

LOOKING FOR COSMOLOGICAL ALFVÉN WAVES IN *WILKINSON MICROWAVE ANISOTROPY PROBE* DATA

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ABSTRACT

A primordial cosmological magnetic field induces and supports vorticity or Alfvén waves, which in turn generate cosmic microwave background (CMB) anisotropies. A homogeneous primordial magnetic field with fixed direction induces correlations between the $a_{l-1,m}$ and $a_{l+1,m}$ multipole coefficients of the CMB temperature anisotropy field. We discuss the constraints that can be placed on the strength of such a primordial magnetic field using CMB anisotropy data from the *Wilkinson Microwave Anisotropy Probe* experiment. We place 3σ upper limits on the strength of the magnetic field of $B < 15$ nG for vector perturbation spectral index $n = -5$ and $B < 1.7$ nG for $n = -7$.

Subject headings: cosmic microwave background — cosmology: observations — methods: statistical — MHD — waves

1. INTRODUCTION

The origin of the large-scale part of observed galactic magnetic fields, of order microgauss strength and apparently coherent over ~ 10 kpc scales, is unknown. They could be the consequence of nonlinear amplification of a tiny seed field by galactic dynamo processes. An alternate possibility is amplification of a weak seed field through anisotropic protogalactic collapse and subsequent further amplification via galactic differential rotation. In both cases, a primordial seed field of strength exceeding 10^{-13} to 10^{-12} G, coherent over megaparsec scales, is apparently needed, and it is often suggested that up to a nanogauss strength seed field might be required. See Kulsrud (1999), Widrow (2002), and Giovannini (2004) for reviews of the state of the art in this area. A primordial magnetic field of order nanogauss present strength can leave observable signatures in the cosmic microwave background (CMB) anisotropy.

In standard cosmologies vorticity perturbations decay and so do not contribute to CMB temperature or polarization anisotropies. The presence of a cosmological magnetic field generated during an earlier epoch⁴ changes this situation: a primordial magnetic field induces and supports vorticity or Alfvén waves (Adams et al. 1996; Durrer et al. 1998, hereafter DKY98). These vector perturbations generate CMB anisotropies.⁵ The presence of a preferred direction due to a homogeneous magnetic field background leads to an m -dependence of $\langle a_{lm}a_{lm}^* \rangle$ and induces correlations between the $a_{l+1,m}$ and $a_{l-1,m}$ multipole coefficients of the CMB temperature anisotropy field.

Since the CMB anisotropies are observed to be random Gaussian,⁶ it is known that such a contribution can only be subdominant.

We use the observed $\overline{\langle a_{l-1,m}a_{l+1,m}^* \rangle}$ correlations measured by the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) to place constraints on the strength of a homogeneous primordial magnetic field. The angular brackets here denote an ensemble average, and the overbar indicates an average over m for each l . Limited by cosmic variance uncertainties, this would be a useful measure for characterizing the signature of a homogeneous primordial magnetic field.

The model on which we base our analysis is introduced in § 2. Our analysis of the *WMAP* data and our results are discussed in § 3. We conclude in § 4.

2. CMB TEMPERATURE ANISOTROPIES GENERATED BY ALFVÉN WAVES

We assume that the homogeneous magnetic field \mathbf{B} is generated prior to the time of recombination. Such a field could be generated during the electroweak phase transition (see, e.g., Vachaspati 1991; Sigl et al. 1997; Giovannini & Shaposhnikov 1998) or by an α -effect dynamo driven by collective neutrino-plasma interactions (Semikoz & Sokoloff 2004). The energy density of this field, $B^2/(4\pi)$, must be small, to prevent a violation of the cosmological principle, and so can be treated as a first-order perturbation. Accounting for the

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⁴ Quantum fluctuations during an early epoch of inflation can generate a primordial nanogauss magnetic field, coherent over very large scales (see, e.g., Ratra 1992; Bamba & Yokoyama 2004).

