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Gamma ray burst constraints on ultraviolet Lorentz invariance violation

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Abstract

We present a unified general formalism for ultraviolet Lorentz invariance violation (LV) testing through electromagnetic wave propagation, based on both dispersion and rotation measure data. This allows for a direct comparison of the efficacy of different data to constrain LV. As an example we study the signature of LV on the rotation of the polarization plane of γ-rays from gamma ray bursts in a LV model. Here γ-ray polarization data can provide a strong constraint on LV, 13 orders of magnitude more restrictive than a potential constraint from the rotation of the cosmic microwave background polarization proposed by Gamboa, López-Sarrión, and Polychronakos [J. Gamboa, J. López-Sarrión, A.P. Polychronakos, Phys. Lett. B 634 (2006) 471].
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Lorentz invariance violation (LV) has been proposed as a possible modification of the standard model of particle physics and cosmology (for recent reviews see Refs. [2–4]). Various LV mechanisms have been considered, including those motivated by phenomenological quantum gravity, string theory, non-commutative geometry, and through a Chern–Simons coupling (for a review see Section 2 of Ref. [3]). LV can influence particle propagation (the dispersion relation), result in rotation of linear polarization (birefringence), and affect the interaction of particles (including resulting in photon decay and vacuum Čerenkov radiation) [4]. These effects can be used to probe LV; for reviews of current and future tests see Refs. [2–4].

The assumed LV mechanism determines the kind of measurements required to test the model. Here we study frequency-dependent Faraday-like rotation of gamma ray burst (GRB) γ-ray and X-ray photon polarization in the context of ultraviolet LV. For discussions of such high energy LV see Refs. [5–7]. Refs. [1,8,9] study a generalized electromagnetism motivated by this kind of LV. On the other hand, LV associated with a Chern–Simons interaction [10,11] affects the complete spectrum of electromagnetic radiation, not just the high-frequency part, and induces a frequency-independent polarization-plane rotation (see Section 4 of Ref. [12]).

In this Letter we present a general formalism for LV testing that encompasses both rotation measure (RM) and photon dispersion measure (DM)[1] observations. This formalism is based on an analogy with electromagnetic (EM) wave propagation in a magnetized medium, and extends previous works [5,10,15,20]. We show that the Gamboa et al. (GLP), [1], LV model is more tightly constrained by RM data than by DM data. The LV model of Myers and Pospelov (MP), [7], can be tightly constrained by GRB γ-ray DM and RM observations. The highly-variable γ-ray flux of energetic GRB photons propagating over cosmological distances make GRBs a powerful cosmological probe [13] (for reviews of cosmological tests involving GRBs, see Refs. [3,21]). Testing LV through RM observations of GRB

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[1] The DM test is based on the LV effect of a phenomenological energy-dependent photon speed [13] or a modified electron dispersion relation. See Ref. [14] for reviews and Refs. [15–18] for recent studies of this effect; related early discussion include Ref. [19]. (Refs. [13,16,18] consider LV models in which rotational and translational invariance are preserved but boost invariance is broken.)

polarization was proposed, [22,23], after the reported observation of highly linearly polarized γ-rays from GRB021206 [24]; this measurement has been strongly contested [25]. On the other hand, Ref. [26] recently presented evidence that the γ-ray flux from GRB 930131 and GRB 960924 is consistent with polarization degree $> 35\%$ and $> 50\%$, respectively. Since the issue of polarization of GRB γ-rays still remains uncertain,[2] we also discuss using future X-ray RM observations.[3] See Refs. [31] for other RM tests.

