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Effects of primordial helicity on CMB

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Abstract

I present here a brief overview of the effects caused by parity violating cosmological sources (such as magnetic or kinetic helicity) on the CMB fluctuations. I discuss also primordial helicity induced relic gravitational waves. All these effects can serve as cosmological tests for primordial helicity detection.

Keywords: Linear cosmological perturbations; CMB fluctuations; Magnetic fields; Turbulence

Contents

1. Introduction

Current and future measurements of cosmic microwave background (CMB) temperature and polarization anisotropies (Page et al., 2006; Spergel et al., 2006) provides probes physical processes in the early universe and cosmological models. There are several astrophysical observations that indicate the presence of an helical magnetic field in clusters of galaxies (Widrow, 2002; Vallée, 2004; Semikoz and Sokoloff, 2005a). A promising possibility to explain such a magnetic field is to assume primordial helicity generated during an early epoch of the universe expansion (Cornwall, 1997; Giovannini and Shaposhnikov, 1998; Giovannini, 2000; Field and Carroll, 2000; Vachaspati, 2001; Sigl, 2002; Semikoz and Sokoloff, 2005b; Campanelli and Giannotti, 2005). Conventionally we can distinguish two different kinds of helicity, kinetic helicity related to primordial plasma motions and magnetic helicity related to a primordial magnetic field (Brandenburg, 2001; Christensson et al., 2005; Verma and Ayyer, 2003; Boldyrev and Cattaneo, 2004; Subramanian, 2004).

The average energy density and helicity of the magnetic have to be small enough to preserve spatial large-scale isotropy of the universe. Under such assumptions the linear

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perturbation theory of gravitational instability may be used to describe perturbation the dynamics (for a review see Giovannini, 2006a). Of course, the two kinds of helicity are related through magnetohydrodynamical evolution. On the other hand, primordial kinetic helicity influences the dynamics of perturbation (Vishniac and Cho, 2001; Brandenburg, 2001; Kleeorin et al., 2003; Subramanian, 2002; Vishniac et al., 2003; Subramanian and Brandenburg, 2004; Banerjee and Jedamzik, 2004; Subramanian et al., 2005), therefore it should be accounted for when the cosmological effects of a primordial helical magnetic field or/and of primordial helical turbulent motions are studied.

The energy-momentum tensor associated with a primordial helicity source (e.g., a magnetic field or turbulent motions) induces all modes of perturbations (scalar, vector, and tensor). Neglecting second order effects (Bartolo et al., 2004; Lesgourgues et al., 2005) and the coupling between scalar, vector, and tensor modes (which results in non-gaussianity of the CMB fluctuations, Brown and Crittenden, 2005; Chiang et al., 2006), the scalar, vector, and tensor modes can be studied separately.

Here I focus on the effects on CMB temperature and polarization anisotropies induced by primordial helicity. This presentation is based on results obtained in collaboration with (see Mack et al., 2002; Kosowsky et al., 2002; Caprini et al., 2004; Kosowsky et al., 2005; Kahniashvili and Ratra, 2005; Kahniashvili et al., 2005; Kahniashvili and Ratra, 2006). We find that primordial helical sources generate vector and tensor metric perturbations (primordial helicity does not influence the scalar mode of perturbations, Kahniashvili and Ratra, 2006, while the energy density of the corresponding source does) and as a result affect all CMB fluctuations. In particular:

(a) Parity violation in the universe results in an asymmetry in the amplitude of left- and right-handed gravitational waves (Lue et al., 1999). As a result primordial helicity generates circularly polarized stochastic gravitational waves (Caprini et al., 2004; Kahniashvili et al., 2005), which can be directly detected by future space based gravitational wave detection missions. Cosmological helicity also induces parity violating vorticity perturbations (Pogosian et al., 2002; Kahniashvili and Ratra, 2005).

(b) Cosmological helicity reduces the amplitudes of the parity-even CMB fluctuation power spectra compared to the case without primordial helicity (Caprini et al., 2004; Kahniashvili and Ratra, 2005; Kahniashvili et al., 2005).

