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Effects of cosmological magnetic helicity on the CMB

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I present a short review of primordial magnetic helicity effects on Cosmic Microwave Background (CMB) temperature and polarization anisotropies. These effects allow us to test for cosmological magnetic helicity; however, very accurate CMB fluctuation data are required. This scheme for magnetic helicity detection is valid only for a cosmological magnetic field with a present amplitude larger than 10^{-9}–10^{-10} Gauss.

1 Introduction

Recent astrophysical observations indicate that the magnetic fields in the Sun and some galaxies and clusters of galaxies might have an helical structure (for reviews see Widrow 2002; Vallée 2004). One promising possibility to explain the observed magnetic helicity is assuming the presence of *seed* helical magnetic fields. Several mechanisms have been proposed for generating such a primordial helical magnetic field during an early epoch of the Universe (Cornwall 1997; Giovannini & Shaposhnikov 1998; Giovannini 2000; Field & Carroll 2000; Vachaspati 2001; Sigl 2002; Semikoz & Sokoloff 2005; Campanelli & Giannotti 2005). Magnetic helicity can be also be generated via helical turbulent motions if there is an inverse cascade (Brandenburg 2001; Chistensson, Hindmarsh & Brandenburg 2005; Verma & Ayyer 2003; Boldyrev & Cattaneo 2004). On the other hand, primordial helicity influences magneto-hydrodynamical processes in the early plasma as well as cosmological perturbation dynamics (Vishniac & Cho 2001; Brandenburg 2001; Kleorin et al. 2003; Subramanian 2002; Vishniac, Lazarian & Cho 2003; Subramanian & Brandenburg 2004; Banerjee & Jedamzik 2004; Subramanian & Shukurov 2005).

To preserve large-scale isotropy, a seed magnetic field (and so magnetic helicity) has to be small enough to allow treating the average energy density and mean helicity of the magnetic field as first order in perturbation theory.

Even if the energy density of a primordial magnetic field $B^2/(8\pi)$ is five or six magnitude less than that of radiation (CMB photons), taking into account that CMB anisotropy measurements have the same order of accuracy, 10^{-6}–10^{-5}, we expect that such a cosmological magnetic field would leave detectable traces in CMB temperature or polarization anisotropies (see Lewis 2004; Giovannini 2005; and references therein).

Here I focus on the effects on CMB temperature and polarization anisotropies induced by magnetic helicity. This talk is based on results obtained in several collaborations (Mack, Kahniashvili & Kosowsky 2002; Caprini, Durrer & Kahniashvili 2004; Kosowsky et al. 2005; Kahniashvili & Ratra 2005; Kahniashvili, Gogoberidze & Ratra 2005). We find that magnetic helicity generates vector and tensor metric perturbations and as a result affects all CMB fluctuations. In particular: (i) magnetic helicity reduces the amplitudes of parity-even CMB fluctuation power spectra in comparison to the case of a non-helical magnetic field (Caprini et al. 2004; Kahniashvili & Ratra 2005); (ii) the Faraday rotation of the CMB polarization plane is strongly dependent on the average energy density of the magnetic field and is independent of magnetic helicity, see Kosowsky et al. (2005); (iii) magnetic helicity induces parity-odd cross-correl ations of the CMB fluctuations, which vanish for the case of a magnetic field without helicity (Pogosian, Vachaspati & Winitski 2002; Caprini et al. 2004; Kahniashvili & Ratra 2005)[1]; and (iv) magnetic helicity generates circularly polarized stochastic gravitational waves (Kahniashvili et al. 2005).

2 Magnetic source for metric perturbations

We assume the existence of a cosmological magnetic field generated during or prior to the radiation-dominated epoch, with the energy density of the field a first-order perturbation to the standard Friedmann-Lemaître-Robertson-Walker homogeneous cosmological spacetime model. Neglecting fluid back-reaction onto the magnetic field, the spatial and temporal dependence of the field separates, $\mathbf{B}(t,\mathbf{x}) =$

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[1] This is not true for a homogeneous magnetic field (Scoccola, Harrari & Mollerach 2004).