We first consider the ultraviolet LV model of GLP [1]. Breaking Lorentz invariance leads to a modification of the Maxwell equations [7,9], and in vacuum they become [1]

$$\nabla\cdot\mathbf{B}=0,\qquad \nabla\times\mathbf{B}=\dot{\mathbf{E}},$$
$$\nabla\cdot\mathbf{E}=0,\qquad \nabla\times\mathbf{E}+(\mathbf{g}\cdot\nabla)\dot{\mathbf{E}}=-\dot{\mathbf{B}}. \tag{1}$$

Here an overdot represents a derivative with respect to conformal time t, $\mathbf{g}$ is the LV vector related to the non-zero commutator of gauge potentials [1], $\mathbf{B}$ is the magnetic field, and $\mathbf{E}$ is the electric field that couples to matter in the usual way but is not related to the gauge potential in the usual way [1]. To account for the expansion of the Universe we have to specify how $\mathbf{g}$ scales in the expanding Universe. In conventional electrodynamics the expansion of the Universe is accounted for by a conformal rescaling of physical quantities, i.e. $\mathbf{B},\mathbf{E}\to\mathbf{B},\mathbf{E}a^2$, where a is the scale factor [32]. Assuming that the GLP model is conformally invariant, the expansion may be accounted for by rescaling $\mathbf{g}\to\mathbf{g}/a$, while the components of the physical electric and magnetic field are diluted as $1/a^2$. On the other hand if the GLP model also violates conformal invariance, it is due to a small effect and so the expansion can be accounted for as above. So GLP LV results in only the Bianchi identity being modified.

In this model the equations for EM wave propagation in vacuum are

$$\left[(\omega^2-k^2)\delta_{ij}-i\omega k_l\epsilon_{ijl}\mathbf{k}\cdot\mathbf{g}\right]E_j(\mathbf{k})=0, \tag{2}$$
$$k_jE_j(\mathbf{k})=0. \tag{3}$$

Here ϵ_{ijl} is the totally antisymmetric symbol, Latin indices denote space coordinates, $i\in(1,2,3)$, ω is the angular frequency of the EM wave measured today, and $\mathbf{k}$ is the wavevector. When transforming between position and wavenumber spaces we use

$$E_j(\mathbf{k})=e^{i\omega t}\int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}}E_j(\mathbf{x},t),$$
$$e^{i\omega t}E_j(\mathbf{x},t)=\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot\mathbf{x}}E_j(\mathbf{k}).$$

The $e^{i\omega t}$ prefactor describes rapidly varying (compared to the cosmological expansion time) EM waves.

A linearly polarized wave can be expressed as a superposition of left (L) and right (R) circularly polarized (CP) waves. Using the polarization basis of Section 1.1.3 of Ref. [33], Eqs. (2) become, for LCP (E^+) and RCP (E^-) waves,

$$\left(\omega^2-k^2\mp\omega k^2\hat{\mathbf{k}}\cdot\mathbf{g}\right)E^\pm=0. \tag{4}$$

A similar dispersion relation, in a D-brane recoil model, has been obtained in Ref. [17]. To account for the phenomenological LV of an energy-dependent photon speed [3,4,7,12,22], we add photon-spin-sign-dependent $\mp\gamma(k)k^2E^\pm(k)$ to the left-hand side of Eqs. (2) [23]. Here (Eq. (5) of Ref. [18])

$$\gamma(k)=\left(\frac{\hbar k}{\xi m_{\rm pl}}\right)^q, \tag{5}$$

where $m_{\rm pl}$ is the Planck mass, $\hbar$ is Planck's constant, ξ is a dimensionless constant that determines the LV energy scale,[4] and q is a model-dependent number.[5] This modification may be viewed as an effective photon "mass" that makes the photon speed less (greater) than the low energy speed of light c for the RCP (LCP) waves.

To keep the formalism simple we consider an EM wave propagating in the $\mathbf{z}$ direction with $\mathbf{k}=(0,0,k)$, and with the LV vector oriented along the $\mathbf{z}$ axis, i.e., $\mathbf{g}=(0,0,g)$. Eqs. (4) lead to the dispersion relations

$$\omega^2=k^2\left[1\pm\gamma(k)\pm g\omega\right], \tag{6}$$

and in this case $E^\pm=(E_x\pm iE_y)/\sqrt{2}$. We now draw an analogy with the propagation of a high-frequency EM wave in a magnetized plasma.[6] High-frequency RCP and LCP waves propagating in the $\mathbf{z}$ direction in an homogeneous magnetic field directed along the $\mathbf{z}$ axis obey [36]

$$\left(1-\frac{\varepsilon_1}{n^2}\right)E_x(k)-i\frac{\varepsilon_2}{n^2}E_y(k)=0, \tag{7}$$
$$i\frac{\varepsilon_2}{n^2}E_x(k)+\left(1-\frac{\varepsilon_1}{n^2}\right)E_y(k)=0. \tag{8}$$