(c) Faraday rotation of the CMB polarization plane is strongly dependent on the average energy density of the cosmological magnetic field and is independent of magnetic helicity (Kosowsky et al., 2005). The scalar mode of perturbations does not reflect the presence of primordial helicity (Kahniashvili and Ratra, 2006). These features of primordial helicity can be used as additional tests when using CMB data to constraint primordial helicity.

(d) Cosmological helicity induces parity-odd cross-correlations of the CMB fluctuations, which vanish for the case of a magnetic field or turbulent motions without helicity (Lue et al., 1999; Pogosian et al., 2002; Caprini et al., 2004; Kahniashvili and Ratra, 2005; Kahniashvili et al., 2005).[1]

2. Polarized gravitational waves background

The energy-momentum tensors corresponding to the magnetic field and turbulent motions have the anisotropic stress part which plays a source term role for gravitational waves. If the parity violation (helicity) is present – the induced gravitational waves have a parity-odd spectrum (Lue et al., 1999), i.e. the gravitational waves background is circularly polarized (Caprini et al., 2004; Kahniashvili et al., 2005). Polarized gravitational waves also can be induced from quantum fluctuations through Chern–Simons coupling (Lyth et al., 2005). The polarization degree of such a gravitational wave background strongly depends on the ratio between the helical and symmetric parts of the source two-point correlations function. We define the polarization degree as (Kahniashvili et al., 2005),

$$\begin{aligned}\mathscr{P}(k,t) &= \frac{\mathscr{H}(k,t)}{H(k,t)}\\ &= \frac{\langle h^{+\star}(\mathbf{k},t)h^{+}(\mathbf{k}',t) - h^{-\star}(\mathbf{k},t)h^{-}(\mathbf{k}',t)\rangle}{\langle h^{+\star}(\mathbf{k},t)h^{+}(\mathbf{k}',t) + h^{-\star}(\mathbf{k},t)h^{-}(\mathbf{k}',t)\rangle} \qquad (1)\end{aligned}$$

Here h^+ and h^- defines two states of the gravitational wave, (right- and left-handed circularly polarized gravitational waves), $h_{ij} = h^+e^+_{ij} + h^-e^-_{ij}$, where $e^{\pm}_{ij}$ is polarization basis. $H(k,t)$ and $\mathscr{H}(k,t)$ characterize the gravitational wave amplitude and polarization. An axisymmetric stochastic vector source (non-helical turbulent motion or any other non-helical vector field) induces unpolarized GWs with $|h^+(\mathbf{k},t)| = |h^-(\mathbf{k},t)|$ (Deriagin et al., 1987; Durrer et al., 2000; Kosowsky et al., 2002; Dolgov et al., 2002; Lewis, 2004; Caprini and Durrer, 2006).

For simplicity I present here the polarization degree of gravitational waves in the case of an helical turbulence model in which the turbulent motions $\mathbf{u}\,(\mathbf{x},t)$ are described by a time-dependent two-point correlation function, (Kosowsky et al., 2002; Kahniashvili et al., 2005).[2]

$$\begin{aligned}&\langle u^\star_i(\mathbf{k},t)u_j(\mathbf{k}',t')\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')\\ &[P_{ij}F_S(k,t-t') + i\epsilon_{ijl}\hat{k}_l F_H(k,t-t')], \qquad (2)\end{aligned}$$

[1] This is not true for an homogeneous magnetic field (Scoccola et al., 2004), or in the case of cosmological defects (Lepora, 1998).

[2] A similar analysis for the polarization degree of gravitational waves induced by a stochastic helical magnetic field is given in (Caprini et al., 2004).