$\mathbf{B}(\mathbf{x})/a^2$; here a is the cosmological scale factor. As a phenomenological normalization of the magnetic field, we smooth the field on a comoving length λ with a Gaussian smoothing kernel $\propto \exp[-x^2/\lambda^2]$ to obtain the smoothed magnetic field with average value of squared magnetic field $B_\lambda{}^2 \equiv \langle \mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x}) \rangle|_\lambda$ and magnetic helicity $H_\lambda{}^2 \equiv \lambda|\langle \mathbf{B}(\mathbf{x}) \cdot [\nabla \times \mathbf{B}(\mathbf{x})] \rangle|_\lambda$.

We also assume that the primordial plasma is a perfect conductor on all scales larger than the Silk damping wavelength $\lambda_{\rm S}$ (the thickness of the last scattering surface) set by photon and neutrino diffusion. We model magnetic field damping by an ultraviolet cut-off wavenumber $k_{\rm D} = 2\pi/\lambda_{\rm D}$, Subramanian & Barrow (1998),

$$\left(\frac{k_{\rm D}}{\rm Mpc^{-1}}\right)^{n_B+5} \approx$$
$$\approx 2.9 \times 10^4 \left(\frac{B_\lambda}{10^{-9}\,\rm G}\right)^{-2} \left(\frac{k_\lambda}{\rm Mpc^{-1}}\right)^{n_B+3} h. \qquad (1)$$

Here n_B is the spectral index of the symmetric part of the magnetic field power spectrum (see Eq. 3 below), h is the Hubble constant in units of 100 km sec^{-1} Mpc^{-1}, $k_\lambda = 2\pi/\lambda$ is the smoothing wavenumber, and $\lambda_{\rm D} \ll \lambda_{\rm S}$. This assumes that magnetic field damping is due to the damping of Alfvén waves from photon viscosity.

Assuming that the stochastic magnetic field is Gaussian distributed and accounting for the possible helicity of the field, the magnetic field spectrum in wavenumber space is, Pogosian et al. (2002),

$$\langle B_i^\star(\mathbf{k}) B_j(\mathbf{k}')\rangle =$$
$$= (2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{k}')[P_{ij}(\hat{\mathbf{k}})P_B(k) + i\epsilon_{ijl}\hat{k}_l P_H(k)]. \quad (2)$$

Here $P_{ij}(\hat{\mathbf{k}}) \equiv \delta_{ij} - \hat{k}_i\hat{k}_j$ is the transverse plane projector with unit wavenumber components $\hat{k}_i = k_i/k$, ϵ_{ijl} is the antisymmetric symbol, and $\delta^{(3)}(\mathbf{k}-\mathbf{k}')$ is the Dirac delta function. $P_B(k)$ and $P_H(k)$ are the symmetric and helical parts of the magnetic field power spectrum, assumed to be simple power laws on large scales,

$$P_B(k) \;\equiv\; P_{B0}k^{n_B} = \frac{2\pi^2\lambda^3 B_\lambda^2}{\Gamma(n_B/2+3/2)}(\lambda k)^{n_B},$$
$$P_H(k) \;\equiv\; P_{H0}k^{n_H} = \frac{2\pi^2\lambda^3 H_\lambda^2}{\Gamma(n_H/2+2)}(\lambda k)^{n_H}, \qquad (3)$$

and vanishing on small scales when $k > k_D$. Here Γ is the Euler Gamma function. These power spectra are generically constrained by $P_B(k) \geq |P_H(k)|$, Caprini et al. (2004), which implies $n_H > n_B$. In addition, finiteness of the magnetic field energy density requires $n_B > -3$ (to prevent an infrared divergence of magnetic field energy density). Finiteness of the magnetic field average helicity requires $n_H > -4$; this is automatically satisfied as a consequence of $n_H > n_B > -3$.

To obtain the magnetic field source terms in the equations for vector (transverse peculiar velocity) and tensor (gravitational waves) metric perturbations we need to extract the transverse vector and tensor parts of the magnetic field stress-energy tensor $\tau_{ij}(\mathbf{k})$. This is done through $\Pi_{ij}^{(V)}(\mathbf{k}) = (P_{ib}(\hat{\mathbf{k}})\hat{k}_j + P_{jb}(\hat{\mathbf{k}})\hat{k}_i)\hat{k}_a\tau_{ab}(\mathbf{k})$ (for vector perturbations) and $\Pi_{ij}^{(T)}(\mathbf{k}) = [P_{ia}(\hat{\mathbf{k}})P_{jb}(\hat{\mathbf{k}}) - \frac{1}{2}P_{ij}(\hat{\mathbf{k}})P_{ab}(\hat{\mathbf{k}})]\tau_{ab}(\mathbf{k})$ (for tensor perturbations); for details see Mack et al. (2002).

