Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
New Astronomy Reviews 49 (2005) 79–82
New Astronomy Reviews
www.elsevier.com/locate/newastrev

Cosmological magnetic fields vs. CMB

Tina Kahniashvili

Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66506, USA
Center for Plasma Astrophysics, Abastumani Astrophysical Observatory, A. Kazbegi ave 2a, Tbilisi 380060, Georgia

Available online 10 February 2005

Abstract

I present a short review of the effects of a cosmological magnetic field on the CMB temperature and polarization anisotropies. Various possibilities for constraining the magnetic field amplitude are discussed.

Cosmological primordial seed magnetic fields were proposed to explain the existence of observed $\sim 10^{-6}$ G magnetic fields in galaxies and cluster (see, e.g., Widrow, 2002; Giovannini, 2003, and references therein). To preserve approximate spatial isotropy a seed vector magnetic field has to be small and hence can be treated as a first order term in perturbations theory . If the energy density parameter of a primordial magnetic field, $\Omega_B = B^2/(4\pi\rho_{cr})$ (where ρ_{cr} is the critical density), is five or six of order of magnitude less than that of the radiation (photons), $\Omega_B \sim 10^{-6}$–$10^{-5}\ \Omega_\gamma$, this is still of the order of the current accuracy of CMB measurements (Bennett et al., 2003), so we might expect that such field strength (10^{-8}–10^{-9} G) could leave detectable traces on CMB temperature or polarization anisotropies.

Primordial magnetic fields could be generated during early epochs of the Universe, such as during inflation, or the electroweak phase transition, or might be generated by primordial turbulence (for reviews, see Grasso and Rubinstein, 2001; Widrow, 2002; Giovannini, 2003). Cosmological magnetic fields induce scalar (density), vector (vorticity) and tensor (gravitational waves) fluctuations, and through them influence the CMB temperature and polarization anisotropies (see Mack et al., 2002 and references therein). Hence, precise CMB measurements (Bennett et al., 2003) can be used to constrain primordial magnetic fields. An interesting possibility is to consider the rotation of the CMB polarization plane due to the Faraday effect (Kosowsky and Loeb, 1996).

The simplest illustrative case to consider is a homogeneous magnetic field (Giovannini and Shaposhnikov, 1998), which generates magnetosonic and Alfvén waves. Due to the rescaling of sound velocity in a cosmological model with a homogeneous magnetic field: $c_S^2 \to c_S^2 + v_A^2$ (where $v_A = B/\sqrt{4\pi(\rho + p)}$ is the Alfvén speed), the

E-mail address: tinatin@phys.ksu.edu.

1387-6473/$ - see front matter
doi:10.1016/j.newar.2005.01.006

influence of fast magnetosonic wave propagation on CMB anisotropies consists of shifts in the acoustic peak positions (Adams et al., 1996).

In stardard cosmology vector perturbations decay with time and so do not affect the CMB. The presence of a homogeneous magnetic field alters this situation: such a field supports Alfvén (vorticity) waves, and also breaks spatial statistical isotropy. A homogeneous magnetic field hence induces non-zero off-diagonal correlations between temperature multipole coefficients. In particular, non-zero correlations between l and $l \pm 2$ multipole coefficients are given by (Durrer et al., 1998)

$$D_l(m) = \langle a^*_{l-1,m} a_{l+1,m} \rangle = \langle a^*_{l+1,m} a_{l-1,m} \rangle. \quad (1)$$

Here, the power spectrum $D_l(m)$ depends on the primordial vorticity perturbation spectrum (which we assume to be given by a simple power law $P_\Omega(k) \propto k^n / k_D^{n+3}$), and the Alfvén speed v_A. The presence of a non-zero $D_l(m)$ has a simple physical explanation: the temperature anisotropy correlation between two points on the sky depends not only on the angular separation between the two points but also on their orientation with respect to the magnetic field.

An observational test to detect (or constrain) the presence of a homogeneous cosmological magnetic field is based on computing the D_l spectrum of CMB anisotropy data. Chen et al. (2004) use the WMAP data to constrain the magnetic field amplitude (at illustrative value of vorticity spectral index $n = -7$, and $n = -5$) to be less than about 10^{-8}–10^{-9} G at three SD.

