Extra dimensions and Lorentz invariance violation

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We consider an effective model of photon interaction with a scalar field corresponding to conformal excitations of the internal space (geometrical moduli/gravexcitons). We demonstrate that this interaction results in a modified dispersion relation for photons, and, consequently, the photon group velocity depends on the energy implying the propagation time-delay effect. We suggest the use of the experimental bounds of the time delay of gamma-ray burst photon propagation as an additional constraint for the gravexciton parameters.

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Several modifications of the standard model of particle physics and cosmology (such as M/string theory, Kaluza-Klein models, brane-world models, etc.) lead to Lorentz invariance (LI) violation (LV) [1]. In this paper we investigate the LV test related to photon dispersion measure. This test is based on the LV effect of a phenomenological energy-dependent speed of photon [2–8] (for recent studies see Ref. [9] and references therein).

The formalism that we use is based on the analogy with electromagnetic wave propagation in a magnetized medium and extends previous works [8,10,11]. In our model, instead of propagation in a magnetized medium, the electromagnetic waves are propagating in a vacuum filled with a scalar field ψ. LV occurs because of an interaction term $f(\psi)F^2$, where F is an amplitude of the electromagnetic field. Such an interaction might have different origins. In the string theory, ψ could be a dilaton field [12,13]. The field ψ could be associated with geometrical moduli. In brane-world models, the similar term describes an interaction between the bulk dilaton and the standard model fields on the brane [14]. In Ref. [15], such an interaction was obtained in $N = 4$ supergravity in four dimensions. In Kaluza-Klein models, the term $f(\psi)F^2$ has the pure geometrical origin, and it appears in the effective, dimensionally reduced, four-dimensional action (see, e.g., [16,17]). In particular, in reduced Einstein-Yang-Mills theories, the function $f(\psi)$ coincides (up to a numerical prefactor) with the volume of the internal space. Phenomenological (exactly solvable) models with spherical symmetries were considered in Ref. [18]. To be more specific, we consider the model which is based on the reduced Einstein-Yang-Mills theory [17], where the term $\propto \psi F^2$ describes the interaction between the conformal excitations of the internal space (gravexcitons) and photons. It is clear that the similar LV effect exists for all types of interactions of the form $f(\psi)F^2$ mentioned above.

Obviously, the interaction term $f(\psi)F^2$ modifies the Maxwell equations, and, consequently, results in a modified dispersion relation for photons. We show that this modification has a rather specific form. For example, we demonstrate that refractive indices for the left and right circularly polarized (LCP and RCP) waves coincide with each other. Thus, rotational invariance is preserved. However, the speed of the electromagnetic wave's propagation in vacuum differs from the speed of light c. This difference implies the time-delay effect which can be measured via high-energy gamma-ray burst (GRB) photon propagation over cosmological distances (see, e.g., Ref. [9]). It is clear that gravexcitons should not overclose the Universe and should not result in variations of the fine structure constant. These demands lead to a certain constraint for gravexcitons (see Refs. [17,19]). We use the time-delay effect, caused by the interaction between photons and gravexcitons, to get additional bounds on the parameters of gravexcitons.

The starting point of our investigation is the Abelian part of D-dimensional action of the Einstein-Yang-Mills theory:

$$S_{EM} = -\frac{1}{2}\int_M d^D x\sqrt{|g|}F_{MN}F^{MN}, \tag{1}$$

where the D-dimensional metric, $g = g_{MN}(X)dX^M \otimes dX^N = g^{(0)}(x)_{\mu\nu}dx^\mu \otimes dx^\nu + a_1^2(x)g^{(1)}$, is defined on the product manifold $M = M_0 \times M_1$. Here, M_0 is the $(D_0 = d_0 + 1)$-dimensional external space. The d_1-dimensional internal space M_1 has a constant curvature with the scale factor $a_1(x) \equiv L_{Pl}\exp\beta^1(x)$. Dimensional reduction of the action (1) results in the following effective D_0-dimensional action [17]:

