by these phase transitions driven by MHD turbulence. In Sec. V we present and discuss our work. In Sec. VI we give our conclusions.

II. MHD TURBULENCE MODEL

Our objective is to derive the gravitational wave energy density, $\rho_{\rm GW}$, that is produced by the EW or QCD phase transition generated magnetic field, **B**. In the first subsection, Sec. II A, we discuss the basic MHD theory leading to turbulence, and in the second subsection, Sec. II B we review the MHD turbulence model that we use for our cosmological applications.

A. MHD turbulence and the Alfvén velocity

An essential aspect of our model is that the magnetic field created after bubble collisions couples to the plasma and creates MHD turbulence, which then generates via the anisotropic stress the gravitational waves. In other words, the magnetic energy density ρ_B is transformed to the gravitational waves energy density $\rho_{\rm GW}$. Since the coupling between the magnetic and gravitational waves energy occurs through the Newton gravitational constant G, due to the small value of G the efficiency of the gravitational wave production directly from the magnetic field is small (see also Refs. [15]), but even accounting for the small efficiency, the gravitational wave signal can possibly be detectable.

To show coupling between an initial phase transition generated magnetic field to the plasma we give here the basic MHD equations for an incompressible, conducting fluid [35]

$$\left[\frac{\partial}{\partial\eta} + (\mathbf{v}\cdot\nabla) - \nu\nabla^2\right]\mathbf{v} = (\mathbf{b}\cdot\nabla)\mathbf{b} - \nabla p + \mathbf{f}, \quad (1)$$

$$\left[\frac{\partial}{\partial\eta} - \eta_{\rm re}\nabla^2\right]\mathbf{b} = -(\mathbf{v}\cdot\nabla)\mathbf{b} + (\mathbf{b}\cdot\nabla)\mathbf{v}, \quad (2)$$

$$\nabla\mathbf{v} = \nabla\mathbf{b} = 0, \quad (3)$$

[1] In what follows we use the natural units $c = \hbar = k_B = 1$ and MKS system of units [a physical system of units that expresses any given measurement using fundamental units of the metre, kilogram, and/or second (MKS); the electron charge is 1.6×10^{-19} Coulomb] for the electromagnetic quantities.

where η is the conformal time, $\mathbf{v}(\mathbf{x}, \eta)$ is the fluid velocity, $\mathbf{b}(\mathbf{x}, \eta) \equiv \mathbf{B}(\mathbf{x}, \eta)/\sqrt{4\pi \mathrm{w}}$ is the normalized magnetic field, $\mathbf{f}(\mathbf{x}, \eta)$ is an external force driving the flow, ν is the comoving viscosity of the fluid, η_{re} is the comoving resistivity, and $w = \rho + p$, ρ, and p the enthalpy, energy density, and pressure of the plasma.

The coupling of the magnetic field to the plasma [see Eqs. (1)–(3)] leads to Alfvén turbulence development with the characteristic velocity, v_A [37,38],

$$v_A = \frac{B}{\sqrt{4\pi \mathrm{w}}} = \sqrt{\frac{3\rho_B}{2\rho}}. \tag{4}$$

Since both ρ_B and $\rho = \rho_{\mathrm{rad}}$ scale as $1/a^4(t)$ (where $a(t)$ is the cosmic scale factor), if there is no damping of the magnetic field (additional ρ_B temporal dependence), the value of v_A is not affected by the expansion of the Universe. v_A is an essential parameter in the generation of gravitational waves in the magnetized turbulent model considered here (for details of the hydroturbulence model, see Ref. [13]). To develop the turbulence picture it is assumed *a priori* that the EW or QCD bubble collisions lead to the vorticity fluctuations, i.e. the presence of the kinetic turbulence, with a characteristic velocity (associated with the largest size bubble) v_0. Equipartition between the magnetic and kinetic energy densities implies $v_A \simeq v_0$.[2] We also define the energy density of the plasma to be equal of the radiation energy density, ρ_{rad}, and ρ_{rad} at the moment of the phase transition with temperature $T_\star$, is given by

$$\rho_{\mathrm{rad}}(\mathrm{at}\ T_\star) = \frac{\pi^2}{30} g_*(T_*)^4, \tag{6}$$

where $g_\star$ is the number of relativistic degrees of freedom at temperature $T_\star$ [39]. Using the BBN bound on the total magnetic energy density $\rho_B < 0.1\rho_{\mathrm{rad}}$, the Alfvén velocity must satisfy $v_A \leq 0.4$.