For vector perturbations the $\Pi_{ij}^{(V)}$ tensor is related to the vector (divergenceless and transverse) part of the Lorentz force $L_i^{(V)}(\mathbf{k}) = k_j\Pi_{ij}(\mathbf{k}) = P_{ib}(\hat{\mathbf{k}})k_a\tau_{ab}(\mathbf{k})$. For the normalized Lorentz force vector, $\Pi_i \equiv L_i^{(V)}/k$, the general spectrum of $\langle \Pi_i^\star(\mathbf{k})\Pi_j(\mathbf{k}')\rangle$ in wavenumber space is similar to Eq. (2), and has two parts, symmetric and helical. Both contain contributions from $P_H(K)$, and so magnetic helicity affects the vector magnetic source term, Kahniashvili & Ratra (2005).

The tensor magnetic source term is obtained through the source two-point function $\langle \Pi_{ij}^{(T)\star}(\mathbf{k})\Pi_{lm}^{(T)}(\mathbf{k}')\rangle$. This is determined by the magnetic field two-point function, and like the vector magnetic source term has symmetric and helical parts (Caprini et al. 2004; Kahniashvili et al. 2005). The helical (parity-odd) piece results in circular polarization of the induced gravitational waves. For a maximally helical magnetic field with $P_H(k) \simeq P_B(k)$, the polarization degree is high enough to allow us to consider the possibility of testing for magnetic helicity through a measurement of the polarization of relic gravitational waves (this might be possible with future gravitational waves detectors); see Kahniashvili et al. (2005).

In both cases (vector and tensor perturbations) the contribution of magnetic field helicity to the symmetric part of the magnetic source is negative. It is clear that the magnetic source terms vanish on scales smaller than the cutoff scale λ_D because of magnetic field damping. Depending on spectral indexes n_B and n_H the magnetic source terms are dominated either by small wavenumber, $\propto k^{2n_B+3}$ for $n_B < -3/2$, or by the high frequency cut-off and so $\propto k_{\rm D}^{2n_B+3}$ for $n_B > -3/2$.

3 CMB anisotropies

For our computations we use the formalism given in Mack et al. (2002), extending it to account for magnetic field helicity. To compute CMB temperature and polarization anisotropy power spectra, we use the total angular momentum method of Hu & White (1997). Our analytical approximations are derived in Caprini et al. (2004) and Kahniashvili & Ratra (2005).

3.1 Parity-even CMB fluctuations

Cosmological magnetic field vector and tensor mode contributions to CMB fluctuations at large angular scales (with multipole number $l < 100$) are of the same order of magnitude, Mack et al. (2002), while for small angular scales (where $l > 100$) CMB fluctuations are dominated by the vector mode contribution because of gravitational wave damping (Lewis 2004).

The complete parity-even CMB fluctuation power spectra may be expressed as,

$$C_l^{\mathcal{X}\mathcal{X}'} = C_{(S)l}^{\mathcal{X}\mathcal{X}'} - C_{(A)l}^{\mathcal{X}\mathcal{X}'}, \qquad (4)$$