where the time $t - t'$ dependence of the functions F_S and F_H reflects the assumption of time translation invariance.According our assumption energy is injected continuously, at $t = t' \in (t_{in}, t_{fi})$, $F_S(k,0) = P_S(k)$ and $F_H(k,0) = P_H(k)$.$P_S(k)$ and $P_H(k)$ are the symmetric (related to the kinetic energy density per unit enthalpy of the fluid) and helical (related to the average kinetic helicity $\langle\, u \cdot (\nabla \times \mathbf{u})\rangle$) parts of the velocity power spectrum (Pogosian et al., 2002; Kahniashvili and Ratra, 2005).We model the decay of helical turbulence by a monotonically decreasing functions $D_1(k)$ and $D_2(k)$, so $F_S(k,t) = P_S(k)D_1(t)$ and $F_H(k,t) = P_H(k)\ D_2(t)$.We model the power spectra by power laws, $P_S(k) \propto k^{n_S}$ and $P_H(k) \propto k^{n_H}$.For non-helical hydrodynamical turbulence the Kolmogorov spectrum has $n_S = -11/3$.The presence of hydrodynamical helicity makes the situation more complex.Two possibilities have been discussed.First, with a forward cascade (from large to small scales) of both energy and helicity (dominated by energy dissipation on small scales) one has spectral indices $n_S = -11/3$ and $n_H = -14/3$ (the helical Kolmogorov (HK) spectrum), p.243 of Lesieur, 1997.Second, if helicity transfer and small-scale helicity dissipation dominate, $n_S = n_H = -13/3$ (the helicity transfer (HT) spectrum), Moiseev and Chkhetiani, 1996.Based on our assumptions (for the details see Kahniashvili et al., 2005) we model the primordial spectra as $P_S(k) = S_0 k^{n_S}$ and $P_H(k) = A_0 k_S^{n_S - n_H} k^{n_H}$, where:(i) for the HK case $S_0 = \pi^2 C_k \bar{\varepsilon}^{2/3}$ and$A_0 = \pi^2 C_k \bar{\delta}/(\bar{\varepsilon}^{1/3} k_S)$ (Ditlevsen and Giuliani, 2001), implying $A_0/S_0 = \bar{\delta}/(\bar{\varepsilon} k_S)$; and, (ii) for the HT case $S_0 = \pi^2 C_s \bar{\delta}^{2/3}$ and $A_0 = \pi^2 C_a \bar{\delta}^{2/3}$ (Moiseev and Chkhetiani, 1996). Here $\bar{\varepsilon}$ and $\bar{\delta}$ are the energy and mean helicity dissipation rates per unit enthalpy, and C_k (the Kolmogorov constant), C_s, and C_a are constants of order unity. Fig. 1 and other numerical results show that for maximal helicity turbulence (when $A_0 = S_0$) with equal spectral indices $n_H = n_S < -3$, the polarization degree $\mathscr{P}(k) \simeq 1$ (upper solid line). For weaker helical turbulence (when $A_0 < S_0$) with $n_H \simeq n_S < -3$, $\mathscr{P}(k) \to CA_0/S_0$, where $1 < C(n_S, n_H) < 2$ is a numerical factor that depends on the spectral indices. For HT turbulence with $n_S = n_H = -13/3$, $C \approx 1.50$, while for Iroshnikov–Kraichnan MHD turbulence ($n_S = n_H = -7/2$), $C \approx 1.39$. It is unlikely that such kind of polarized gravitational waves will be detected in the near future, however, gravitational waves generated by helical turbulence will have an enough high degree of circular polarization and future detector configurations may well be able to. On the other hand, the polarized gravitational waves might leave an observable traces on CMB anisotropies, in particular parity-violating cross correlations between B-polarization and temperature and E-B polarization.