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$$\bar{S}_{EM} = -\frac{1}{2}\int_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}[(1-\mathcal{D}\kappa_0\psi)F_{\mu\nu}F^{\mu\nu}], \quad (2)$$

which is written in the Einstein frame with the D_0-dimensional metric, $\tilde{g}^{(0)}_{\mu\nu} = (\exp d_1\bar{\beta}^1)^{-2/(D_0-2)}g^{(0)}_{\mu\nu}$. Here, $\kappa_0\psi \equiv -\bar{\beta}^1\sqrt{(D_0-2)/d_1(D-2)} \ll 1$ and $\bar{\beta}^1 \equiv \beta^1 - \beta^1_0$ are small fluctuations of the internal space scale factor over the stable background β^1_0 (0 subscript denotes the present-day value). These internal space scale-factor small fluctuations/oscillations have the form of a scalar field (so-called gravexciton [20]) with a mass m_ψ defined by the curvature of the effective potential (for details see [20]). Action (2) is defined under the approximation $\kappa_0\psi < 1$ that obviously holds for the condition[1] $\psi < M_{Pl}$. $\kappa_0^2 = 8\pi/M_{Pl}^2$ is a four-dimensional gravitational constant, M_{Pl} is the Plank mass, and $\mathcal{D} = 2\sqrt{d_1/[(D_0-1)(D-1)]}$ is a model-dependent constant. The Lagrangian density for the scalar field ψ reads: $\mathcal{L}_\psi = \sqrt{|\tilde{g}^{(0)}|}(-\tilde{g}^{\mu\nu}\psi_{,\mu}\psi_{,\nu} - m_\psi^2\psi\psi)/2$. For simplicity we assume that $\tilde{g}^0$ is the flat Friedman-Lemaitre-Robertson-Walker (FLRW) metric with the scale factor $a(t)$.

Let us consider Eq. (2). It is worth of noting that the D_0-dimensional field strength tensor, $F_{\mu\nu}$, is gauge invariant.[2] Second, action (2) is conformally invariant in the case when $D_0 = 4$. The transform to the Einstein frame does not break gauge invariance of the action (2), and the electromagnetic field is antisymmetric as usual, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Varying (2) with respect to the electromagnetic vector potential,

$$\partial_\nu[\sqrt{-g}(1-\mathcal{D}\kappa_0\psi)F^{\mu\nu}] = 0. \quad (3)$$

The second term in the round brackets $\mathcal{D}\kappa_0\psi F^{\mu\nu}$ reflects the interaction between photons and the scalar field ψ, and, as we show below, it is responsible for LV. In particular, coupling between photons and the scalar field ψ makes the speed of photons different from the standard speed of light. Equation (3) together with the Bianchi identity (which is preserved in the considered model due to gauge invariance of the tensor, $F_{\mu\nu}$ [17]) defines a complete set of the generalized Maxwell equations. As we noted, action (2) is conformally invariant in the 4D-dimensional space-time. So, it is convenient to present the flat FLRW metric $\tilde{g}^0$ in the conformally flat form: $\tilde{g}^0_{\mu\nu} = a^2\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric.

Using the standard definition of the electromagnetic field tensor, $F_{\mu\nu}$, we obtain the complete set of the Maxwell equations in vacuum:

[1] In the brane-world model, the prefactor κ_0 in the expression for $\kappa_0\psi$ is replaced by the parameter proportional to M_{EW}^{-1} [14]. Thus, the smallness condition holds for $\psi < M_{EW}$.

[2] Equation (2) can be rewritten in the more familiar form $\bar{S}_{EM} = -(1/2)\int_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}$ [17]. The field strength tensor $\bar{F}_{\mu\nu}$ is not gauge invariant here.

$$\nabla\cdot\mathbf{B} = 0, \quad (4)$$

$$\nabla\cdot\mathbf{E} = \frac{\mathcal{D}\kappa_0}{1-\mathcal{D}\kappa_0\psi}(\nabla\psi\cdot\mathbf{E}), \quad (5)$$

$$\nabla\times\mathbf{B} = \frac{\partial\mathbf{E}}{\partial\eta} - \frac{\mathcal{D}\kappa_0\dot{\psi}}{1-\mathcal{D}\kappa_0\psi}\mathbf{E} + \frac{\mathcal{D}\kappa_0}{1-\mathcal{D}\kappa_0\psi}[\nabla\psi\times\mathbf{B}], \quad (6)$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial\eta}, \quad (7)$$

where all operations are performed in the Minkowski space-time, η denotes conformal time related to physical time t as $dt = a(\eta)d\eta$, and an overdot represents a derivative with respect to conformal time η.

