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# Testing Lorentz invariance violation with Wilkinson Microwave Anisotropy Probe five year data

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We consider different renormalizable models of Lorentz invariance violation. We show that the limits on birefringence of the propagation of cosmic microwave background photons from the five year data of the Wilkinson Microwave Anisotropy Probe (WMAP) can be translated into a limit of Lorentz symmetry violation. The obtained limits on Lorentz invariance violation are stronger than other published limits. We also cast them in terms of limits on a birefringent effective photo mass and on a polarization dependence of the speed of light.

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## I. INTRODUCTION

The principal spacetime symmetry of particle interactions in the standard model is Lorentz invariance. Experiments confirm Lorentz symmetry at all currently accessible energy scales of up to 2 TeV. This scale will be extended shortly to 14 TeV with the Large Hadron Collider (LHC) at CERN. Although present experiments confirm Lorentz invariance to a good precision, it can be broken in the very early Universe when energies approach the Planck scale. There are a number of extensions of the standard model of particle physics and cosmology that violate Lorentz invariance (for reviews see Refs. [1–5]).

As can be expected, Lorentz invariance violation (LV) affects photon propagation (the dispersion relation) and generically results in a rotation of linear polarization (birefringence). Other effects include new particle interactions such as a photon decay and vacuum Cherenkov radiation [4]. All these effects can be used to probe Lorentz invariance. The dispersion measure test is based on a phenomenological energy dependence of the photon velocity [6] (see also Ref. [7] for reviews and Refs. [8–10] for recent studies of this effect; early discussions include Ref. [11]; Refs. [6,9,10] consider Lorentz symmetry violating models which preserve rotational and translational invariance but break boost invariance).

Several models of LV predict frequency dependent effects. Such high energy Lorentz invariance breakings are discussed in Refs. [12–14]. References [15] study generalizations of electromagnetism, motivated by this kind of Lorentz invariance violation. On the other hand, LV associated with a Chern-Simons interaction [16,17] affects the entire spectrum of electromagnetic radiation, not just the high frequency part, and induces a frequency-independent

rotation of polarization (see Sec. 4 of Ref. [2] and Sec. III of this work).

To determine the effects induced by Lorentz symmetry violation, it is useful to consider the analogy with the propagation of electromagnetic waves in a magnetized plasma as outlined in Refs. [8,12,16,18,19]. Using the well-known formalism for the propagation of light in a magnetized plasma, it is easy to see that for Lorentz symmetry violating models which depend also on polarization and not only on frequency, the rotation measure constrains the symmetry breaking scale more tightly than the dispersion measure, see Refs. [14,15,19].

The propagation of ultrahigh energy photons represents a promising possibility to probe Lorentz symmetry [20]. Gamma ray bursts (GRB) are astrophysical objects located at cosmological distances which emit very energetic photons [6]; reviews describing cosmological tests involving GRBs are e.g. Refs. [3,21], for recent studies see [22]. After the observation of highly linearly polarized  $\gamma$  rays from GRB021206 has been reported [23], Refs. [24,25] have proposed to test Lorentz symmetry violation with the rotation measure by the analysis of GRB polarization. Even though this measurement has been strongly contested [26], there is evidence that the  $\gamma$ -ray flux from GRB930131 and GRB960924 is consistent with more than 35% and 50% polarization, respectively [27]. However, the issue of polarization of GRB  $\gamma$  rays is still under debate and additional x-ray studies are needed to either confirm or disprove polarization of  $\gamma$  rays from GRB's [28].

In this paper we mainly consider renormalizable models of LV as described in Ref. [2]. We use the very well understood and measured temperature anisotropy and polarization of the cosmic microwave background (CMB) to constrain Lorentz symmetry violation. These data have been proposed as a probe of Lorentz invariance in the Universe in Refs. [29–32]. In our study we use the WMAP five year limits on birefringence [33] and obtain

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limits which are significantly more stringent than those obtained from radio galaxy polarimetry [16].

As we shall see below, generically Lorentz symmetry violation leads to birefringence, i.e. a photon dispersion relation which depends on polarization. This leads to a rotation of the CMB polarization which induces parity-odd cross correlations, such as temperature-*B*-polarization and *E-B*-polarization [29]. These correlators vanish in models which preserve parity. Generally speaking, the effect is similar to that induced by a homogeneous magnetic field [34,35]. In this paper we use the WMAP five year limit on the rotation measure [33] to constrain Lorentz invariance violating theories.