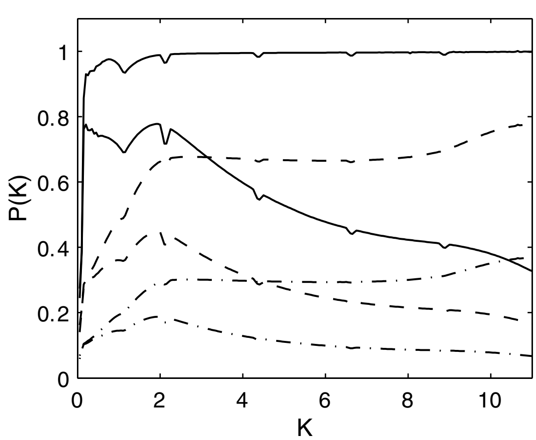

Fig. 1. GW polarization degree $\mathscr{P}(K, t_{fi})$, Eq. (1), as a function of scaled wave-number $K = k/k_S$ relative to the large-scale wave-number k_S on which energy is pumped into the turbulence. This is evaluated at time t_{fi}, after the turbulence has switched off, and remains unchanged to the present epoch. It has been computed for a damping wave-number $k_D = 10k_S$. Three pairs of curves are shown. Solid lines correspond to the amplitude ratio $A_0/S_0 = 1$, dashed lines to $A_0/S_0 = 0.5$, and dot-dashed lines are for $A_0/S_0 = 0.2$. The upper line in each pair corresponds to HT turbulence with $n_S = n_H = -13/3$ and the lower line to HK turbulence with $n_S = -11/3$ and $n_H = -14/3$. Even for helical turbulence with $A_0/S_0 \leqslant 0.5$, for large wave-numbers $k \sim k_D$, $n_S = n_H = -13/3$ is unlikely so the large K part of the lower dashed and dot-dashed HT curves are unrealistic. The large $k \sim k_D$ decay of the HK curves is a consequence of vanishing helicity transfer at large k.

3. CMB fluctuations

Lets consider another parity violating source which might be present in the early universe – a stochastic helical cosmological magnetic field.

Neglecting fluid back-reaction onto the magnetic field, the spatial and temporal dependence of the field separates, $\mathbf{B}(t,\mathbf{x}) = \mathbf{B}\ (\mathbf{x})/a^2$; here a is the cosmological scale factor. Assuming that the primordial plasma is a perfect conductor we model magnetic field damping by an ultraviolet cut-off wavenumber $k_D = 2\pi/\lambda_D$ (Subramanian and Barrow, 1998). Gaussianly distributed an helical magnetic field two-point correlation function is

$$\langle B_i^{\star}(\mathbf{k}) B_j(\mathbf{k}')\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')[P_{ij}(\hat{\mathbf{k}}) P_B(k) + i\epsilon_{ijl} \hat{k}_l P_H(k)] \quad (3)$$

where $P_B(k)$ and $P_H(k)$ are the symmetric and helical parts of the magnetic field power spectrum, assumed to be simple power laws on large scales:

$$P_B(k) \equiv P_{B_0} k^{n_B} = \frac{2\pi^2 \lambda^3 B_\lambda^2}{\Gamma(n_B/2 + 3/2)} (\lambda k)^{n_B},$$
$$P_H(k) \equiv P_{H_0} k^{n_H} = \frac{2\pi^2 \lambda^3 H_\lambda^2}{\Gamma(n_H/2 + 2)} (\lambda k)^{n_H}, \quad (4)$$

and vanishing on small scales when $k > k_D$. B_λ^2 is the squared smoothed magnetic field amplitude at the λ scale.

The symmetric part of the magnetic field spectrum can be reconstructed from measurements of Faraday rotation of the CMB polarization plane (Kosowsky et al., 2005). This is because magnetic helicity does not contribute to the Faraday rotation effect (Ensslin and Vogt, 2003; Campanelli et al., 2004; Kosowsky et al., 2005). Faraday rotation by an helical magnetic field induces a B-polarization signal that peaks at very high multipole number, $l \sim 15000$ (see Fig. 2). The position of this peak makes it