⁵ In the future one can hope to constrain vector modes through their effect on CMB temperature and polarization anisotropy spectra. CMB polarization spectra that result from vector perturbations due to a primordial magnetic field have been discussed by Seshadri & Subramanian (2001), Pogosian et al. (2002), Mack et al. (2002), and Subramanian et al. (2003), while Lewis (2004) considers the case of vector modes supported by free-streaming neutrinos.

⁶ Colley et al. (1996), Mukherjee et al. (2000), and Park et al. (2001) give some early discussions of the Gaussianity of the CMB anisotropy. More recent discussions of the Gaussianity of the *WMAP* CMB anisotropy data are in Komatsu et al. (2003), Colley & Gott (2003), Chiang et al. (2003), Park (2004), Eriksen et al. (2004a, 2004b), Coles et al. (2004), Vielva et al. (2004), Copi et al. (2003), Hansen et al. (2004), Gurzadyan et al. (2004), and Mukherjee & Wang (2004). The simplest inflation models predict Gaussian fluctuations (see, e.g., Fischler et al. 1985; Ratra 1985), and this is consistent with most observational indications (see, e.g., Peebles & Ratra 2003). While there are indications of mild peculiarities in some subsets of the *WMAP* data, for example, the apparent paucity of large-scale power (Spergel et al. 2003; see Górski et al. 1998b for a similar indication from *COBE* data) and the differences between data from different parts of the sky (see papers cited above), foreground contamination (see, e.g., Park et al. 2002; Mukherjee et al. 2002, 2003; de Oliveira-Costa et al. 2004; Bennett et al. 2003b; Tegmark et al. 2003) and other systematics might be responsible for part of this.

high conductivity of the primordial nonrelativistic plasma (with $v \ll 1$, where v is the velocity field of the plasma), we can use the infinite-conductivity, frozen-in condition $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$. We also assume that charged particles are tightly coupled to the radiation. We write $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$, where \mathbf{B}_1 denotes the first-order vector perturbation in the magnetic field (where $\nabla \cdot \mathbf{B}_1 = 0$), and $\mathbf{v} = 0 + \mathbf{\Omega}$, with $\mathbf{\Omega}$ being the first-order vector perturbation in the fluid velocity (where $\nabla \cdot \mathbf{\Omega} = 0$).

As a consequence of magnetic flux conservation, the field lines in an expanding universe are conformally diluted, $B_0 \propto 1/a^2$, and the Alfvén velocity in the photon-baryon plasma during the photon-dominated epoch, when the energy density $\rho_R = \rho_\gamma + \rho_b \simeq \rho_\gamma$ until recombination, is

$$v_A = B_0 / \sqrt{4\pi(\rho_R + p_R)} = 4 \times 10^{-4} (B_0 / 10^{-9} \text{ G})$$

and is time independent. Rescaling physical quantities according to the expansion of the universe, the MHD equations result in an equation describing Alfvén wave propagation with velocity $v_A(\mathbf{b} \cdot \hat{\mathbf{k}}) \equiv v_A \mu$:

$$\ddot{\mathbf{\Omega}} = v_A^2 (\mathbf{b} \cdot \mathbf{k})^2 \mathbf{\Omega} \quad (1)$$

(DKY98). Here $\mathbf{b} \equiv \mathbf{B}_0 / B_0$ is the unit vector in the direction of the magnetic field and an overdot denotes a derivative with respect to conformal time η . Choosing only the sine mode, to satisfy the initial condition $\mathbf{\Omega}(\mathbf{k}, \eta = 0) = 0$, we have

$$\mathbf{\Omega}(\mathbf{k}, \eta) \simeq \Omega_0 v_A k \mu \eta, \quad |\Omega_0| = \frac{v_A}{B_0} |\mathbf{B}_1|. \quad (2)$$

In equation (1) we have neglected viscosity, so the vorticity solution in equation (2) is applicable only on scales bigger than the damping scale. Assuming that the initial vector perturbation is generated by a random process, the two-point correlation function of the vorticity field can be written as (Pogosian et al. 2002)

$$\langle \Omega_{0i}^*(\mathbf{k}) \Omega_{0j}(\mathbf{k}') \rangle = [(\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + i \epsilon_{ijl} \hat{k}_l A(k)] \delta(\mathbf{k} - \mathbf{k}'). \quad (3)$$