where $\mathcal{X}$ is either Θ, E, or B, which represent respectively the temperature, E-polarization, and B-polarization anisotropies, and $C_{(A)l}^{\mathcal{X}\mathcal{X}'}$ are the antisymmetric power spectra induced by magnetic helicity. The minus sign reflects the negative contribution of magnetic helicity to the total parity-even CMB fluctuation power spectra (from terms proportional to $\int d^3p\, P_H(p) P_H(|\mathbf{k}-\mathbf{p}|)$). For large angular scales this result holds for both (vector and tensor) modes, while for $l > 100$ it applies only for vector perturbations. The fractional differences $\kappa_l^{\mathcal{X}\mathcal{X}'} \equiv 1 - C_{(A)l}^{\mathcal{X}\mathcal{X}'}/C_{(S)l}^{\mathcal{X}\mathcal{X}'}$, where $0 < \kappa_l^{\mathcal{X}\mathcal{X}'} < 1$, can be used to characterize the reduction of the parity-even CMB fluctuation power spectra amplitudes as a consequence of non-zero magnetic helicity. The ratio $C_{(A)l}^{\mathcal{X}\mathcal{X}'}/C_{(S)l}^{\mathcal{X}\mathcal{X}'}$ may be expressed in terms of P_{0H}/P_{0B} and spectral indexes n_H and n_B (Caprini et al. 2004; Kahniashvili & Ratra 2005). While the reduction of parity-even power spectra amplitudes are significant for a maximally helical field, the most interesting effects from magnetic helicity are the generation of parity-odd CMB fluctuations, such as the cross-correlations between temperature and B-polarization, and between E- and B-polarizations.

3.2 Parity-odd CMB fluctuations

Magnetic helicity induces parity-odd cross correlations between the E- and B-polarization anisotropies, as well as between temperature and B-polarization anisotropies, (Pogosian et al. 2002; Caprini et al. 2004; Kahniashvili & Ratra 2005). Such off-diagonal parity-odd cross correlations occur also in the case of an homogeneous magnetic field from the Faraday rotation effect, Scoccola et al. (2004), but not in the case of Faraday rotation in a stochastic magnetic field, even one with non-zero helicity, Kosowsky et al. (2005). Faraday rotation measurements used to measure a magnetic field amplitude cannot be used to detect magnetic helicity (Enßlin & Vogt 2003; Campanelli et al. 2004; Kosowsky et al. 2005). A possible way of detecting magnetic helicity directly from CMB fluctuation data is to detect the parity-odd CMB fluctuation cross-correlations or/and to detect the effects magnetic helicity has on parity-even CMB fluctuations, Kahniashvili & Ratra (2005).

3.2.1 Temperature B-polarization cross-correlations

At large angular scales ($l < 100$) where the contribution from the tensor mode is significant, for $n_B + n_H > -2$ the vector mode $C_l^{\Theta B(V)}$ and the tensor mode $C_l^{\Theta B(T)}$ have the same l dependence $\propto l^2$. For all other values of spectral indexes n_B and n_H, the growth rate (with l) of $C_l^{\Theta B(V)}$ is faster than $C_l^{\Theta B(T)}$. The ratio between temperature B-polarization signals from vector and tensor modes is independent of the amplitudes of the average magnetic field (B_λ) and average magnetic helicity (H_λ).

For small angular scales ($l > 100$) where the tensor mode signal vanishes, for a maximally helical magnetic field with $n_H \simeq n_B$, due to the suppression factor $L_{\gamma,\rm dec}/\eta_0$ (where $L_{\gamma,\rm dec}$ is the photon mean free path at decoupling and η_0 conformal time today) the temperature E-polarization cross-correlation power spectrum, $C_l^{\Theta E}$, is smaller than the temperature B-polarization cross-correlation power spectrum, $C_l^{\Theta B}$, but both are $\propto l^2$, if $n_B + n_H > -5$. The same suppression factor makes $C_l^{\Theta B}$ smaller than $C_l^{\Theta\Theta}$. For an arbitrary helical field $C_l^{\Theta B}/C_l^{\Theta E}$ depends on the ratio $(P_{H0}/P_{B0})k_{\rm D}^{n_H - n_B}$ and order unity prefactors that depend on n_B and n_H. A dependence on l appears only if $n_B + n_H < -5$, when the ratio, $C_l^{\Theta B}/C_l^{\Theta E}$ decreases as $\propto l^{n_B+n_H+5}$ (Kahniashvili & Ratra 2005).

3.2.2 E and B-polarization cross-correlations

For a tensor mode signal at large angular scales ($l < 100$), E- and B-polarization cross-correlation C_l^{EB} is of the same order of magnitude as the tensor mode temperature B-polarization anisotropy cross-correlation spectrum, $C_l^{\Theta B}$, Caprini et al. (2004). The situation is different for a vector mode which survives up to small angular scales (e.g., Subramanian & Barrow 1998; Mack et al. 2002; Lewis 2004; Giovannini 2005). In this case, the E- and B-polarization anisotropy cross-correlation power spectrum has a suppression factor of $kL_{\gamma,\rm dec}$ implying that $C_l^{EB} \ll C_l^{\Theta B}$. This is consistent with the result of Hu & White (1997).