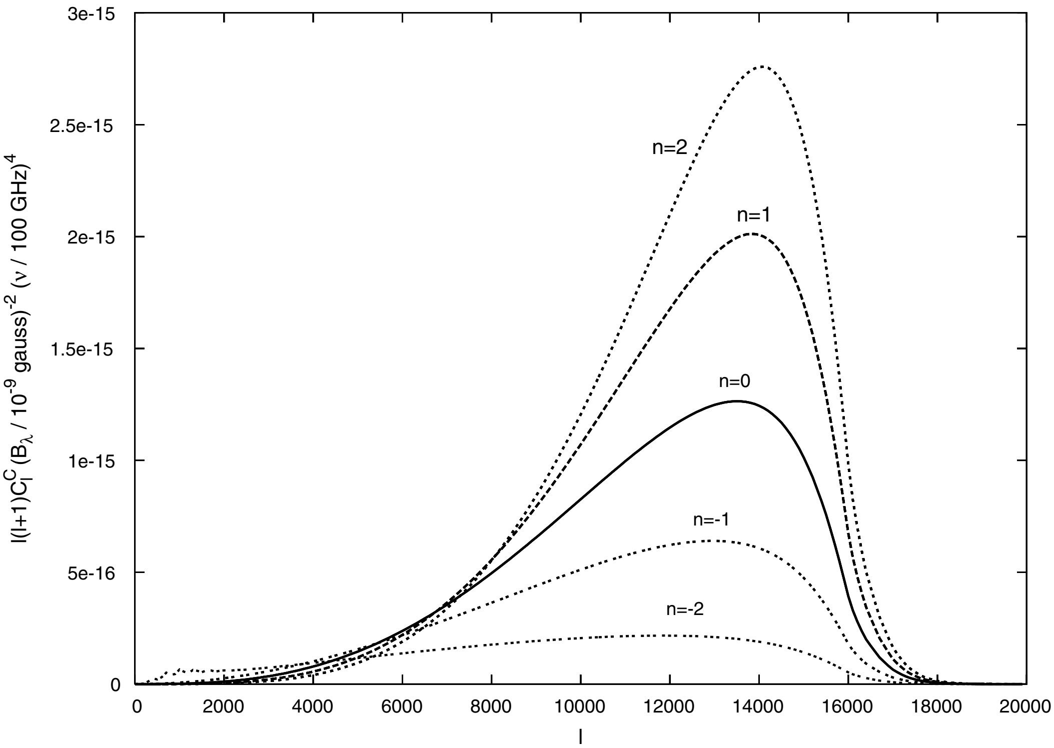

Fig. 2. The B-polarization power spectrum of the microwave background induced by the Faraday rotation field. The curves in order of decreasing amplitude on the right side of the plot correspond to magnetic field power spectral indices $n_{\mathrm{B}} = 2$, 1, 0, −1, and −2. The magnetic fields have been normalized to a nanogauss at the smoothing scale $\lambda = 1$ Mpc.

possible to distinguish the Faraday-rotation induced B-polarization signal from the signal arising from the presence of the vector and tensor modes which peak around $l \sim 2000$ (Lewis, 2004; Challinor and Lewis, 2005).

The helical part of the magnetic field spectrum induces parity-odd cross correlations between temperature and B-polarization anisotropies, and between E- and B-polarization anisotropies. Below I discuss explicity these parity-odd cross correlations.

For our computations we use the formalism by Mack et al. (2002), extending it to account for magnetic field helicity. To compute CMB temperature and polarization anisotropy power spectra we use the total angular momentum method by Hu and White (1997). Our results (Caprini et al., 2004; Kahniashvili and Ratra, 2006) are obtained using analytic approximations and for the vector mode are valid for $l < 500$. We emphasis that for the tensor mode the usage of our approximations are limited by $l < 60$ because of the fast decay of the gravitational wave source in the matter-dominated epoch. Our results might be presented in terms of a ratio between CMB fluctuation contributions from the symmetric and helical parts of the magnetic field power spectrum.

To obtain the magnetic field source terms in the equations for vector (transverse peculiar velocity) and tensor (gravitational waves) metric perturbations we need to extract the transverse vector and tensor parts of the magnetic field stress-energy tensor $\tau_{ij}(k)$. This is done through $\Pi_{ij}^{(V)}(\mathbf{k}) = (P_{i\mathrm{b}}(\hat{\mathbf{k}})\hat{k}_j + P_{j\mathrm{b}}(\hat{\mathbf{k}})\hat{k}_i)\hat{k}_{\mathrm{a}}\tau_{\mathrm{ab}}(\mathbf{k})$ (for vector perturbations) and $\Pi_{ij}^{(T)}(\mathbf{k}) = [P_{i\mathrm{a}}(\hat{\mathbf{k}})P_{j\mathrm{b}}(\hat{\mathbf{k}}) - \frac{1}{2}P_{ij}(\hat{\mathbf{k}})P_{\mathrm{ab}}(\hat{\mathbf{k}})]\tau_{\mathrm{ab}}(\mathbf{k})$ (for tensor perturbations); for details, see Mack et al. (2002). In both cases (vector and tensor perturbations) the contribution of magnetic field helicity to the symmetric part of the magnetic source is negative.