Here ϵ_{ijl} is the totally antisymmetric tensor, and the $S(k)$ and $A(k)$ power spectra describe the symmetric and helical parts of the two-point correlation function. We assume that the spectra $S(k) [=|\Omega_0(k)|^2]$ and $A(k)$ are given by simple power laws of the scale $1/k$ on scales larger than the perturbation damping scale $1/k_D$, i.e., for $k < k_D$ we take $S(k) = S_0 k^n / k_D^{n+3}$ and $A(k) = A_0 k^m / k_D^{m+4}$. Here S_0 and A_0 are dimensionless normalization constants with $S_0 \geq A_0$, and n and m are spectral indices. The cutoff scale $1/k_D$ is the scale below which the magnetic field is damped away, because of the finite value of the conductivity. Using $T_{\text{dec}} \sim 0.3$ eV and $t_{\text{dec}} \sim 10^{23}$ cm, the comoving magnetic field damping wavenumber at decoupling is $k_D(t = t_{\text{dec}}) \sim 3 \times 10^{-10} \text{ cm}^{-1}$ (DKY98).

The CMB fractional temperature anisotropy, in the direction \mathbf{n} on the sky, induced by a vorticity perturbation (ignoring a possible dipole contribution from the vector perturbation) is (DKY98)

$$\frac{\Delta T}{T}(\mathbf{n}, \mathbf{k}) \simeq \mathbf{n} \cdot \mathbf{\Omega}(\mathbf{k}, \eta) = \mathbf{n} \cdot \mathbf{\Omega}_0 v_A \mu (k \eta_{\text{dec}}). \quad (4)$$

Decomposing the CMB fractional temperature anisotropy into a spherical harmonic expansion,

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\mathbf{n}), \quad (5)$$

and using the definition of the power spectrum C_l ,

$$\left\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \right\rangle = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\mathbf{n} \cdot \mathbf{n}'), \quad (6)$$

where P_l is the Legendre polynomial, we obtain, in the isotropic case, $C_l = \langle a_{lm}^* a_{lm} \rangle$. Here the angular brackets denote a theoretical (averaging) expectation value over an ensemble of statistically identical universes. In Fourier space this expectation value can be replaced by integration over all possible wavenumbers, i.e., $\langle \dots \rangle \rightarrow \int d^3 \mathbf{k} / (2\pi)^3$.

Computing $\langle a_{lm}^* a_{l'm'} \rangle$, it can be shown that the helical part of the vorticity does not contribute (see Pogosian et al. 2002). Hence, in what follows we consider only the symmetric part of the spectrum. Detailed computation of C_l for vorticity perturbations is described in DKY98, in which it has been shown that the presence of a homogeneous magnetic field induces off-diagonal correlations in multipole space, in particular, correlations between l and $l \pm 2$ multipole coefficients. To quantify this we introduce a second power spectrum defined by

$$D_l(m) = \langle a_{l-1,m}^* a_{l+1,m} \rangle = \langle a_{l+1,m}^* a_{l-1,m} \rangle. \quad (7)$$

The two power spectra, $C_l(m)$ and $D_l(m)$, depend on the spectral index n , the normalization constant S_0 , the Alfvén velocity v_A , and the perturbation damping wavenumber k_D . The power spectra are defined only for $n > -7$ (the quadrupole diverges at small k for $n \leq -7$), and for $n > -1$ the results are determined by the damping wavenumber k_D . The case $n = -5$ corresponds to the Harrison-Peebles-Yu-Zel'dovich scale-invariant spectrum ($C_l, D_l \sim l^2$). See DKY98 for a more detailed discussion.