# II. LORENTZ INVARIANCE VIOLATION: GENERAL DESCRIPTION

For methodological purpose let us first briefly summarize the usual Faraday rotation effect. We consider an electromagnetic wave with frequency  $\omega$  and spatial wave vector  $\mathbf{k}$ ,  $k \equiv |\mathbf{k}|$  propagating in a magnetized plasma. A linearly polarized wave can be expressed as superposition of left (-) and right (+) circularly polarized waves. In a magnetized plasma, a homogeneous magnetic field induces a difference in the phase velocity of left and right-handed waves. This causes a rotation of the polarization, called Faraday rotation [36]. The group velocity of the wave also differs from c. These two effects can be expressed in terms of the refractive indices defined by  $k_{\pm} = n_{\pm}\omega$  where  $k_{\pm}$  denotes the wave number for right and left-handed waves. The indices  $n_{\pm}$  are [36]

$$n_{\pm}^2 = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c)} \simeq 1 - \frac{\omega_p^2}{\omega^2} \pm \frac{\omega_p^2 \omega_c}{\omega^3}.$$
 (1)

Here  $\omega_p = 4\pi e^2 n_e/m_e$  is the plasma frequency and  $\omega_c = eB/m_e$  is the electron cyclotron frequency for the magnetic field *B* (see Sec. 4.9 of Ref. [36]).

The magnitude of both the dispersion measure, due to the different group velocities, and the rotation measure, i.e., the rotation of polarization, are proportional to the photon travel distance  $\Delta l$ ,

$$\Delta t_{\pm} = \Delta l \left( 1 - \frac{\partial k_{\pm}}{\partial \omega} \right), \tag{2}$$

$$\Delta \alpha = \frac{1}{2}(k_+ - k_-)\Delta l. \tag{3}$$

Here,  $\Delta t_{\pm}$  is the difference between the travel time of a right-handed (left-handed) photon and that of a photon traveling at the speed of light, and  $\Delta \alpha$  is the rotation of the angle of polarization.

Faraday rotation is widely used in astrophysics to measure magnetic fields in galaxies and clusters (see Ref. [37] for a review and references therein). In cosmology, Faraday rotation of CMB photons [34,38] has been used to constrain the amplitude of a homogeneous as well as a sto-

chastic cosmological magnetic field [39] (for review on cosmological magnetic fields, see Ref. [40]).

In the following, we show that Lorentz symmetry violation leads to a modification of Maxwell's equations [14,15] analogous to the modifications described above.

Following Ref. [2], the most general renormalizable form of Lorentz symmetry violation can be expressed by two additional terms in the action (we set  $\hbar = c = 1$ )

$$\Gamma_{\rm LV} = \int d^4x \sqrt{-g} \left[ K_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho} - \frac{1}{4} L^{\mu} A^{\nu} \tilde{F}_{\mu\nu} \right], \quad (4)$$

where Greek indices  $(\mu, \nu, \lambda, \rho)$  denote time-space coordinates,  $F^{\mu\nu}$  is the electromagnetic field strength tensor,  $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu}{}^{\lambda\rho} F_{\lambda\rho}$  is its dual,  $\epsilon_{\mu\nu\lambda\rho}$  is the totally antisymmetric tensor normalized such that  $\epsilon_{0123} = \sqrt{-g}$ , and  $A^{\nu}$  is the vector potential. The four-vector  $(L_{\mu}) = (L_0, \mathbf{L})$  has the dimension of mass and describes a super-renormalizable (dimension 3) coupling and  $K_{\mu\nu\lambda\rho}$  is a renormalizable, dimensionless coupling giving rise to a dimension 4 operator. We want to break Lorentz symmetry but keep conformal invariance of electrodynamics in this work. For this we have to ask that the components of  $K_{\mu\nu}{}^{\lambda\rho}$ and  $L_{\mu}$  be independent of conformal transformations of the metric. In the cosmological setup with  $g_{\mu\nu} = a^2(t)\eta_{\mu\nu}$ , the above action is then independent of the scale factor a(t). I.e. in a conformally flat spacetime the action is like in flat space. To see this note that the forms  $A_{\mu}$  and  $F_{\mu\nu}$  are independent of the metric hence  $K_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}$  and  $L^{\mu}A^{\nu}\tilde{F}_{\mu\nu}$  scale like  $a^{-4}$  which is canceled by  $\sqrt{-g} = a^4$ .