The Afvén velocity v_A as well as the bubble kinetic motion velocity v_0 can be related to the phase transitions parameters, α_{PT}, the ratio between the latent heat and the thermal energy, and the efficiency, κ_{PT}, which determines what part of the vacuum energy is transferred to the kinetic energy of the bubble motions as opposed to the thermal energy. Ref. [5] presents the estimate for the largest size turbulent bubble velocity, $v_A \simeq v_0 = \sqrt{\kappa_{\mathrm{PT}}\alpha_{\mathrm{PT}}/(4/3 + \kappa_{\mathrm{PT}}\alpha_{\mathrm{PT}})}$, and thus accounting for $v_A \simeq v_0$, BBN bound leads $\kappa_{\mathrm{PT}}\alpha_{\mathrm{PT}} \leq 0.2$.

[2]Accounting for the stochastic nature of the magnetic field, the Alfvén velocity is scale dependent, and if it is not specified, v_A is associated with the largest size magnetic eddy. On the other hand, v_A can be expressed in terms of the magnetic field comoving amplitude as

$$v_A \simeq 4 \times 10^{-4} \left(\frac{B}{10^{-9}\ \mathrm{Gauss}}\right)\left(\frac{g_\star}{100}\right)^{-1/6}. \tag{5}$$

Here we used Eqs. (4) and (6). The MHD turbulence description presumes that the magnetic turbulent energy density is saturated when the Alfvén velocity reaches the kinetic velocity of the plasma, i.e. $v_A \simeq v_0$.

B. Direct cascade and magnetic energy density

After being coupled to the fluid, the magnetic field energy density is injected into the plasma at characteristic comoving length scale λ_0 (which is, of course, inside the comoving Hubble radius λ_H at the moment of the field generation). The magnetic field energy density due to coupling with the fluid motions is redistributed spatially through the following regimes [38], $k < k_0$ ($k_0 = 2\pi/\lambda_0$), $k_0 < k < k_D$ (with $k_D = 2\pi/\lambda_D$ the wave number corresponding to the magnetic field damping scale due to the plasma viscosity) and $k > k_D$. Inside the so-called inertial range, $k_0 < k < k_D$, the selective turbulence decay occurs and the magnetic energy flows from the large to the small scales according to the direct cascade Kolmogoroff law [40], resulting in the magnetic field spectral energy density $E_M(k) \propto k^{-5/3}$, while at large scales ($k < k_0$) the free turbulence decay takes place, leading to the magnetic field spectrum $E_M(k) \propto k^{\alpha_T}$, with $\alpha_T \geq 4$ [41]. The initial Batchelor spectrum $\alpha_T = 4$ can be transformed via the nonlinear processes to the Kazantzev spectrum, $\alpha_T = 3/2$ [42]. Another numerical realization of large scale turbulence can be the white noise spectrum, $\alpha_T = 2$, (Saffman) spectrum [43]. Using the fact that the total energy density of the magnetic field $E_M = \int_0^\infty dk E_M(k)$ cannot be larger than the initial magnetic energy density, ρ_B, it is straightforward to obtain the maximal allowed values for $E_M(k)$, and get the magnetic field limits at large scales [44].

In our present work MHD turbulence is created by bubble collisions. It is physically justified to assume that the typical injection scale of the magnetic energy is associated with the phase transition largest bubble sizes, which are given by the bubble wall velocity v_b and phase transition time scales β^{-1} for the EW and QCD phase transitions (determined through the bubble nucleation rate), i.e. $\gamma \equiv \lambda_0/\lambda_H = v_b(\beta/H_\star)^{-1} \ll 1$ [6]. With these assumptions one finds for the Kolmogoroff power law and the wave numbers [13]

$$E_M(k, t) = C_M \varepsilon^{2/3} k^{-5/3} \tag{7}$$

over the range of wave numbers $k_0 < k < k_D$, where $k_D = k_0 Re^{3/4}$, $C_M \simeq 1$, $R \gg 1$ is the turbulence Reynolds number at the temperature $T_\star$, and $\varepsilon = (2/3)^{3/2} k_0 v_A^3$ is the comoving magnetic energy dissipation rate per unit enthalpy.