As we have already noted, a possible way to detect magnetic helicity directly from CMB fluctuation data is to observe the parity-odd CMB fluctuation cross-correlations. As an additional cross-check it may be possible to detect the effects that magnetic helicity has on parity-even CMB fluctuations (Caprini et al., 2004; Kahniashvili and Ratra, 2005). Since we find (Kahniashvili and Ratra, 2006) that magnetic field induced density perturbations are independent on magnetic helicity, if one can extract the scalar mode contribution from the total magnetic field sourced CMB fluctuations, that will allow for a determination of the symmetric part of the magnetic field spectrum (see also Giovannini, 2006a,b), and result in a more accurate estimate of magnetic helicity from parity-odd CMB anisotropies.

At large angular scales ($l < 100$) where the contribution from the tensor mode is significant, for $n_{\mathrm{B}} + n_{\mathrm{H}} > -2$ the vector mode $C_l^{\Theta\mathrm{B(V)}}$ and the tensor mode $C_l^{\Theta\mathrm{B(T)}}$ have the same l dependence $\propto > l^2$. For all other values of spectral indexes n_{B} and n_{H}, the growth rate (with l) of $C_l^{\Theta\mathrm{B(V)}}$ is faster than $C_l^{\Theta\mathrm{B(T)}}$. The ratio between temperature-B-polarization signals from vector and tensor modes is independent of the amplitudes of the average magnetic field (B_λ) and average magnetic helicity (H_λ).

For small angular scales ($l > 100$) where the tensor mode signal vanishes, for a maximally helical magnetic field with $n_{\rm H} \simeq n_{\rm B}$, due to the suppression factor $L_{\gamma,{\rm dec}}/\eta_0$ (where $L_{\gamma,{\rm dec}}$ is the photon mean free path at decoupling and η_0 conformal time today) the temperature-E-polarization cross-correlation power spectrum, $C_l^{\Theta {\rm E}}$, is smaller than the temperature-B-polarization cross-correlation power spectrum, $C_l^{\Theta {\rm B}}$, but both are $\propto < l^2$, if $n_{\rm B} + n_{\rm H} > -5$. The same suppression factor makes $C_l^{\Theta {\rm B}}$ smaller than $C_l^{\Theta\Theta}$. For an arbitrary helical field $C_l^{\Theta {\rm B}}/C_l^{\Theta {\rm E}}$ depends on the ratio $(P_{{\rm H}_0}/P_{{\rm B}_0})k_D^{n_{\rm H}-n_{\rm B}}$ and order unity pre-factors that depend on $n_{\rm B}$ and $n_{\rm H}$. A dependence on l appears only if $n_{\rm B} + n_H < -5$, when the ratio, $C_l^{\Theta {\rm B}}/C_l^{\Theta {\rm E}}$ decreases as $\propto l^{n_{\rm B}+n_{\rm H}+5}$ (Kahniashvili and Ratra, 2005).

For a tensor mode signal at large angular scales ($l < 100$), E- and B-polarization cross-correlation $C_l^{\rm EB}$ is of the same order of magnitude as the tensor mode temperature-B-polarization anisotropy cross-correlation spectrum, $C_l^{\Theta {\rm B}}$ (Caprini et al., 2004). The situation is different for a vector mode which survives up to small angular scales (e.g., Subramanian and Barrow, 1998; Mack et al., 2002; Lewis, 2004; Giovannini, 2006a). In this case, the E- and B-polarization anisotropy cross-correlation power spectrum has a suppression factor of $kL_{\gamma,{\rm dec}}$ implying that $C_l^{\rm EB} \ll C_l^{\Theta {\rm B}}$. This is consistent with the result of Hu and White (1997).

4. Conclusion

I have discussed the cosmological effects of primordial helicity. In particular, I examined CMB parity-violating fluctuations that arise from helical sources. These CMB fluctuations should be detectable (if the current magnetic field amplitude is at least 10^{-10} or 10^{-9} G on Mpc scales – such a magnetic field can be generated during inflation from quantum fluctuations, see Ratra, 1992; Bamba and Yokoyama, 2004) by near future CMB polarization measurements (from WMAP, PLANK, CMBPol and others). As a specific imprint of primordial kinetic helicity I discussed polarization of relic gravitational waves, possibly detectable by future space missions (Smith et al., 2006; Chongchitnan and Efstathiou, 2006).

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