The tensor  $K_{\mu\nu\lambda\rho}$  has the same symmetries as the Riemann tensor and we only consider its trace-free part which is analog to the Weyl tensor (the trace part also leads to dispersion measure but not to birefringence, we therefore do not consider it here). Even though we apply the formalism used for the Weyl tensor below, we do not consider  $K_{\mu\nu\lambda\rho}$  to be the Weyl tensor which of course vanishes in a (unperturbed) Friedmann universe. The most plausible origin for the Lorentz violating terms in (4) is that  $K_{\mu\nu\lambda\rho}$  or the vector  $L_{\mu}$  stem from the nonvanishing vacuum expectation value of some dynamical field and the action (4) therefore represents a spontaneous rather than explicit breaking of Lorentz symmetry. However, for the following discussion the origin of the Lorentz violating terms is not relevant.

Both terms in Eq. (4) lead to birefringence but the frequency dependence is different. The first term in the action  $\Gamma_{\rm LV}$  can be computed within the Newman-Penrose formalism, which is usually applied for the Weyl tensor [2]. We consider a plane wave with conformal wave vector  $(k_{\mu}) = (\omega, \mathbf{k})$ . We normalize the scale factor to unity today,  $a_0 = a(t_0) = 1$ , so that conformal frequencies or length scales correspond to physical scales today. In terms of the conformal wave vector, the dispersion relation is like in flat space where it has been derived in Ref. [2],

$$\omega^2 = k^2 \mp 8\omega^2 |\Psi_0|. \tag{5}$$

Here  $\Psi_0$  is the analog of the Newman-Penrose scalar (for more details see [2]),

$$\Psi_0 = -a^{-4} [K_{0i0j} - K_{0ilj} n^l - K_{kilj} n^k n^l] m^i m^j,$$

where **m** and  $\bar{\mathbf{m}}$  represent the left and right circular polarization basis vectors and  $\mathbf{n} = \mathbf{k}/k$  is the photon propagation direction. We normalize **n** and **m** with the flat metric,  $n^i n^j \delta_{ij} = m^i \bar{m}^j \delta_{ij} = 1$ , and multiply the expression with the correct power of the scale factor,  $a^{-4}$ , so that, given the scaling of the tensor K, one sees explicitly that  $\Psi_0$  is independent of the scale factor. (Latin indices indicate spatial components of a vector or tensor.)

The second term in the action  $\Gamma_{\rm LV}$  leads to the dispersion relation [2,16]

$$(k_{\mu}k^{\mu})^2 + (k_{\mu}k^{\mu})(L_{\nu}L^{\nu}) - (L_{\mu}k^{\mu})^2 = 0. \tag{6}$$

To first order in the small parameters  $L_0$  and  $\sqrt{\delta^{ij}L_iL_j} \equiv L$  one has

$$\omega^2 = k^2 \mp \omega (L_0 - L \cos \phi), \tag{7}$$

where  $\phi$  is an angle between the photon propagation direction and the vector  $\mathbf{L}$ ,  $\cos \phi = (\mathbf{L} \cdot \mathbf{n})/L$ . Note the similarity of the expressions (5) and (7) with the corresponding ones following from Eq. (1).

To be as general as possible, we rewrite the dispersion relation for both types of Lorentz symmetry violation in the form (see also [19]),

$$k^2 = \omega^2 \left[ 1 \pm \left( \frac{M}{M_{\rm PL}} \right) \left( \frac{\omega}{M_{\rm PL}} \right)^{N-4} \right], \tag{8}$$

where  $M_{\rm PL}$  is the Planck mass,  $M_{\rm PL} \simeq 1.2 \times 10^{19}$  GeV, N is the dimension of the Lorentz symmetry violating operator, and M is a mass scale of the model. For N=4, the birefringent part is independent of the photon energy and we have  $8\Psi_0 = M/M_{\rm PL}$ . For N=3 the Planck mass cancels out and the birefringent term is inversely proportional to the photon energy. The mass scale is  $M=L_0-L\cos\phi$ . Generally speaking, the smaller M, the weaker the LV. For N=4, the LV is frequency independent and the amplitude of the effect is of order  $M/M_{\rm Pl}$ , while for the super-renormalizable case, N=3, the LV is strongest at low frequencies,  $\omega < M$ . Our aim is to limit the function

$$\gamma(\omega) \equiv \left(\frac{M}{M_{\rm PL}}\right) \left(\frac{\omega}{M_{\rm PL}}\right)^{N-4}$$

from CMB birefringence. This ansatz can also be applied to nonrenormalizable models with higher dimension operators. For  $N \ge 5$ ,  $M \ne 0$  indicates that there is LV at frequencies  $\omega \ge M_{\rm Pl}(M_{\rm Pl}/M)^{1/(N-4)}$ .