We consider only the inertial (direct cascade) range due to following reasons [38]: (i) helicity vanishes (for the effects of initial kinetic or magnetic helicity see

Refs. [12]) in our symmetric treatment of bubble collisions for the phase transitions; (ii) we presume that the contribution from the large scales ($k < k_0$) into the gravitational waves signal will not exceed significantly (or even being smaller than) that which comes from the inertial range, due to the free decay of nonhelical turbulence and the small amount of magnetic energy density present at large scales (we will address this issue in the separate work Ref. [45]). Another important assumption that we make is equipartition between the kinetic and magnetic energy densities, which allows us to use the direct analogy between the hydro and magnetized turbulence.

The direct cascade turbulence is characterized not only by the spatial structure (k dependence), but it is important to take into account the time dependence of the turbulent quantity correlations. First, we assume that the source lasts enough time to allow us to consider the developed turbulence, so we do not include in our calculations the pulselike source [15]. To compute the direct cascade duration time we use the fluctuation time decorrelation function $\eta(k) \simeq \sqrt{2\pi}(k/k_0)^{2/3}\tau_0^{-1}$ [46], where $\tau_0 = l_0/v_0$ is the largest turbulent eddy turn-over time. Note $l_0 = v_b\beta^{-1}$ is the physical length scale of the largest size eddy (bubble). As a result the time dependence of the magnetic field two-point correlation function within the inertial range is given by the function $f(\eta(x), \tau)$ [46]. The function $f(\eta(x), \tau)$ is such that it becomes negligibly small for $\tau \gg 1/\eta(k)$, and from dimensional analysis one finds $f[\eta(k), \tau] = \exp[-\pi\eta^2(k)\tau^2/4]$ [46]. It is clear that the temporal decorrelation function $\eta(k) \propto (k/k_0)^{2/3}$ reaches its maximum equal to $\sqrt{2\pi}/\tau_0 = \sqrt{2\pi}\beta v_A v_b^{-1}$ within the inertial range for the maximal size eddy, $k = k_0$. As a consequence, the smallest eddies are decorrelated first, and the turbulence cascade time is determined by the largest size eddy turn-over time τ_0. As a result the comoving (measured today) peak frequency of the induced gravitational waves is given by $f_{\rm peak} \simeq v_A/\lambda_0$ (see below).

The proper consideration of the temporal decorrelation leads to the fast damping of the gravitational wave signal amplitude for frequencies $f \gg f_{\rm peak}$, i.e. larger than that associated with the direct cascade turbulence induced peak frequency [13,14]. Several previous studies, Refs. [10,11], did not account for the temporal exponential decorrelation function, and as a result the shape of the gravitational wave at high frequencies was given by power law, without having steep exponential damping [13]. We also underline that our description does not apply for any pulselike sources [15]. In the former case, the characteristic comoving gravitational wave peak frequency is determined by $1/\lambda_0$ [11], so it is higher than that in the case considered here.

Another consequence of the temporal decorrelation is that the turbulence cascade time scale is much shorter than the Hubble time scale, and thus we are allowed neglect the expansion of the Universe during the gravitational waves generation process [45]. We account for the expansion of the Universe only when computing the gravitational wave amplitude $h_C(f)$ measured today (or the corresponding spectral energy density parameter $\Omega_{\rm GW}(f)$), as a function of the linear frequency f measured today.[3]

III. MAGNETIC FIELDS GENERATED BY EW AND QCD PHASE TRANSITIONS

A. Electroweak phase transition

First we review the magnetic field created during EW phase transition. In a suitable MSSM model the EW phase transition is first order, which results in bubble nucleation and collisions. The MSSM EW theory for bubble nucleation is developed in Ref. [28], a Weinberg-Salam model with all supersymmetric partners integrated out except the top squark, the partner to top quark, has the form

$$\begin{aligned}
\mathcal{L}^{\rm MSSM} &= \mathcal{L}^1 + \mathcal{L}^2 + \mathcal{L}^3 + \text{leptonic and quark interactions},\\
\mathcal{L}^1 &= -\frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},\\
\mathcal{L}^2 &= \left|\left(i\partial_\mu - \frac{g}{2}\tau\cdot W_\mu - \frac{g'}{2}B_\mu\right)\Phi\right|^2 - V(\Phi),\\
\mathcal{L}^3 &= \left|\left(i\partial_\mu - \frac{g_s}{2}\lambda^a C^a_\mu\right)\Phi_s\right|^2 - V_{hs}(\Phi_s, \Phi),
\end{aligned} \tag{10}$$

with

$$\begin{aligned}
W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu,\\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,
\end{aligned} \tag{11}$$

where the W^i, with $i = (1, 2)$, are the W^+, W^- fields, C^a_μ is an SU(3) gauge field, (Φ, Φ_s) are the (Higgs, right-handed top squark fields), (τ^i, λ^a) are the (SU(2), SU(3)) generators, and the electromagnetic (EM) and Z fields are defined as