The nonzero correlation of temperature for unequal l -values has a simple physical explanation: the presence of a preferred spatial direction, that of the magnetic field \mathbf{B}_0 , breaks the spatial isotropy of the CMB map, leading not only to an m -dependence of the correlators but also to nonzero off-diagonal (in l -space) correlations.⁷ The temperature perturbation correlation between two points on the sky depends not only on the angular separation between the two points but also on their orientation with respect to the magnetic field.

A simple observational test to detect (or constrain) the presence of a homogeneous magnetic field in the universe is based on computing the D_l spectrum of CMB anisotropy data.⁸ For this it is useful to introduce the arithmetic mean over m of the two power spectra,

$$\begin{aligned} \bar{C}_l &\equiv \overline{\langle a_{lm}^* a_{lm} \rangle} = \frac{1}{2l+1} \sum_{m=-l}^l \langle a_{lm}^* a_{lm} \rangle, \\ \bar{D}_l &\equiv \overline{\langle a_{l-1,m}^* a_{l+1,m} \rangle} = \frac{1}{2l+1} \sum_{m=-l}^l \langle a_{l-1,m}^* a_{l+1,m} \rangle. \end{aligned} \quad (8)$$

⁷ In a recent paper, Bershadskii & Sreenivasan (2004) show that collisions between Alfvén wave packets and their cascades could generate arcminute-scale CMB temperature anisotropies and argue that this is consistent with the *WMAP* data.

⁸ Other statistics, related to the D_l -values here, could also provide useful tests (e.g., Hajian & Souradeep 2003).

According to DKY98,

$$\bar{C}_l \simeq S_0 \left(\frac{\eta_{\text{dec}}}{\eta_0} \right)^2 (k_D \eta_0)^{-(n+3)} v_A^2 \frac{2^{n+1} \Gamma(-n-1)}{3 \Gamma(-n/2)^2} l^{n+3}, \quad n < -1, \quad (9)$$

$$\frac{\bar{C}_l}{\bar{D}_l} = |n+1| \left[\frac{\Gamma(-(n+1)/2)}{\Gamma(-n/2)} \right]^2, \quad n < -1, \quad (10)$$

$$\bar{C}_l \simeq \bar{D}_l \simeq S_0 \left(\frac{\eta_{\text{dec}}}{\eta_0} \right)^2 (k_D \eta_0)^{-2} v_A^2 \frac{1}{n+1} l^2, \quad n > -1. \quad (11)$$

Using $B_1 \leq B_0$, we have $|\Omega_0|^2 k^3 \leq v_A^2$ (see eq. [2]). This inequality must hold on all scales inside the Hubble radius at decoupling, $k \geq 1/\eta_{\text{dec}}$. With the $S(k)$ spectrum definition in equation (3), we therefore get $2S_0(k/k_D)^{n+3} \leq v_A^2$ for $1/\eta_{\text{dec}} \leq k \leq k_D$, implying

$$2S_0(k_D \eta_{\text{dec}})^{-(n+3)} \leq v_A^2, \quad n \leq -3, \quad (12)$$

$$2S_0 \leq v_A^2, \quad n \geq -3. \quad (13)$$

Therefore, for $n \leq -3$ the result is independent of the damping wavenumber k_D .

We now estimate an upper limiting value of l , l_C , beyond which our approximation is no longer valid. Just as for scalar perturbations (Peebles 1980, p. 356), vector perturbations are affected by collisionless damping. Adding a photon drag force term on the right-hand side of the vorticity equation (1), we can see that there are no oscillations in the vector perturbation case and that damping occurs on scales slightly larger than the damping scale for scalar perturbations, when $k_C \eta_{\text{dec}} \sim 10$ (DKY98), corresponding to $l_C \sim 500$, beyond which our approximation breaks down.

