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On a periodic boundary value problem for cyclic feedback type linear functional differential systems

By

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Abstract. Nonimprovable effective sufficient conditions are established for the unique solvability of the periodic problem

$$u_i'(t) = \ell_i(u_{i+1})(t) + q_i(t) \quad (i = \overline{1, n-1}),$$
$$u_n'(t) = \ell_n(u_1)(t) + q_n(t),$$
$$u_j(0) = u_j(\omega) \quad (j = \overline{1, n}),$$

where $\omega > 0$, $\ell_i : C([0,\omega]) \to L([0,\omega])$ are the linear bounded operators, and $q_i \in L([0,\omega])$.

1. Statement of Problem and Formulation of Main Results. Consider on $[0, \omega]$ the system

$$u_i'(t) = \ell_i(u_{i+1})(t) + q_i(t) \quad (i = \overline{1, n-1}),$$
$$u_n'(t) = \ell_n(u_1)(t) + q_n(t), \tag{1.1}$$

with the periodic boundary conditions

$$u_j(0) = u_j(\omega)(j = \overline{1, n}), \tag{1.2}$$

where $\omega > 0$, $\ell_i : C([0,\omega]) \to L([0,\omega])$ are linear bounded operators and $q_i \in L([0,\omega])$.

By a solution of the problem (1.1), (1.2) we understand a function $u \in \widetilde{C}([0,\omega])$ which satisfies the system (1.1) almost everywhere on $[0,\omega]$ and satisfies the conditions (1.2).

Much work has been carried out on the existence and uniqueness of the solution for a periodic boundary value problem for systems of ordinary differential equations and many interesting results have been obtained (see, for instance, [1], [2], [3], [4], [5], [6], [7], [8] and the references therein). However, the analogous problem for functional differential equations, even in the case of linear equations, remains little investigated.

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