[3]The amplitude and the energy density of the gravitational wave is related through [1],

$$h_C(f) = 1.26\times 10^{-18}\left(\frac{\rm Hz}{f}\right)[h_0^2\Omega_{\rm GW}(f)]^{1/2}, \tag{8}$$

where f is the linear frequency measured today, and it is given by $f = (2\pi)^{-1}\omega$, with $\omega = (a_\star/a_0)\omega_\star$, where $\omega_\star$ is the gravitational wave frequency at the time of generation. Because of the Universe expansion the freely propagating gravitational wave amplitude and frequency are rescaled by a factor

$$\frac{a_\star}{a_0} \simeq 8\times 10^{-16}\left(\frac{100\ {\rm GeV}}{T_\star}\right)\left(\frac{100}{g_\star}\right)^{1/3}, \tag{9}$$

where $a_\star$ and a_0 are the scale factors at the time of generation and today, respectively. See Ref. [1] for the definitions and discussions on gravitational wave direct detection experiments.

$$A_\mu^{\rm em} = \frac{1}{\sqrt{g^2 + g'^2}}(g' W_\mu^3 + g B_\mu),$$
$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g W_\mu^3 - g' B_\mu). \quad (12)$$

The parameters used here are $g = e/\sin\theta_W = 0.646$ and $g' = g\tan\theta_W = 0.343$. The equations of motion were solved using an SU(2) I-spin ansatz for the gauge fields, and it was found that the nucleation velocity of the bubble walls is $v_b = 1/2$. This is a very important parameter for our present work. Using EW MSSM theory, we find v_b to be larger than found using models for EW bubble walls, $v_b = 0.05$, [47]. The recent study, Ref. [48], estimates the bubble wall velocity within the first order EW phase transitions, ranging from $0.01 \leq v_b \leq 0.1$ in the case of the model with the strong coupling between the extra fermion and the Higgs boson, and $0.1 \leq v_b < 0.6$ in the case of the model with the extra bosons. This is consistent with Ref. [28] The high bubble wall velocity leads to the strong gravitational wave signal (see Refs. [4,10,13] for and Sec. IV below).

Starting from the Lagrangian given by Eq. (10) for the MSSM EW phase transition, from which bubbles are formed, to compute bubble collisions we assume that the top squark is integrated out, so the $\mathcal{L}^3$ term is not included. The magnetic field created during the collision of two bubbles was estimated by Refs. [30,31]. Our model is based on the assumption that the final magnetic field is created by the final two bubbles colliding. The time scale is from 10^{-11} to 10^{-10} s, and the critical temperature (EW phase transition energy scale) is $T_* \simeq M_H$, with the Higgs mass $M_H \simeq 110$ 130 GeV [49]. The magnitude of the magnetic field created in this final collision of two bubbles, $B^{\rm (EW)}$, is found in our new calculations [50], based on Ref. [31], but with the colliding bubbles having larger overlap. That is the bubble collision has been followed to the stage of overlap of the bubbles such that the B field in the Universe just after the EW phase transition has been determined. The result for the B field is

$$B_\star^{\rm (EW)}({\rm at}\ T_\star^{\rm (EW)}) \simeq 10 M_W^2 = 6.4 \times 10^4 M_{W,80}^2\ {\rm GeV}^2, \quad (13)$$

where $M_{W,80} = M_W/80$ GeV is the normalized W-boson mass. Even though the above estimate is given for a two-bubble collision, we extend our consideration presuming that there is a continuous creation of the magnetic field through the bubble collisions, and the total magnetic energy density released during the MSSM EW phase transition can be approximated by $\rho_B^{\rm EW} = (B^{\rm (EW)})^2/8\pi$. In reality, of course, there are many bubble collisions,[4] which leads to the establishment of the stochastic nature of the resulting magnetic field. The coupling between this initial field with the fluid results in MHD turbulence [37]. Using Eqs. (4)–(6) we find that the Alfvén velocity is

$$v_A^{\rm (EW)} \simeq 0.27 M_{W,80}^2 \left(\frac{T_\star}{100\ {\rm GeV}}\right)^{-2} \left(\frac{g_\star}{100}\right)^{-1/2}. \quad (14)$$