Inserting the limiting values given for S_0 in equations (12) and (13) into equation (9), we find for $n = -5$,

$$\bar{C}_l = 9.04 \times 10^{-16} l^{-2} \left(\frac{B}{1 \text{ nG}} \right)^4, \quad \bar{D}_l = \frac{\bar{C}_l}{2.26}, \quad (14)$$

and for $n = -7$,

$$\bar{C}_l = 8.61 \times 10^{-10} l^{-4} \left(\frac{B}{1 \text{ nG}} \right)^4, \quad \bar{D}_l = \frac{\bar{C}_l}{2.17}. \quad (15)$$

In this paper we use $n = -5$ and $n = -7$ as two illustrative cases. These two cases are interesting, as they correspond to a Harrison-Peebles-Yu-Zel'dovich scale-invariant spectrum result for C_l and D_l and to a possible inflation-model primordial vorticity field perturbation spectrum, respectively. These two cases also span the range of constraints that can be placed on B using this method in the range $-3 \geq n \geq -7$, i.e., $n = -5$ gives the weakest and $n = -7$ the strongest constraint on B . With $k_D \eta_0 \sim 0.4 \times 10^{14}$, from equations (9) and (11) B is not meaningfully constrained for $n > -3$.

3. ANALYSIS AND RESULTS

We use the foreground cleaned Q -, V -, and, W -band co-added *WMAP* data (Bennett et al. 2003a) to determine the

off-diagonal correlations. The data are available in the HEALPix format (Górski et al. 1998a) at resolution $N_{\text{side}} = 512$. For each value of the magnetic field amplitude B , we generate 5000 simulations of the CMB sky, and each time we apply the Kp2 Galactic cut mask prior to computing the model D_l -values. The expected model D_l -values, obtained from the mean of these simulations, is then compared with the D_l -values obtained similarly from the *WMAP* data, using the χ^2 statistic. Confidence levels on the field strength are derived from the resulting likelihood function.

Specifically, each simulation is a realization of the CMB with power spectrum C_l given by the best-fit flat- Λ cold dark matter model with a power-law primordial power spectrum (Spergel et al. 2003) and with D_l the same as that predicted by equation (10) (or more specifically, eqs. [14] or [15]) for a given value of B . In other words, we generate a_{lm} -values such that they satisfy

$$\langle a_{lm}^* a_{l'm'} \rangle = \delta_{m,m'} [\delta_{l,l'} C_l + (\delta_{l+1,l'-1} + \delta_{l-1,l'+1}) \bar{D}_l] \quad (16)$$

instead of

$$\langle a_{lm}^* a_{l'm'} \rangle = \delta_{m,m'} \delta_{l,l'} C_l. \quad (17)$$

The a_{lm} -values are generated up to an l_{max} of 512 (corresponding to HEALPix resolution $N_{\text{side}} = 256$, since the *WMAP* data are expected to contain useful cosmological information up to such an l_{max}). These are then convolved with the beam functions of each of the Q , V , and W radiometer channels (eight in all) to produce eight maps at HEALPix resolution $N_{\text{side}} = 512$ (because the noise maps have this resolution). Independent Gaussian noise realizations of rms $\sigma_0/N_{\text{obs}}^{1/2}$ from *WMAP* are added to the maps, and the eight maps are co-added, weighted by $N_{\text{obs}}/\sigma_0^2$, where the effective number of observations N_{obs} varies across the sky and σ_0 is different for each radiometer channel. This is how each simulation is created. Hereafter, the same analysis procedure that is applied to the data map is applied to each of the simulations. This consists of bringing the co-added map down to the HEALPix resolution $N_{\text{side}} = 256$ (since we mostly use D_l -values only up to an l_{max} of 300 in the subsequent analysis) and applying the Kp2 sky cut prior to computing D_l . The whole analysis can be repeated for different values of the spectral index n that characterizes the spectrum of the magnetic field perturbations.

Figure 1 shows the D_l -values obtained from the *WMAP* data (*crosses*) and the median and 68% confidence range contours from simulations for two illustrative values of B . The spread of about 10^{-3} mK^2 in the 68% confidence contours for $l(l+1)D_l$ is consistent with what is expected from cosmic variance alone.