A more realistic case [1] to consider is a stochastic magnetic field with a (Gaussian random) two-point correlation spectrum (Pogosian et al., 2002):

$$\langle B^*_i(\mathbf{k}) B_j(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') [P_{ij} P_B(k) + \mathrm{i}\epsilon_{ijl} \hat{k}_l P_H(k)], \quad (2)$$

where $P_B(k)(\propto \langle |\mathbf{B}|^2 \rangle)$ and $P_H(k)(\propto \langle \mathbf{B} \cdot (\nabla \times \mathbf{B}) \rangle)$ are the symmetric and helical magnetic field power spectra, respectively (we assume that both are given by simple power laws), the plane projector $P_{ij} \equiv \delta_{ij} - \hat{k}_i \hat{k}_j$, ϵ_{ijl} is the totally antisymmetric tensor, and $\hat{k}_i = k_i / k$. The possibility of generating helical magnetic fields is discussed in Vachaspati (2001) and Sigl (2002). The symmetric part of the magnetic field in Eq. (2) contributes to the CMB temperature and polarization anisotropies via induced vector and tensor perturbations (for details, see Mack et al., 2002). [2] $P_B(k)$ induces parity-even CMB fluctuations, with the following maximum rate of growth with respect of l

$$C_l^{\theta\theta(V)} \propto l^2, \quad C_l^{EE(V)} \propto l^2, \quad C_l^{BB(V)} \propto l^2$$
$$C_l^{\theta E(V)} \propto l^2. \quad (3)$$

For vector perturbations, the BB-power spectrum is slightly larger than the EE one, whereas the EE and θE power spectra are approximately equal. [3] While $n_S \to -3$ corresponds to the symmetric part magnetic field power spectrum being scale-invariant, the CMB vector power spectra are not flat for this value.

For tensor perturbations, the parity-even CMB power spectra generated from the symmetric magnetic field power spectrum are (Durrer et al., 2000; Mack et al., 2002):

$$C_l^{\theta\theta(T)} \propto l, \quad C_l^{EE(T)} \propto l, \quad C_l^{BB,(T)} \propto l, \quad C_l^{\theta E(T)} \propto l \quad (4)$$

For magnetic field induced gravitational wave contribution to the CMB anisotropies, the E polarization power spectrum is slightly larger than the B one. For $n_S > -3/2$ the polarization power

[1] For cosmological magnetic field generation mechanisms see, e.g. Turner and Widrow (1988), Carroll and Field (1991), Vachaspati (1991), Ratra (1992), Dolgov and Silk (1993), and Enqvist and Olesen (1994).

[2] CMB temperature and polarization anisotropy vector mode contributions for a magnetic field spectrum peaked at a fixed value of k are given in Subramanian and Barrow (1998), Seshadri and Subramanian (2001), and Subramanian et al. (2003). CMB temperature anisotropy induced by gravitational waves generated by a magnetic field are discussed in Durrer et al. (2000).

[3] Temperature anisotropies are dominated by the vector dipole term, which correlates poorly with the radial function corresponding to E polarization (Hu and White, 1997), so the θE power spectrum is dominated by a subdominant temperature contribution arising from the vector quadropole term, which then coincidentally renders the spectrum in a form approximately identical to the E polarization power spectrum itself.

spectra are comparable to the temperature power spectra. This is due to the fact that both the temperature and polarization fluctuations are dominated by the intrinsic temperature quadropole moments, which arise from the gravitational wave solution $\dot{h}$ instead of being induced via free streaming dipoles as in the case of a vector perturbations. Also, for $n_S > -3/2$ the magnetic source term for the tensor mode is approximately independent of k and the resulting power spectra have the well known behaviour for a white noise source: $C_l l^2 \propto l^3$. As expected, for a scale-invariant magnetic field with $n_S \to -3$, the tensor part of CMB power spectra is flat. Note that our analytical approximations are valid for $l < 500$ for the vector mode and for $l < 100$ for the tensor mode (due to the damping of gravitational waves when they enter horizon at decoupling), see Caprini et al. (2004) for detailed discussion. Comparison with the WMAP CMB data (Bennett et al., 2003) constrains the magnetic field amplitude to be less than about 10^{-9} G. [4]