4 Conclusion

I have discussed how cosmological magnetic helicity affects CMB fluctuations. Even for a cosmological magnetic field with maximal helicity, such effects may be detectable only if the current magnetic field amplitude is at least 10^{-10} or 10^{-9} G on Mpc scales. A non-helical seed field of this amplitude can be generated during inflation (Ratra 1992; Bamba & Yokoyama 2004). A cosmological magnetic field generates a B-polarization signal via induced vector and/or tensor modes, so detection of such a signal may indicate the presence of a cosmological magnetic field. However, it has to be emphasized that a B-polarization anisotropy signal can also arise in other ways, such as from primordial tensor perturbations, gravitational lensing, or Faraday rotation of the CMB anisotropy polarization plane; for a review see Subramanian (2004). The peak position of B-polarization anisotropy power spectrum, $l^2 C_l^{BB}$, may help to identify the B-polarization source. For example, cosmological-magnetic-field-induced tensor perturbations only contribute on large angular scales $l < 100$, while the B-polarization anisotropy from gravitational lensing has a peak amplitude $l^2 C_l^{BB} \sim 10^{-14}$ at $l \sim 1000$ (Challinor & Lewis 2005). The Faraday rotation B-polarization anisotropy signal from a field with $B_\lambda = 10^{-9}$ G (at $\lambda = 1$ Mpc) and spectral index $n_B = -2$ peaks at a substantially smaller scale $l \sim 10^4$ with a frequency-dependent peak amplitude $l^2 C_l^{BB} \sim 10^{-12}$ (at

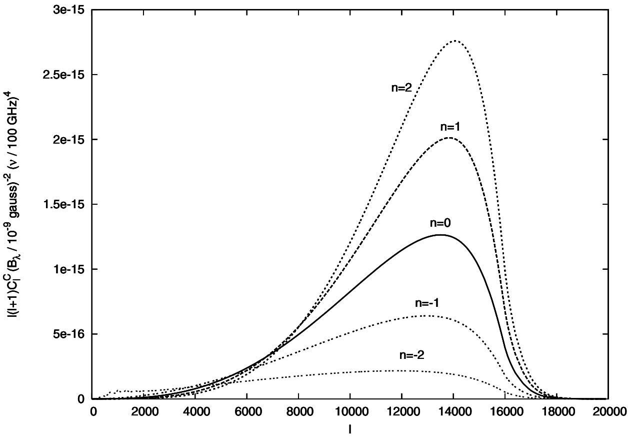

Fig. 1 The C-polarization power spectrum of the microwave background induced by the Faraday rotation effect, Kosowsky et al. (2005). The curves in order of decreasing amplitude on the right side of the plot correspond to magnetic field power spectral indices $n_B = 2, 1, 0, -1$, and -2. The magnetic fields have been normalized to 1 nG at the smoothing scale $\lambda = 1$ Mpc.

10 GHz) and $l^2 C_l^{BB} \sim 10^{-14}$ (at 30 GHz); see Fig. 1. A non-helical cosmological magnetic field with $B_\lambda = 10^{-9}$ G at $\lambda = 1$ Mpc induces a B-polarization anisotropy signal via the vector perturbation mode with a peak amplitude $l^2 C_l^{BB} \sim 10^{-13}$ at $l \sim 1000$, Lewis (2004). We have shown that a magnetic field with maximal helicity results in the reduction of the B-polarization anisotropy signal on all scales by a factor of $1/3$ for $-3/2 < n_B \simeq n_H$, relative to the non-helical magnetic field case, Kahniashvili & Ratra (2005). Summarizing, we argue that cosmological magnetic helicity in the case of a magnetic field larger than 10^{-9} G affects CMB anisotropies, in addition to the effects it has on MHD dynamo amplification and processes in the early Universe (Cornwall 1997; Banerjee & Jedamzik 2004). To measure primordial magnetic helicity through these effects very accurate CMB fluctuation data is required.

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