For the EW phase transition with $g_* \simeq 100$ and $T_* \simeq 100$ GeV, one finds that $v_A^{\rm (EW)} \simeq 0.27$, which is below the BBN bound discussed above. On the other hand the value of $v_A \simeq 0.27$ means that 7.4% of the radiation energy density is in the form of the magnetic energy density. In terms of the parameters $\kappa_{\rm PT}$ and $\alpha_{\rm PT}$, we obtain $\kappa_{\rm PT}\alpha_{\rm PT} \simeq 0.1$ which is high, but still reasonable for the first order first transitions.[5]

B. QCD phase transition

Recent lattice QCD studies have shown that the QCD phase transition is first order [19]. In Ref. [34] the magnetic field created by the QCD phase transition bubble collisions was derived. A gluonic wall is created as two bubbles collide, and a magnetic wall is formed by the interaction of the nucleons with the gluonic wall, with electromagnetic interaction Lagrangian

$$\mathcal{L}^{\rm int} = -e\bar{\Psi}\gamma^\mu A_\mu^{\rm em}\Psi, \quad (15)$$

where Ψ is the nucleon field operator and $A^{\rm em}$ is the electromagnetic 4-potential. In Ref. [34] it was shown that the interaction of the quarks in the nucleons with the gluonic wall aligns the nucleons magnetic dipole moments, producing a B field orthogonal to the gluonic wall.

Using an instanton model, for the gluonic instanton wall oriented in the x-y direction one obtains for $B_z \equiv B_\star^{\rm (QCD)}$ within the wall, of thickness ζ,

$$B_\star^{\rm (QCD)}({\rm at}\ T_\star^{\rm (QCD)}) \simeq \frac{1}{\zeta\Lambda_{\rm QCD}}\frac{e}{2M_n}\langle\bar{\Psi}\sigma_{21}\gamma_5\Psi\rangle, \quad (16)$$

where $\Lambda_{\rm QCD}$ is the QCD momentum scale. A similar form had been derived earlier using a domain wall model [51]. The value for $B^{\rm (QCD)}$ was found to be

$$B_\star^{\rm (QCD)} \simeq 0.39\frac{e}{\pi}\Lambda_{\rm QCD}^2 \simeq 1.5 \times 10^{-3}\ {\rm GeV}^2. \quad (17)$$

Equations (4)–(6) with $g_* = 15$, $T_\star = 0.15$ GeV give

$$v_A^{\rm (QCD)} \simeq 8.4 \times 10^{-3}. \quad (18)$$

In the case of the QCD phase transitions the efficiency of the magnetic field is very low, and as a consequence $\kappa_{\rm PT}\alpha_{\rm PT} \ll 1$.

[4]It is clear that within the Hubble radius we have several areas where the magnetic field is generated. The size of the colliding bubble determines the correlation length of the magnetic field, as we noted above.

[5]Recall that $\alpha_{\rm PT} = \rho_{\rm vac}/\rho_{\rm thermal}$ and $\kappa_{\rm PT}\rho_{\rm vac} = \rho_{\rm turb} \simeq \rho_B$, for the case described above, 10% goes equally into the magnetic and turbulent energy, 25% is available for bubble wall motions.

IV. GRAVITATIONAL WAVES GENERATED BY MAGNETIC FIELDS

In this section we derive the strain amplitude measured today, $h_C(f)$, of the gravitational waves generated by MHD turbulence developed during the EW or QCD phase transitions. The gravitational waves are generated by the transverse and traceless part, S_{ij}, of the stress-energy tensor, T_{ij} [52].

$$S_{ij}(\mathbf{x}, t) = T_{ij}(\mathbf{x}, t) - \frac{1}{3}\delta_{ij}T^k_k(\mathbf{x}, t). \tag{19}$$

When considering gravitational waves generated by the magnetic field, the source S_{ij} is associated with the magnetic anisotropic stress [9]. On the other hand, if the duration of the source is short enough when comparing with the Hubble time at the moment of the generation, $H_\star^{-1}$, we can neglect the expansion of the Universe and the gravitational wave generation is described by the simplified equation and solution

$$\nabla^2 h_{ij}(\mathbf{x}, t) - \frac{\partial^2}{\partial t^2}h_{ij}(\mathbf{x}, t) = -16\pi G S_{ij}(\mathbf{x}, t),$$
$$h_{ij}(\mathbf{x}, t) = \int d^3\mathbf{x}' \frac{S_{ij}(\mathbf{x}', t)}{|\mathbf{x}' - \mathbf{x}|}. \tag{20}$$

Here $h_{ij}(\mathbf{x}, t)$ is the metric tensor perturbation which satisfies the following conditions: $h_{ii} = 0$ and $\partial h_{ij}/\partial x^j = 0$.