To compare the likelihoods for different B -values, we use the diagonal χ^2 statistic,

$$\chi^2 = \sum_{l=2}^{300} \frac{(D_l^W - \bar{D}_l^S)^2}{\sigma_l^2}, \quad (18)$$

where D_l^W is the *WMAP* data value, and \bar{D}_l^S and σ_l are the average and standard deviation of D_l^S , both obtained from 5000 model simulations, for each value of B . Note that we do not use the full covariance matrix for D_l but rather just the diagonal terms. This is because 5000 simulations are not

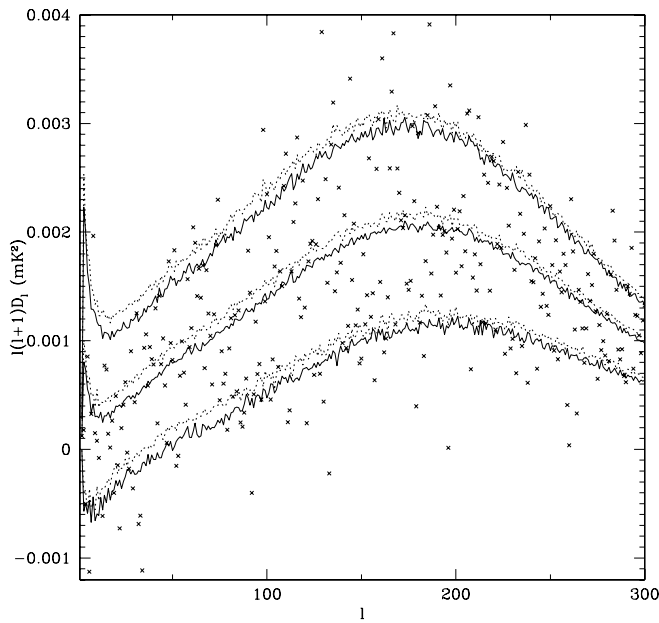


FIG. 1.—Off-diagonal power spectra obtained from the *WMAP* data (crosses) and the median and 68% confidence contours obtained from model simulations with magnetic field strengths $B = 0$ (solid lines) and 16 nG (dotted lines), for a magnetic field perturbation spectral index $n = -5$.

sufficient to produce a reliably converged full covariance matrix for D_l . As also noted by Eriksen et al. (2004b), for example, even the above χ^2 test can provide a just comparison between data and simulations. The likelihood is proportional to $e^{-\chi^2/2}$. We calculate the likelihood for a few different values of B .

The likelihood function obtained for the $n = -5$ case is shown in Figure 2. After integration, we get a 3σ confidence upper limit of $B < 15$ nG. As we can see from Figure 2, $B = 0$ G, corresponding to pure Gaussian primordial fluctuations,

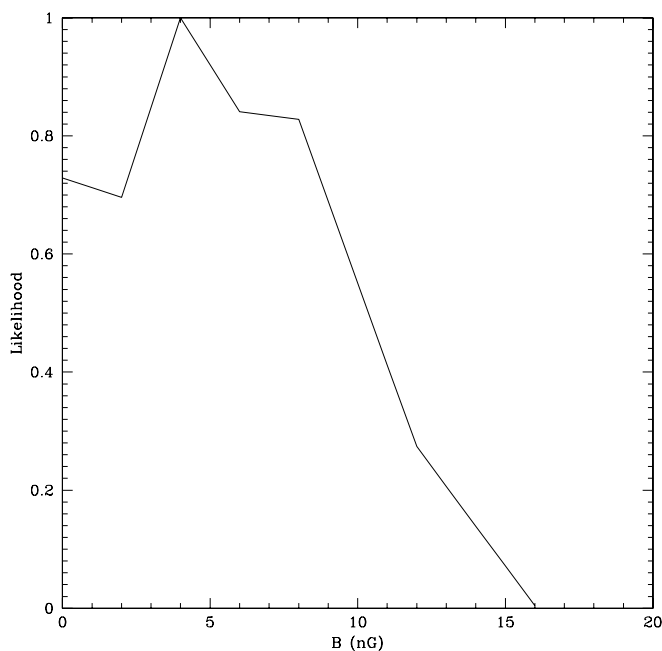


FIG. 2.—Likelihood as a function of the strength of the magnetic field for a magnetic field perturbation spectral index $n = -5$.