## III. RESULTS

To compute the CMB polarization rotation angle induced by Lorentz symmetry violation, we follow the analogy with photon propagation in a magnetized medium which yields  $n_{\pm} = 1 \pm \gamma(\omega)/2$ . Using Eq. (3), we obtain

$$\Delta \alpha^{(LV)} = \frac{1}{2} \omega \gamma(\omega) \Delta l. \tag{9}$$

In the case N=4,  $\gamma$  is frequency independent, hence  $\Delta\alpha^{(\mathrm{LV})}$  grows linearly with frequency. In this case, and for all models with higher dimension operators, the best limits can in principle be obtained from high frequency photons (for example GRB  $\gamma$  rays [24,25]), while CMB photons are less affected. However, the fact that the theory of CMB anisotropies and polarization yields that both temperature–B-polarization cross correlations (TB) and E- and B-polarization cross correlations (EB) have to vanish in standard cosmology, while the polarization of GRB's is still under debate, at present, a test using CMB data is to be preferred. Another advantage is that for the CMB the distance  $\Delta l \simeq H_0^{-1}$  is maximal.

In the dimension 3 model,  $\Delta\alpha^{(\mathrm{LV})} = -\frac{1}{2}(L_0 - L\cos\phi)\Delta l$  is frequency independent. In Ref. [16] the above result is applied to polarization data from distant radio galaxies,  $\Delta\alpha < 6^\circ$  at 95% C.L. at redshift  $z \sim 0.4$ . The constraint obtained if Ref. [16] is  $|L_0 - L\cos\phi| \leq 1.7 \times 10^{-42} h_0$  GeV, where  $h_0 \simeq 0.7$  is the present Hubble parameter in units of 100 km s<sup>-1</sup> Mpc<sup>-1</sup>.

We use the recent WMAP five year constraints on the rotation angle of the CMB polarization plane (combined constraints from the low and high multipole number, l, data assuming a constant  $\Delta\alpha$  across the entire multipole range),  $-5.9^{\circ} < \Delta\alpha < 2.4^{\circ}$  at 95% C.L., and  $\Delta\alpha = -1.7^{\circ} \pm 2.1^{\circ}$  at 68% C.L. [33] (Sec. 4.3). Assuming Gaussian errors, it is straightforward to convert this to the following limits on the absolute value of rotation angle,

$$|\Delta\alpha|_{\rm obs} \le 4.90^{\circ}$$
 at 95% C.L., (10)

$$|\Delta \alpha|_{\text{obs}} \le 2.52^{\circ}$$
 at 68% C.L. (11)

We adopt  $\Delta l \simeq 9.8 \times 10^9 h_0^{-1}$  years. We express our results in terms of  $\nu_{100} = \nu/100$  GHz to keep them as independent of the CMB band frequency as possible.

Using Eq. (9), we find the following limit on the function  $\gamma(\nu)$  with  $\omega=2\pi\nu$ :

$$\gamma(\nu) \le 8.6 \times 10^{-31} \nu_{100}^{-1} h_0$$
 at 95% C.L., (12)

$$\gamma(\nu) \le 4.4 \times 10^{-31} \nu_{100}^{-1} h_0$$
 at 68% C.L. (13)

We can also express the limit on  $\gamma$  in terms of a limit for the mass scale M or the dimensionless parameter  $M/M_{\rm Pl}$ 

$$\frac{M}{M_{\rm Pl}} \lesssim 8.6 \times 10^{-31} (3 \times 10^{31})^{(N-4)} \nu_{100}^{3-N} h_0$$
 at 95% C.L. (14)

For N > 4, these limits are not very interesting, while for N = 4 or N = 3 "naturally expected" values of the parameters are ruled out. More precisely, for the models considered we constrain the dimensionless scalar  $\Psi_0$  for the N = 4 case,

$$|\Psi_0| \le 1.1 \times 10^{-31} h_0 \nu_{100}^{-1}$$
 at 95% C.L.,

while we find for N = 3

$$|L_0 - L\cos\phi| \le 3.6 \times 10^{-43} h_0 \text{ GeV}$$
 at 95% C.L.

This is almost an order of magnitude better than the limit obtained in Ref. [16].

