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On periodic solutions of two-dimensional nonautonomous differential systems

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Abstract

Nonimprovable conditions are established for the existence and uniqueness of an ω-periodic solution of the nonautonomous differential systems

$$u_i' = p_{i1}(t)u_1 + p_{i2}(t)u_2 + q_i(t) \quad (i = 1, 2)$$

and

$$u_i' = f_i(t, u_1, u_2) \quad (i = 1, 2),$$

where $p_{ik} : R \to R$, $q_i : R \to R$ $(i, k = 1, 2)$ are ω-periodic functions, Lebesgue integrable on $[0, \omega]$, and $f_i : R \times R^2 \to R$ $(i = 1, 2)$ are functions from the Carathéodory class such that

$$f_i(t + \omega, x_1, x_2) \equiv f_i(t, x_1, x_2) \quad (i = 1, 2).$$

MSC: 34C25.

Keywords: Two-dimensional nonautonomous differential system; ω-Periodic solution; Existence and uniqueness theorems

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1. Formulation of the main results. Examples

Problems on the existence and uniqueness of a periodic solution of systems of nonautonomous ordinary differential equations have long been attracting the attention of mathematicians and used as the subject of many studies (see, for example, [1–20] and the references therein). And all the same these problems still remain topical for two-dimensional linear and nonlinear differential systems

$$u_i' = p_{i1}(t)u_1 + p_{i2}(t)u_2 + q_i(t) \quad (i = 1, 2) \tag{1.1}$$

and

$$u_i' = f_i(t, u_1, u_2) \quad (i = 1, 2). \tag{1.2}$$

In this paper, an attempt is made to fill to a certain extent the gap existing in this area. More exactly, new and, in a certain sense, optimal sufficient conditions are established for the existence and uniqueness of a periodic solution of systems (1.1) and (1.2) with a period $\omega > 0$.

Throughout the paper the following notation is used.

R^m is the m-dimensional real Eucledean space.

L_ω is the space of ω-periodic and Lebesgue integrable on $[0, \omega]$ functions $p : R \to R$ with the norm

$$\|p\| = \int_0^\omega |p(t)|\,\mathrm{d}t.$$

$K_\omega(R \times R^m)$ is the space of functions $f : R \times R^m \to R$, which are ω-periodic in the first argument and satisfy the local Carathéodory conditions. Consequently, the notation $f \in K_\omega(R \times R^m)$ means that $f(t, \cdot, \ldots, \cdot) : R^m \to R$ is continuous for any $t \in R$, $f(\cdot, x_1, \ldots, x_m) \in L_\omega$ for any $(x_1, \ldots, x_m) \in R^m$ and

$$\max\left\{ |f(\cdot, x_1, \ldots, x_m)| : \sum_{i=1}^m |x_i| \leqslant \rho \right\} \in L_\omega \quad \text{for any } \rho \in]0, +\infty[.$$

For any function $p : R \to R$ the notation $p(t) \not\equiv 0$ means that p is different from zero on the set of positive measure.

Everywhere in the sequel when we consider systems (1.1) and (1.2), it will be assumed that

$$p_{ik} \in L_\omega, \quad q_i \in L_\omega \quad (i, k = 1, 2) \tag{1.3}$$

and

$$f_i \in K_\omega(R \times R^2) \quad (i = 1, 2), \tag{1.4}$$

respectively.