To derive the energy density of the induced gravitational waves, accounting for the stochastic nature of the magnetic turbulence source, and as a consequence the stochastic nature of the gravitational signal, we must compute the autocorrelation function $\langle \partial_t h_{ij}(\mathbf{x}, t)\partial_t h_{ij}(\mathbf{x}, t + \tau)\rangle/32\pi G$ which can be expressed through the two-point correlation function of the source $R_{ijij}(\boldsymbol{\xi}, \tau) = \langle S_{ij}(\mathbf{x}', t)S_{ij}(\mathbf{x}'', t + \tau)\rangle$, where $\boldsymbol{\xi} = \mathbf{x}'' - \mathbf{x}'$ [13]. As the calculations performed in Refs. [13,14] show, the gravitational energy density, $\rho_{\rm GW}$, at the moment of generation is given by the duration time τ_T and the Fourier transform of R_{ijij} tensor, $H_{ijij}(0, \omega_\star)$,

$$\rho_{\rm GW}(\omega_\star) = 16\pi^3\omega_\star^3 G {\rm w}_\star^2 \tau_T H_{ijij}(0, \omega_\star), \tag{21}$$

where $\omega_\star$ is the angular frequency measured at the moment of the gravitational waves generation. To obtain $\rho_{\rm GW}(\omega_\star)$ and the frequency today one must account for the gravitational wave amplitude and frequency rescaling given by Eq. (9).

Since equipartition between kinetic and magnetic energy densities is maintained during Kolmogoroff turbulence, the total source for the gravitational waves (from the magnetic and kinetic turbulence) is simply 2 times the kinetic turbulence source. Using the assumptions made above, the source H_{ijij} tensor is given by [13,14],

$$H_{ijij}(0, \omega_\star) = \frac{7C_M^2\bar{\varepsilon}}{6\pi^{3/2}}\int_{\bar{k}_0}^{\bar{k}_D}\frac{d\bar{k}}{\bar{k}^6}\exp\left(-\frac{\omega_\star^2}{\bar{\varepsilon}^{2/3}\bar{k}^{4/3}}\right)$$
$$\times \operatorname{erfc}\left(-\frac{\omega_\star}{\bar{\varepsilon}^{1/3}\bar{k}^{2/3}}\right). \tag{22}$$

Here, $\operatorname{erfc}(x)$ is the complementary error function defined as $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$, where $\operatorname{erf}(x) = \int_0^x dy \exp(-y^2)$ is the error function. $\bar{\varepsilon} = (a_0/a_\star)\varepsilon$ and $\bar{k} = (a_0/a_\star)k_0$ are the physical energy dissipation rate and the physical wave number, respectively. As can be expected, the integral in Eq. (22) is dominated by the large scale ($\bar{k} \simeq \bar{k}_0$) contribution so, for forward-cascade turbulence, the peak frequency is [13] $\omega_{\max\star} \simeq \bar{k}_0 v_0$. It must be noted that the peak frequency is determined by the time characteristic of turbulence only in the case when the turbulence duration time is enough long ($\tau_T \simeq$ few $\times\, \tau_0$ to insure the applicability of the Proudman argument [40], which has been used to justify the use of the Kolmogoroff model. Otherwise, the peak frequency is determined by the characteristic scales of the pulselike source [15,36].

Taking into account the expansion of the Universe and using Eq. (22) for the H_{ijij} tensor, we find the gravitational wave amplitude as a function of the linear frequency measured today, $f = (a_\star/a_0)f_\star$ with $f_\star = \omega_\star/2\pi$,

$$h_C(f) \simeq 2\times 10^{-14}\left(\frac{100\ {\rm GeV}}{T_*}\right)\left(\frac{100}{g_*}\right)^{1/3}$$
$$\times [\tau_T\omega_\star H_\star^4 H_{ijij}(0, \omega_\star)]^{1/2}, \tag{23}$$

where $f_H = \lambda_H^{-1}$ is the Hubble frequency measured today, $f_H \simeq 1.6\times 10^{-5}\ {\rm Hz}(g_*/100)^{1/6}(T_*/100\ {\rm GeV})$.