Here $n=k/\omega$ is the refractive index and ε_1 and ε_2 are components of the electric permittivity or dielectric tensor ε_{ij},

$$\varepsilon_1=\varepsilon_{xx}=\varepsilon_{yy}=1+\frac{\omega_{\rm p}^2}{\omega_{\rm c}^2-\omega^2},$$
$$\varepsilon_2=\varepsilon_{yx}=-\varepsilon_{xy}=\frac{\omega_{\rm c}}{\omega}\frac{\omega_{\rm p}^2}{\omega_{\rm c}^2-\omega^2}, \tag{9}$$

where $\omega_{\rm p}$ and $\omega_{\rm c}$ are the plasma and electron cyclotron angular frequencies (see Section 4.9 of Ref. [36]).

In the magnetized plasma case an homogeneous magnetic field induces a phase velocity difference between LCP and RCP waves and so causes rotation of the polarization plane. Also, in this case, the group velocity of an EM wave differs from c and so results in time delay. These two independent

[2] For a review of models for generating polarized γ-rays from GRBs see Sections V.F and VI.E of Ref. [21]; more recent discussions include Ref. [27]. For discussions of hard X-ray and γ-ray polarimetry see Refs. [28,29].

[3] Ref. [30] predicts linearly polarized X-rays from flares following prompt GRB γ-ray emission.

[4] In this case the modification of Maxwell equations does not preserve conformal invariance [34].

[5] Ref. [35] argues that the much-studied $q=1$ case is almost ruled out by Crab nebula X-ray polarimetry data.

[6] This is motivated by the fact that LV generates an homogeneous magnetic field [9,34].

DM and RM effects can be expressed in terms of refractive indices, $n_{L,R} = (\varepsilon_1 \mp \varepsilon_2)^{1/2}$, where the sum (lower sign) corresponds to the RCP wave [36].[7] As a consequence the LCP and RCP wavevectors are $k_{L,R} = \omega n_{L,R}$. Both DM and RM effects depend on the photon travel distance Δl and are expressed through

$$\Delta t_{L,R} = \Delta l\left(1 - \frac{\partial k_{L,R}}{\partial \omega}\right), \tag{10}$$

$$\Delta\phi = \frac{1}{2}(k_L - k_R)\Delta l. \tag{11}$$

Here $\Delta t_{L,R}$ is the difference between the LCP (RCP) photon travel time and that for a "photon" which travels at c, and $\Delta\phi$ is the polarization-plane rotation angle.

We can rewrite Eqs. (2) and (3) for the LV case in a form similar to Eqs. (7) and (8) for a magnetized plasma. Define two dimensionless quantities $\varepsilon_1^{\pm} = 1/(1 \pm \gamma(k))$ and $\varepsilon_2^{\pm} = -gk^2/[\omega(1 \pm \gamma(k))]$. Experimentally LV is small so we simplify by taking $\gamma(k)$ and $g\omega$ to be small and work to linear order in these quantities. To linear order, $\varepsilon_2^{\pm} \approx -g\omega$ and is independent of the photon spin sign, while $\varepsilon_1^{\pm} \approx 1 \mp \gamma$ and depends on the photon spin sign. The corresponding (L, upper sign) and (R, lower sign) refractive indices are $n_{L,R} = (1 \pm \gamma \pm g\omega)^{1/2}$. Both kinds of LV (scalar γ and vector $\mathbf{g}$) induce DM and RM effects. There are two different regimes of interest, when $\gamma \gg g\omega$ and when $\gamma \ll g\omega$.