A magnetic field with non-zero helicity ($P_H(k)$) will induce additional effects (Pogosian et al., 2002). In particular, the presence of a helical part results in non-zero parity-odd CMB power spectra, such as C_l^{EB} and $C_l^{\theta B}$. Also, a helical magnetic field will generate gravitational waves with parity odd spectra (Caprini et al., 2004). Using the linear polarization basis, $e^T_{ij} = (\mathbf{e_1} \times \mathbf{e_1} - \mathbf{e_2} \times \mathbf{e_2})_{ij}$, $e^\times_{ij} = (\mathbf{e_1} \times \mathbf{e_2} + \mathbf{e_2} \times \mathbf{e_1})_{ij}$, the helical part of the magnetic tensor source $g(k)$ can be directly connected with gravitational waves h^T and $h^\times$ ($h_{ij} = e^T_{ij} h^T + e^\times_{ij} h^\times$) correlations:

$$\langle h^{*T}(\mathbf{k}) h^\times(\mathbf{k}') - h^{*T}(\mathbf{k}) h^\times(\mathbf{k}')\rangle \propto \mathrm{i}\delta(\mathbf{k} - \mathbf{k}') g(k). \tag{5}$$

A magnetic field with helicity will also induce non-decaying vorticity waves (Pogosian et al., 2002). Both modes (vector and tensor) generate CMB temperature and polarization anisotropies. The helical part contributions to parity-even total CMB power spectra are negative, but due to the causality restriction $P_B(k) > |P_S(k)|$ and $n_S \leqslant n_A$, the total C_l's are positive. The ratio between the helical and symmetric part contributions to the parity-even CMB power spectra $C_{l,H}/C_{l,S}$ depends on the corresponding indices n_H and n_S, as well as on $P_H(k)$ and $P_B(k)$. The parity-odd power spectra are generated by $P_H(k)$, but are dependent on both the spectral amplitude and index. (for the tensor mode see: Caprini et al. (2004), and for the vector mode paper is in preparation).

As mentioned above, the presence of a cosmological magnetic field results also in the rotation of the CMB polarization plane via Faraday effect (Kosowsky and Loeb, 1996). Assuming that the rotational effect on polarization generated by the magnetic field itself is a second order effect, and also that only scalar perturbations are present, Faraday rotation will generates B-polarization. In current project (Kosowsky et al., 2004) we study the Faraday rotation effect (and resulting B-polarization signal) due to a stochastic magnetic field. We show that an average rotational measure is independent of a helical part of magnetic field. Hence, precise rotation measure can constrain the symmetric part of magnetic field spectrum. The resulting B-polarization depends on the initial polarization spectrum ($C^{EE}_{l,\mathrm{in}}$) and on the rotation angle power spectrum ($C_l^{\alpha\alpha}$) as (Kosowsky et al., 2004):

$$\langle a^{B'*}_{l'm'} a^{B'}_{lm}\rangle = \delta_{ll'}\delta_{mm'} N_l^2 \sum_{l_1 l_2} N_{l_2}^2 K(l, l_1, l_2)^2 C^{EE}_{l_2,\mathrm{in}} C^{\alpha\alpha}_{l_1} \times \frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)} \left(C^{l0}_{l_1 0 l_2 0}\right)^2, \tag{6}$$

where

$$K(l, l_1, l_2) \equiv -\frac{1}{2}(L^2 + L_1^2 + L_2^2 - 2L_1L_2 - 2L_1L + 2L_1 - 2L_2 - 2L) \tag{7}$$

with $L = l(l+1)$, $N_l \equiv (2(l-2)!/(l+2)!)^{1/2}$, and $C^{l0}_{l_1 0 l_2 0}$ are Clebsh–Gordon coefficients.

Assuming precise measurements of CMB temperature and polarization anisotropies, Faraday rotation allow us to reconstruct symmetric magnetic field spectrum, as since the total CMB power spectra depend on both symmetric and helical parts of the magnetic field spectrum, we can also

[4] Constraints of a similar magnitude result from considering resonant photon-graviton conversion (Chen (1995)), and from the distortion of the CMB (Jedamzik et al., 2000).

constrain magnetic helicity. Also, there is the theoretical possibility of reconstructing magnetic helicity from the magnetic-field-generated gravitational wave spectrum.