Equation (23) allows us to predict the gravitational wave spectral properties: (i) The low-frequency ($f \ll f_{\rm peak}$) dependence is $h_C(f) \propto f^{1/2}$, leading to $\Omega_{\rm GW}(f) \propto f^3$. Such a behavior is common for all causal sources [13,15] and it is true for any kind of the waves, including the sound waves generation by turbulence [53]. (ii) The peak position is determined by the time duration of the source, and it is equal either to $v_A\gamma f_H$ (developed stationary source) [13] or γf_H (pulselike source) [11,36]. (iii) At higher frequencies, $f \gg f_H$, the gravitational waves amplitude is damped exponentially due to the exponential temporal decorrelation of fluctuations [13] as opposed to the power law slow damping shape of the gravitational waves generated by the bubble collisions [5,6,8,15,24]. (iv) For the turbulence generated gravitational waves the power law shape takes place in the vicinity of the peak, i.e. for $f_{\rm peak} < f \leq {\rm R}^{1/2}f_H$, $h_C(f) \propto f^{-13/4}$ [13].

V. RESULTS AND DISCUSSION

The results for $h_C(f)$ for the EW phase transitions are shown in Fig. 1. For a semianalytical estimate it is straightforward to get the peak frequency of the amplitude of the gravitational waves emitted during direct cascade to be

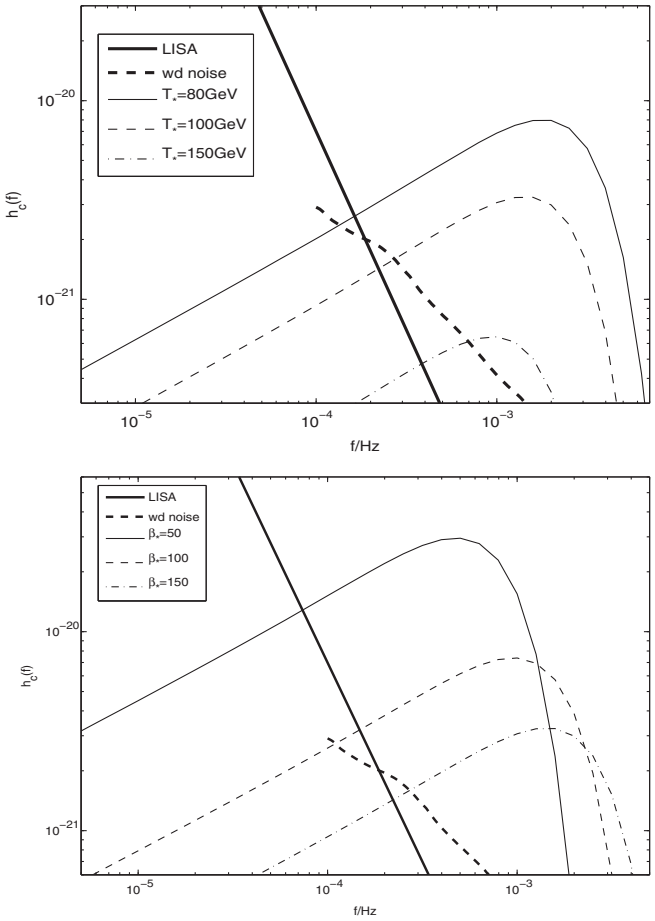

FIG. 1. $h_C(f)$ for the EW phase transitions with $g_* = 100$, $v_b = 1/2$, and $v_0 = v_A$ with zero magnetic helicity. Top panel: $T_* = 80$ GeV (solid line), $T_* = 100$ GeV (dashed line), and $T_\star = 150$ GeV (dash-dotted line) with $\beta = 100H_\star$. Bottom panal: $T_\star = 100$ GeV and $\beta = 50H_\star$ (solid line), $\beta = 100H_\star$ (dashed line), and $\beta = 150H_\star$ (dash-dotted line). In both panels the bold solid line corresponds to the 1-year, 5σ LISA design sensitivity curve [55] including confusion noise from white dwarf binaries, bold dash line [56]. For other parameters, for example, in the case of the low value of v_b [47,48], the gravitational wave spectrum peak and the amplitude must be rescaled according to Eqs. (24) and (25).