When $\gamma(k) \gg g\omega$, as in the MP model [7], Eqs. (10) and (11) become

$$\Delta t_{L,R} \simeq \mp\frac{\Delta l}{2}(1+q)\gamma(k), \tag{12}$$

$$\Delta\phi \simeq \frac{\Delta l}{2}\omega\gamma(k). \tag{13}$$

These expressions agree with those obtained earlier in Refs. [18,22,23]. DM and RM measurements can be used to constrain γ. DM testing of LV through the time delay of GRBs photons has been widely discussed (for a recent review see Ref. [4]) and so is not discussed here.

When $\gamma(k) \ll g\omega$, as in the GLP model [1], Eq. (11) yields (see also the Conclusion of Ref. [1]),

$$\Delta\phi \simeq \omega^2 g\frac{\Delta l}{2}, \tag{14}$$

and Eq. (10) for the time delay gives

$$\Delta t_{L,R} \approx \mp g\omega\Delta l. \tag{15}$$

DM and RM measurements constrain the value of ε_2 (or $g\omega$), but the dependence on frequency is different, with the constraints from the RM test being the strongest for high-frequency waves.

For "classical" Faraday rotation $\Delta\phi \sim \omega^{-2}$, [37], and the effect is the strongest for low-frequency waves. For GLP LV $\Delta\phi \sim \omega^2$ and the effect is the strongest for high-frequency waves. Ref. [1] suggests using cosmic microwave background (CMB) polarization data to test GLP LV, as was previously proposed to detect a primordial cosmological magnetic field [32, 37] and test for CPT violation [38]. We argue below that GRB γ-rays polarization measurements will give a much stronger bound on this kind of LV. On the other hand, lower-frequency CMB polarization data may be used to constrain LV induced by a Chern–Simons coupling since in this case the RM is frequency independent [10] (this will complement the limit obtained from radio galaxy RM data [10]).

It should be possible to measure a $\Delta\phi \sim 10^{-2}$ rad. For CMB radiation with $\omega \sim 10^{11}$ Hz and for photon travel distance $\Delta l \sim 1.3 \times 10^{10}$ y, the RM GLP LV constraint, Eq. (14), indicates that one may probe to

$$g_{CMB} \sim 10^{-18}\ \text{GeV}^{-1}. \tag{16}$$

For GRB γ-rays with $\omega \sim 10^{19}$ Hz and $\Delta l \sim 3$–5×10^9 y, even with less accurate RM data with, say, $\Delta\phi \sim 1$ rad, Eq. (14) shows that there is detectable LV down to

$$g_{GRB} \sim 10^{-31}\ \text{GeV}^{-1}. \tag{17}$$

In the GLP model GRB γ-ray data can probe 13 orders of magnitude higher in energy than can CMB data. Note that synchrotron radiation RM data at $\omega = 340$ GHz [39] from Sagittarius A* at $\Delta l \simeq 2.5 \times 10^4$ y with $\Delta\phi \simeq 0.5$ rad gives the weaker constraint $g_{Sag} \approx 10^{-11}$ GeV^{-1}. The polarization data at the optical band from active galactic nuclei give 8 magnitudes weaker limits than GRB future data.

To compare the relative efficacy of RM and DM data at probing LV, we consider the ratios of the same-source DM and RM data LV limits for the two characteristic LV quantities ξ^{-1} and g,

$$r_\xi = \frac{\xi_{DM}^{-1}}{\xi_{RM}^{-1}}, \qquad r_g = \frac{g_{DM}}{g_{RM}}. \tag{18}$$

The constraints on ξ^{-1} in the case when $\gamma \gg g\omega$ and $k \simeq \omega$ can be obtained from Eqs. (5), (12), and (13),

$$\xi_{DM}^{L,R} = \frac{\hbar\omega}{m_{pl}}\left[\frac{(q+1)\Delta l}{\mp 2\Delta t_{L,R}}\right]^{1/q},$$

$$\xi_{RM} = \frac{\hbar\omega^{1+1/q}}{m_{pl}}\left[\frac{\Delta l}{2\Delta\phi}\right]^{1/q}. \tag{19}$$