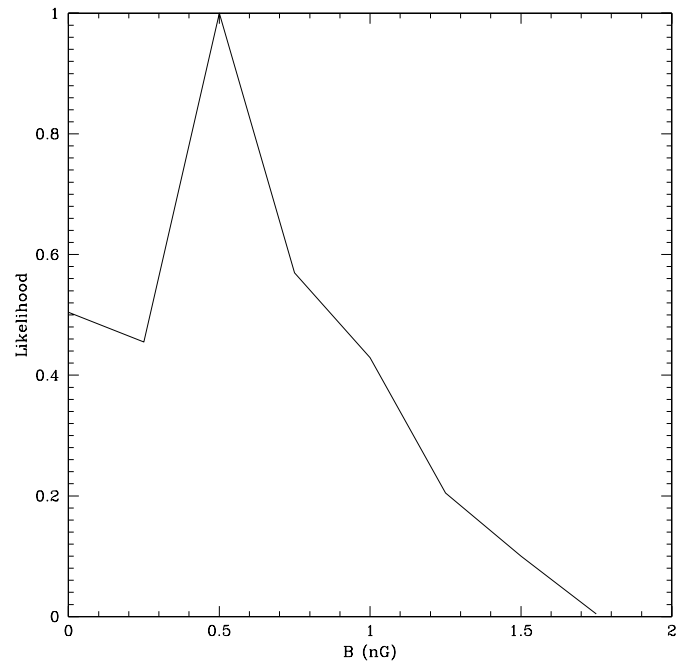


FIG. 3.—Likelihood as a function of the strength of the magnetic field for a magnetic field perturbation spectral index $n = -7$.

is within the 1σ confidence range from the peak of the likelihood function (this 1σ range corresponds to δB of 3.9 nG). The jaggedness in the likelihood function is from the fluctuation of the mean D_l used in calculating χ^2 (see Fig. 1). With 5000 simulations, the fluctuation of the mean D_l should be on the order of $\sigma_l/5000^{1/2}$. This results in a fluctuation of 0.1 in the χ^2 values, or a 5% uncertainty in the estimated likelihood. This does not much affect the results of our analysis (for example, for the 3σ limit on B we look for a $\Delta\chi^2$ of 9, which is not very sensitive to a 5% uncertainty in the estimated likelihoods).

For the $n = -7$ case (see Fig. 3), the 3σ confidence limit is $B < 1.7$ nG, and again the pure Gaussian primordial fluctuation case with $B = 0$ G is not far from the 1σ confidence range from the peak. In this case 1σ corresponds to $\delta B = 0.4$ nG. More stringent limits are obtained in this case, as expected (eq. [15] shows that larger D_l -values with a stronger l -dependence are predicted by the model for $n = -7$).

The conditions of homogeneity and unidirectionality of the primordial magnetic field may be a better approximation on some scales rather than others. In each of the above cases for n , other ranges in l , such as 2–100, 101–200, 201–300, or 2–500, did not indicate anything qualitatively different, i.e., $B = 0$ G remains a satisfactory fit.

4. CONCLUSIONS

We study off-diagonal correlations of the form $D_l = \langle a_{l-1,m} a_{l+1,m}^* \rangle$ in the first-year *WMAP* CMB anisotropy data. Such correlations can result from a homogeneous primordial magnetic field. We do not find significant off-diagonal correlations in the data, which appear to be satisfactorily fitted by a zero primordial magnetic field hypothesis. We place 3σ upper limits on the strength of the magnetic field of $B < 15$ nG for spectral index $n = -5$ and $B < 1.7$ nG for $n = -7$. These two cases are interesting, as they correspond to a Harrison-Peebles-Yu-Zel'dovich scale-invariant spectrum result for the

C_l - and D_l -values and to a possible inflation-model primordial magnetic field perturbation spectrum, respectively. These two cases also span the range of constraints that can be placed on B using this method. Future CMB anisotropy data should allow for tighter constraints on a primordial cosmological magnetic field.

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