Equations (4) and (7) correspond to the Bianchi identity, and since it is preserved, Eqs. (4) and (7) keep their usual forms. Equations (5) and (6) are modified due to interactions between photons and gravexcitons ($\propto \kappa_0\psi$). These modifications have simple physical meaning: the interaction between photons and the scalar field ψ acts as an effective electric charge $e_{\rm eff}$. This effective charge is proportional to the scalar product of the ψ field gradient and the $\mathbf{E}$ field, and it vanishes for a homogeneous ψ field. The modification of Eq. (6) corresponds to an effective current $\mathbf{J}_{\rm eff}$, which depends on both electric and magnetic fields. This effective current is determined by variations of the ψ field over the time ($\dot{\psi}$) and space ($\nabla\psi$). For the case of a homogeneous ψ field the effective current is still present and LV takes place. The modified Maxwell equations are conformally invariant. To account for the expansion of the Universe we rescale the field components as $\mathbf{B}, \mathbf{E} \to \mathbf{B}, \mathbf{E}\, a^2$ [21].

To obtain a dispersion relation for photons, we use the Fourier transform between position and wave number spaces as $\mathbf{F}(\mathbf{k},\omega) = \iint d\eta\ d^3x e^{-i(\omega\eta - \mathbf{k}\cdot\mathbf{x})}\mathbf{F}(\mathbf{x},\eta)$. Here, $\mathbf{F}$ is a vector function describing either the electric or the magnetic field, ω is the angular frequency of the electromagnetic wave measured today, and $\mathbf{k}$ is the wave vector. We assume that the field ψ is an oscillatory field with the frequency ω_ψ and the momentum $\mathbf{q}$, so $\psi(\mathbf{x},\eta) = Ce^{i(\omega_\psi\eta - \mathbf{q}\cdot\mathbf{x})}$, $C = \text{const}$. Equation (4) implies $\mathbf{B}\perp\mathbf{k}$. Without losing generality, and for simplicity of description, we assume that the wave vector $\mathbf{k}$ is oriented along the $\mathbf{z}$ axis. Using Eq. (7) we get $\mathbf{E}\perp\mathbf{B}$.

A linearly polarized wave can be expressed as a superposition of left (L, −) and right (R, +) circularly polarized waves. Using the polarization basis of Sec. 1.1.3 of Ref. [22], we derive $E^\pm = (E_x \pm iE_y)/\sqrt{2}$. Rewriting Eqs. (4)–(7) in the components[3] for LCP and RCP waves we get

[3] We have defined the system of six equations with respect to six components of the vectors $\mathbf{E}$ and $\mathbf{B}$. This system has nontrivial solutions only if its determinant is nonzero. From this condition we get the dispersion relation. The Faraday rotation effect is absent if the matrix has a diagonal form.

$$(1 - n_+^2)E^+ = 0, \qquad (1 - n_-^2)E^- = 0, \tag{8}$$

where n_+ and n_- are refractive indices for RCP and LCP electromagnetic waves

$$n_+^2 = \frac{k^2[1 - \mathcal{D}\kappa_0\psi(1 + q_z/k)]}{\omega^2[1 - \mathcal{D}\kappa_0\psi(1 + \omega_\psi/\omega)]} = n_-^2. \tag{9}$$

In the case when LI is preserved, the electromagnetic waves propagating in vacuum have $n_+ = n_- = n = k/\omega \equiv 1$. For the electromagnetic waves propagating in the magnetized plasma, $k/\omega \neq 1$, and the difference between the LCP and RCP refractive indices describes the Faraday rotation effect, $\alpha \propto \omega(n_+ - n_-)$ [23]. In the considered model, since $n_+ = n_-$ the rotation effect is absent, but the speed of electromagnetic wave propagation in vacuum differs from the speed of light c (see also Ref. [24] for LV induced by electromagnetic field coupling to other generic fields). This difference implies the propagation time-delay effect, $\Delta t = \Delta l(1 - \partial k/\partial\omega)$ (Δl is a propagation distance); Δt is the difference between the photon travel time and that for a "photon" which travels at the speed of light c. Here, t is physical synchronous time. This formula does not take into account the evolution of the Universe. However, it is easy to show that the effect of the Universe expansion is negligibly small.