Our conclusions are as follows:

The homogeneous magnetic field, via generated Alfvén waves, induces non-zero off-diagonal correlations of multipoles coefficients. The magnetic field can thus be constrained by testing CMB data for non-gaussianity.

A helical magnetic field generates gravitational waves with parity odd spectra. This could serve, in principle, as a method of detection of the helicity of the magnetic field.

The Faraday rotation measurement cannot constrain magnetic helicity. Thus, only the symmetric part of magnetic field spectrum could be reconstructed from rotation measurements.

Current CMB data constrain the amplitude of a cosmological magnetic field to be less than order 10^{-9} G.

Acknowledgements

The author thanks the Dark Matter 2004 conference organising committee for hospitality. The author acknowledges her collaborators C. Caprini, G. Chen, R. Durrer, P. Ferreira, A. Kosowsky, G. Lavrelashvili, A. Mack, P. Mukherjee, B. Ratra, Y. Wang, A. Yates, and thanks P. Chen and T. Vachaspati for discussions. This work is supported by NSF CAREER Grant AST-9875031, DOE EPSCoR Grant DE-FG02-00ER45824, and CRDF-GRDF Grant 3316.

References

Adams, J., Danielsson, U.H., Grasso, D., Rubinstein, H., 1996. Phys. Lett. B 388, 253.
Bennett, C.L., et al., 2003. Ap. J. S. 148, 1.
Caprini, C., Durrer, R., Kahniashvili, T., 2004. Phys. Rev. D 69, 063006.
Carroll, S.M., Field, G.B., 1991. Phys. Rev. D 43, 3789.
Chen, G., Mukherjee, P., Kahniashvili, T., Ratra, B., Wang, Y., 2004. astro-ph/0403695, Ap. J. (in press).
Chen, P., 1995. Phys. Rev. Lett. 74, 634.
Dolgov, A.D., Silk, J., 1993. Phys. Rev. D 47, 3144.
Durrer, R., Kahniashvili, T., Yates, A., 1998. Phys. Rev. D 58, 123004.
Durrer, R., Ferreira, P., Kahniashvili, T., 2000. Phys. Rev. D. 61, 043001.
Enqvist, K., Olesen, P., 1994. Phys. Lett. B 329, 195.
Giovannini, M., 2003. astro-ph/0312614.
Giovannini, M., Shaposhnikov, M., 1998. Phys. Rev. Lett. 80, 22.
Grasso, D., Rubinstein, H.R., 2001. Phys. Rep. 348, 163.
Hu, W., White, M., 1997. Phys. Rev. D 56, 596.
Jedamzik, J., Katalinić, V., Olinto, A.V., 2000. Phys. Rev. Lett. 85, 700.
Kosowsky, A., Loeb, A., 1996. Ap. J. 469, 1.
Kosowsky, A., Kahniashvili, T., Lavrelashvili, G., Ratra, B., 2004. (in preparation).
Mack, A., Kahniashvili, T., Kosowsky, A., 2002. Phys. Rev. D 65, 123004.
Pogosian, L., Vachaspati, T., Winitzki, S., 2002. Phys. Rev. D 65, 083502.
Ratra, B., 1992. Ap. J. 391, L1.
Seshadri, T.R., Subramanian, K., 2001. Phys. Rev. D 87, 101301.
Sigl, G., 2002. Phys. Rev. D 66, 123002.
Subramanian, K., Barrow, J.D., 1998. Phys. Rev. Lett. 81, 3575.
Subramanian, K., Seshadri, T.R., Barrow, J.D., 2003. MNRAS 344, L31.
Turner, M.S., Widrow, L.M., 1988. Phys. Rev. D 37, 2743.
Vachaspati, T., 1991. Phys. Lett. B 265, 258.
Vachaspati, T., 2001. Phys. Rev. Lett. 87, 251302.
Widrow, L.A., 2002. Rev. Mod. Phys. 74, 775.