The constraints on g when $\gamma \ll g\omega$ can be obtained from Eqs. (14) and (15),

$$g_{DM}^{L,R} = \mp\frac{\Delta t_{L,R}}{\omega\Delta l}, \qquad g_{RM} = \frac{2\Delta\phi}{\omega^2\Delta l}. \tag{20}$$

We first consider GLP LV where $\gamma \ll g\omega$. Using the GRB γ-ray parameters mentioned above, taking $|\Delta t_{L,R}| = 10^{-4}$ s as the current accuracy of time delay data [18], and assuming $\Delta\phi = 1$ rad, $|r_g^{GRB}| = \omega|\Delta t_{L,R}|/(2\Delta\phi) \sim 10^{14}$. So in this case the limit from RM data is the strongest. If one wishes to constrain g using GRB DM and CMB RM data, then $|r_g^\star| = |g_{DM}^{GRB}|/g_{RM}^{CMB} \approx 0.2$, so both are almost equally good tests for LV.

[7] The LCP and RCP EM wave electric fields obey $[n^2 - (\varepsilon_1 \pm \varepsilon_2)]E^{\pm} = 0$ [36]. The basis vectors $(\mathbf{e}^+, \mathbf{e}^-, \hat{\mathbf{z}})$ satisfy $\mathbf{e}^{\pm} \cdot \mathbf{e}^{\mp} = 1$, $\mathbf{e}^{\pm} \cdot \mathbf{e}^{\pm} = 0$, $\mathbf{e}^{\pm}(\hat{\mathbf{z}}) = \mathbf{e}^{\mp}(-\hat{\mathbf{z}})$, and $\pm\mathbf{e}^{\pm} = i\hat{\mathbf{z}} \times \mathbf{e}^{\pm}$ [33].

In the opposite case when $\gamma \gg g\omega$, if DM and RM data from the same source are used,

$$r_\xi^{\mathrm{L,R}} = \left[\frac{\mp\omega\Delta t_{\mathrm{L,R}}}{(q+1)\Delta\phi}\right]^{1/q}. \tag{21}$$

Conventionally two cases are considered, the linear case with $q = 1$ [15,18], and the quadratic case with $q = 2$ [18,22]. For $q = 1$ Eq. (21) reads for GRB γ-rays $|r_\xi^{\mathrm{GRB}}| = \omega|\Delta t_{\mathrm{L,R}}|/(2\Delta\phi) \sim 10^{14}$. Note that our ξ is the inverse of the ξ of Ref. [15] and coincides with the ξ of Ref. [18]. Using the GRB γ-ray parameters considered above, we see that CMB polarization RM data may slightly improve the ξ limit obtained from GRB γ-ray DM data [18]. The improvement will be much more significant if GRB γ-ray RM data is used [4,22]. For the $q = 1$ MP model [7] r_g and r_ξ are the same order of magnitude; i.e., RM data used for frequency $\omega > 2\Delta\phi/|\Delta t_{\mathrm{L,R}}|$ results in similar limits on g and ξ^{-1}. With $q = 1$, as a consequence of the frequency dependence $|r_{g,\xi}| \propto \omega$, high-frequency data result in more restrictive constraints. For the $q = 2$ case, $r_\xi \propto \sqrt{\omega}$ and $r_g \propto \omega$, so the potential limit on ξ^{-1} from GRB γ-ray RM data is 6–7 orders of magnitude better than that from DM data [22].

In summary, we present a unified general treatment of both LV DM and RM tests by analogy with EM wave propagation in a magnetized plasma. This treatment does not depend on the LV model, and allows simultaneous consideration of different LV mechanisms. We considered conventional ultraviolet LV, i.e. linear MP, quadratic MP, and GLP models. For these models, RM data provide better limits than DM data, (the improvement is ~ 100 for linear MP and GLP LV, ~ 10 for quadratic MP LV, if $\omega > 100$ kHz), and the improvement increases by using higher frequency EM wave data (for an arbitrary MP model $r_\xi \propto \omega^{1/q}$ and thus RM test efficacy decreases as q increases). Future γ- and X-ray RM data from distant objects, such as GRBs, quasars, or blazars hold great promise for testing and strongly constraining LV.

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