Solving the dispersion relation as a square equation,

$$\frac{\partial k}{\partial \omega} \simeq \pm\left\{1 + \frac{1}{2}\left[\frac{\omega_\psi^2 - q_z^2}{4\omega^2}\right](\mathcal{D}\kappa_0\psi)^2\right\}, \tag{10}$$

where $\pm$ signs correspond to the forward and backward directions of photons, respectively.

The modified inverse group velocity (10) shows that the LV effect can be measured if we know the gravexciton frequency ω_ψ, z component of the momentum q_z, and its amplitude ψ. For our estimates, we assume that ψ is the oscillatory field, satisfying (in local Lorentz frame) the dispersion relation, $\omega_\psi^2 = m_\psi^2 + \mathbf{q}^2$, where m_ψ is the mass of gravexcitons. Unfortunately, we do not have any information concerning parameters of gravexcitons (some estimates can be found in [17,19]). Thus, we intend to use possible LV effects (supposing they are caused by interaction between photons and gravexcitons) to set limits on gravexciton parameters. We can easily get the following estimate for the upper limit of the amplitude of gravexciton oscillations:

$$|\psi| \approx \frac{1}{\sqrt{\pi}\mathcal{D}}\sqrt{\left|\frac{\Delta t}{\Delta l}\right|\frac{\omega}{m_\psi}}M_{Pl}, \tag{11}$$

where for ω and m_ψ we can use their physical values. In the case of GRB with $\omega \sim 10^{21}$–10^{22} Hz (10^{-4}–10^{-3} GeV), and $\Delta l \sim 3$–5×10^9 y $\sim 10^{17}$ sec, the typical upper limit for the time delay is $\Delta t \sim 10^{-4}$ sec [9]. For these values the upper limit on gravexciton amplitude of oscillations is

$$|\kappa_0\psi| \approx \frac{10^{-13}\ \mathrm{GeV}}{m_\psi}. \tag{12}$$

This estimate shows that our approximation $\kappa_0\psi < 1$ works for gravexciton masses $m_\psi > 10^{-13}$ GeV. Future measurements of the time-delay effect for GRBs at frequencies $\omega \sim 1$–10 GeV would increase significantly the limit up to $m_\psi > 10^{-9}$ GeV. On the other hand, Cavendish-type experiments [25,26] exclude fifth-force particles with masses $m_\psi \lesssim 1/(10^{-2}\ \mathrm{cm}) \sim 10^{-12}$ GeV, which is rather close to our lower bound for ψ field masses. Respectively, we slightly shift the considered mass lower limit to be $m_\psi \geq 10^{-12}$ GeV. These masses are considerably higher than the mass corresponding to the equality between the energy densities of the matter and radiation (matter/radiation equality), $m_{\mathrm{eq}} \sim H_{\mathrm{eq}} \sim 10^{-37}$ GeV, where H_{eq} is the Hubble "constant" at matter/radiation equality. It means that such ψ particles start to oscillate during the radiation-dominated epoch (for details see Refs. [17,19]. Another bound on the ψ particles' masses comes from the condition of their stability. With respect to decay $\psi \to \gamma\gamma$ the lifetime of ψ particles is $\tau \sim (M_{Pl}/m_\psi)^3 t_{Pl}$ [17], and the stability conditions require that the decay time should be greater than the age of the Universe. According to this we consider light gravexcitons with masses $m_\psi \lesssim 10^{-21} M_{Pl} \sim 10^{-2}\ \mathrm{GeV} \sim 20 m_e$ (where m_e is the electron mass).