[13]

$$f_{\rm peak} = \left(\frac{v_A}{v_b}\right)\left(\frac{\beta}{H_\star}\right) f_H. \tag{24}$$

The peak frequency is shifted to the lower frequencies with $T_\star$ increasing due to f_H dependence on $T_\star$. Using the definition of the $H_{ijij}(0, \omega_\star)$ tensor, Eq. (22), the gravitational wave signal reaches its maximal amplitude approximately at $f = \gamma^{-1} v_A f_H$, and then,

$$h_C(f_{\rm peak}) \simeq 10^{-15} v_A^{3/2} v_b^2 \left(\frac{\beta}{H_\star}\right)^{-2} \left(\frac{100\ {\rm GeV}}{T_\star}\right)\left(\frac{100}{g_\star}\right)^{1/2}. \tag{25}$$

According to Eqs. (24) and (25), the peak amplitude of the EW phase transition gravitational signal is order of 5×10^{-21} for $T_\star = 100$ GeV, $\beta = 100H_\star$, and $g_\star = 100$, with $f^{\rm EW}_{\rm peak} \simeq 10^{-3}$ Hz, which is in agreement with Fig. 1. For the stochastic gravitational waves the real LISA sensitivity will be lower. Even accounting for this, the gravitational signal from the EW phase transition should be detectable, since it significantly exceeds the LISA noise around 2 mHz, if we adopt the model described here with $v_b = 1/2$, and $v_A \simeq 0.3$. Rewriting this result in terms of the gravitational wave spectral energy density parameter, we obtain $\Omega_{\rm GW}(f = f_{\rm peak}) \simeq 2 v_A^5 \gamma^2 (100/g_\star)^{2/3} \times 10^{-4}$. Note $\Omega_{\rm GW}(f = f_{\rm peak})$ is temperature independent for a given value of v_A and v_b, as can be shown from dimensional analysis [6], and only slightly depends on $g_\star$.

In the different model of the EW phase transitions resulting in $v_b = 0.05$ [47] the gravitational wave signal amplitude is significantly lower than in the theory presented in Fig. 1, $v_b = 0.5$, approximately by 2 orders of magnitude [see Eq. (25)], while the signal peak is shifted to the higher frequency [see Eq. (24)]. The frequency peak shift occurs since the smaller size magnetic and/or turbulent eddies are formed, and as a consequence the maximal eddy turn-over time is significantly shorter.

The peak frequency for QCD phase transitions with $T_\star = 0.15$ GeV, $\beta = 6H_\star$, and $g_\star = 15$ is

$$f^{\rm QCD}_{\rm peak} = 1.8 \times 10^{-6} f^{\rm EW}_{\rm peak} \simeq 2 \times 10^{-9}\ {\rm Hz}, \tag{26}$$

which is order of 6 magnitudes lower than the LISA low-frequency sensitivity.

We also define the efficiency of the gravitational wave production $\kappa_{\rm GW}$ as the ratio between the magnetic energy density available from the phase transitions and the energy density converted into the gravitational waves, i.e. $\kappa_{\rm GW}(f) \equiv \rho_{\rm GW}/\rho_B$. Since both total energy densities scale the same way with the expansion of the Universe, $\kappa_{\rm GW}$ is invariant (no damping of the magnetic field). It can be shown that $\kappa_{\rm GW} \simeq 2(\beta/H_\star)^{-2} v_b^2 v_A^3 \ll 1$ [13] [for EW phase transitions with the parameters mentioned above $\kappa_{\rm GW} \simeq 3\gamma^2 (\rho_B/\rho_{\rm rad})^{3/2} \simeq 10^{-6}$], so only a small fraction is transferred to the gravitational wave signal. However, the LISA sensitivity is $\Omega_{\rm GW}(f = 1\ {\rm mHz}) \sim 10^{-12}$ and the magnetic field generated signal from the EW phase transition would be still detectable if even 1% of the radiation energy consists of the magnetic energy.

We emphasize that although we present our results for $v_b = 1/2$, they can be easily extended for an arbitrary value of v_b. For example, a different model of the EW MSSM phase transition gives v_b an order of magnitude smaller; see Ref. [54]. Another degree of the freedom is related to the energy scale of the phase transitions. In particular, for the EW phase transition the energy scale is approximately equal to the Higgs mass, which ranges from about 110 to 127 GeV [49], however, such freedom of the

EW phase transition energy scale just slightly affects our final results,

VI. CONCLUSIONS

The model presented above might be applied for different mechanisms [25,27] of the magnetic field generation leading to the substantial magnetic energy presence during phase transitions, but we emphasize that the ours is based on the derivation of the magnetic field amplitude from the fundamental EW MSSM or QCD Lagrangians.

Summarizing, using MHD turbulence model with no helicity we find that the gravitational wave produced during the EW phase transitions is most likely detectable by LISA, while that produced by the QCD phase transitions will not be detectable.

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