We can also introduce an effective photon mass by writing the modified dispersion relation in the form  $\omega^2 = k^2 \pm m_{\gamma}^2$  with

$$m_{\gamma}^2 = \omega^2 \gamma(\omega) = M \omega \left(\frac{\omega}{M_{\rm PL}}\right)^{N-3} = 2 \frac{\Delta \alpha}{\Delta l} \omega.$$

For N>2 this is not a mass in the usual sense of the energy of the particle at rest, but rather a measure for the modification of the dispersion relation which tends to zero with frequency. For the renormalizable dimension 4 and 3 operators considered in this work we have  $m_{\gamma}^{(4D)}(\omega)=2\omega|2\Psi_0|^{1/2}$  and  $m_{\gamma}^{(3D)}=[\omega(|L_0-L\cos\phi|)]^{1/2}$ , respectively. As in Ref. [24] we can interpret our result also in terms of a polarization dependent group velocity,

$$v_{\pm} = 1 \pm \frac{N-2}{2} \frac{M}{M_{\rm Pl}} \left(\frac{\omega}{M_{\rm Pl}}\right)^{N-4} = 1 \pm \frac{N-2}{2} \gamma(\omega).$$
 (15)

Reference [24] only studied the cases  $N \ge 5$ . From Eq. (12) we derive the constraint on the effective birefringent mass,

$$m_{\gamma} \le 3.8 \times 10^{-19} (h_0 \nu_{100})^{1/2} \text{ eV}$$
 at 95% C.L. (16)

Note that left and right-handed photons have effective square masses of opposite sign. For the velocity difference this implies

$$|v_{+} - v_{-}| \le \begin{cases} 8.6 \times 10^{-31} h_{0} v_{100}^{-1} & \text{at } 95\% \text{ C.L., for } N = 3\\ 1.7 \times 10^{-30} h_{0} v_{100}^{-1} & \text{at } 95\% \text{ C.L., for } N = 4. \end{cases}$$
(17)

The limits on  $m_{\gamma}$  are model independent because  $m_{\gamma}$  only depends on the directly measured rotation angle  $\Delta \alpha$  and on the frequency.

If  $L \ll L_0$ , we can safely neglect the angular dependence and assume that  $m_{\gamma}^{(3D)} = \sqrt{\omega L_0}$ . However, if  $L \gg$ 

 $L_0$ , the modification of the photon dispersion becomes direction dependent and must be averaged over all sky for the CMB photons. Then, the rotation angle can be estimated by the two-point correlation function, i.e.,  $\Delta \alpha_{\rm eff} = \langle |\Delta \alpha|^2 \rangle^{1/2}$ . A rough estimate leads to a prefactor  $\sim 1/\sqrt{2}$ . In a more detailed analysis the presence of L breaks rotational symmetry and leads to off-diagonal correlations in the temperature anisotropy and polarization spectra analog to the effects on the CMB by a constant magnetic field [35,38]. To take this fully into account requires to estimate the CMB temperature-B polarization, E- and B-polarization cross correlations, as well as Bpolarization spectra due to the Lorentz symmetry violating vector field L, and to compare theoretical estimates with the corresponding CMB anisotropy and polarization data. Also the scalar  $|\Psi_0|$  of the 4D model breaks rotational symmetry and taking the direction dependence of  $\Delta \alpha$  into account is relatively complicated. This breakdown of statistical isotropy can also be tested using the bipolar power spectrum introduced in Ref. [41].

We shall address this issue in future work, but even though the limits may improve somewhat, we do not expect them to change significantly.

### IV. CONCLUSIONS

The obtained bound on a birefringent effective photon mass is below the limit for a standard photon mass given by the particle data group [42],  $m_{\gamma} \leq 3 \times 10^{-19} < 10^{-18}$  eV, but is less stringent than the limit from galactic magnetic fields which are, however, model dependent [43]. Of course, our photon mass would be measured only when measuring the dispersion relation of a polarized photon beam and would disappear when averaging over polarizations. It is not an ordinary mass.

Another useful bound is the departure of the refraction index in vacuum from unity, i.e.,  $|\Delta n| = |1 - k/\omega| =$  $|\gamma(\omega)|/2$ . In the 4D model,  $|\Delta n^{(4D)}| \simeq 4|\Psi_0|$ , in the 3D model,  $|\Delta n^{(3D)}| \simeq L_0/2\omega$  (when  $L \ll L_0$ ). Generically Eq. (12) implies  $|\Delta n| \le 4.3 \times 10^{-31} h_0 \nu_{100}^{-1}$ . The difference of the refractive index from 1 can be viewed as a difference of the photon speed from 1,  $\Delta c$  at the level of  $10^{-30}$ , which is much more stringent than the (more general) limit obtained in Ref. [44], which is  $\Delta c < 10^{-23}$ . The formalism given here is applicable also for higher dimension operators, but due to the frequency dependence  $|\alpha^{(LV)}| \propto \omega^{N-3}$ , the CMB based limits on the amplitudes for higher dimension operators become much weaker than those given from high energy photons ( $\gamma$  or x rays). Even the bounds obtained from the nearby Crab Nebulae are more promising [45] if  $N \ge 5$ .

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