An additional restriction arises from the condition that such cosmological gravexcitons should not overclose the observable Universe. This reads $m_\psi \lesssim m_{\mathrm{eq}}(M_{Pl}/\psi_{\mathrm{in}})^4$, which implies the following restriction for the amplitude of the initial oscillations: $\psi_{\mathrm{in}} \lesssim (m_{\mathrm{eq}}/m_\psi)^{1/4} M_{PL} \ll M_{Pl}$ [19]. Thus, for the range of masses $10^{-12}\ \mathrm{GeV} \leq m_\psi \leq 10^{-2}$ GeV, we obtain respectively $\psi_{\mathrm{in}} \lesssim 10^{-6} M_{Pl}$ and $\psi_{\mathrm{in}} \lesssim 10^{-9} M_{Pl}$. It is not difficult to estimate (see for details Ref. [19]) for the considered light gravexcitons their amplitude of oscillations at the present time: $|\kappa_0\psi| \sim 10^{-60}(\psi_{\mathrm{in}}/M_{Pl})(M_{Pl}/m_\psi)^{3/4}$. Together with the nonovercloseness condition, we obtain from this expression that $|\kappa_0\psi| \sim 10^{-43}$ for $m_\psi \sim 10^{-12}$ GeV and $\psi_{\mathrm{in}} \sim 10^{-6} M_{Pl}$ and $|\kappa_0\psi| \sim 10^{-53}$ for $m_\psi \sim 10^{-2}$ GeV and $\psi_{\mathrm{in}} \sim 10^{-9} M_{Pl}$. Obviously, it is much less than the upper limit (12). Note, as we mentioned above, gravexcitons with masses $m_\psi \gtrsim 10^{-2}$ GeV can start to decay at the present epoch. However, taking into account the estimate $|\kappa_0\psi| \sim 10^{-53}$, we can easily get that their energy density $\rho_\psi \sim (|\kappa_0\psi|^2/8\pi) M_{Pl}^2 m_\psi^2 \sim 10^{-55}\ \mathrm{g/cm^3}$ is much less than the present energy density of the radiation $\rho_\gamma \sim 10^{-34}\ \mathrm{g/cm^3}$. Thus, ρ_ψ contributes negligibly in ρ_γ. Otherwise, the gravexcitons with masses $m_\psi \gtrsim 10^{-2}$ GeV should be observed at the present time, which, obviously, is not the case. Additionally, it follows from Eq. (42) in Ref. [17] that to avoid the problem of the fine structure constant variation, the amplitude of the initial oscillations should satisfy the condition $\psi_{\mathrm{in}} \lesssim 10^{-5} M_{Pl}$, which, obviously, completely agrees with our upper bound $\psi_{\mathrm{in}} \lesssim 10^{-6}$ GeV.

In summary, we have shown that LV effects can give additional restrictions on parameters of gravexcitons. First, gravexcitons should not be lighter than 10^{-13} GeV. It is very close to the limit following from the fifth-force experiment. Moreover, experiments for GRB at frequencies $\omega > 1$ GeV can result in a significant shift of this lower limit making it much stronger than the fifth-force estimates. Together with the nonovercloseness condition, this estimate leads to $\psi_{\rm in} \lesssim 10^{-6}$ GeV. Thus, the bound on the initial amplitude obtained from the fine structure constant variation is one magnitude weaker than ours, even for the limiting case of the gravexciton masses. This limit becomes stronger for heavier gravexcitons. Our estimates for the present-day amplitude of the gravexciton oscillations, following from the above obtained limitations, show that we cannot use the LV effect for the direct detection of the gravexcitons. Nevertheless, the obtained bounds can be useful for astrophysical and cosmological applications. For example, let us suppose that gravexcitons with masses $m_\psi > 10^{-2}$ GeV are produced during late stages of the Universe expansion in some regions and GRB photons travel to us through these regions. Then, estimate $|\kappa_0\psi| \sim 10^{-60}(\psi_{\rm in}/M_{Pl})(M_{Pl}/m_\psi)^{3/4}$ is not valid for such gravexcitons having astrophysical origin, and the only upper limit on the amplitude of their oscillations (in these regions) follows from Eq. (12). In the case of TeV masses we get $|\kappa_0\psi| \sim 10^{-16}$. If GRB photons have frequencies up to 1 TeV, $\omega \sim 1$ TeV, then this estimate is increased by six orders of magnitude.

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