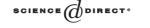


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## On periodic solutions of two-dimensional nonautonomous differential systems

I. Kiguradze\*, S. Mukhigulashvili

A. Razmadze Mathematical Institute of the Georgian Academy of Sciences, 1 M. Aleksidze St., Tbilisi 0193, Georgia

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## Abstract

Nonimprovable conditions are established for the existence and uniqueness of an  $\omega$ -periodic solution of the nonautonomous differential systems

 $u'_i = p_{i1}(t)u_1 + p_{i2}(t)u_2 + q_i(t)$  (i = 1, 2)

and

 $u'_i = f_i(t, u_1, u_2)$  (i = 1, 2),

where  $p_{ik}: R \to R$ ,  $q_i: R \to R$  (i, k = 1, 2) are  $\omega$ -periodic functions, Lebesgue integrable on  $[0, \omega]$ , and  $f_i: R \times R^2 \to R$  (i = 1, 2) are functions from the Carathéodory class such that

 $f_i(t + \omega, x_1, x_2) \equiv f_i(t, x_1, x_2)$  (i = 1, 2).

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<sup>\*</sup> Corresponding author. Tel.: +995-32-33-45-95. *E-mail addresses:* kig@rmi.acnet.ge (I. Kiguradze), smukhig@rmi.acnet.ge (S. Mukhigulashvili).

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## 1. Formulation of the main results. Examples

Problems on the existence and uniqueness of a periodic solution of systems of nonautonomous ordinary differential equations have long been attracting the attention of mathematicians and used as the subject of many studies (see, for example, [1–20] and the references therein). And all the same these problems still remain topical for two-dimensional linear and nonlinear differential systems

$$u'_{i} = p_{i1}(t)u_{1} + p_{i2}(t)u_{2} + q_{i}(t) \quad (i = 1, 2)$$

$$(1.1)$$

and

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$$u'_i = f_i(t, u_1, u_2)$$
  $(i = 1, 2).$  (1.2)

In this paper, an attempt is made to fill to a certain extent the gap existing in this area. More exactly, new and, in a certain sense, optimal sufficient conditions are established for the existence and uniqueness of a periodic solution of systems (1.1) and (1.2) with a period  $\omega > 0$ .

Throughout the paper the following notation is used.

 $R^m$  is the *m*-dimensional real Eucledean space.

 $L_{\omega}$  is the space of  $\omega$ -periodic and Lebesgue integrable on  $[0, \omega]$  functions  $p : R \to R$  with the norm

$$\|p\| = \int_0^\omega |p(t)| \,\mathrm{d}t.$$

 $K_{\omega}(R \times R^m)$  is the space of functions  $f : R \times R^m \to R$ , which are  $\omega$ -periodic in the first argument and satisfy the local Carathéodory conditions. Consequently, the notation  $f \in K_{\omega}(R \times R^m)$  means that  $f(t, \dots, \dots) : R^m \to R$  is continuous for any  $t \in R$ ,  $f(\cdot, x_1, \dots, x_m) \in L_{\omega}$  for any  $(x_1, \dots, x_m) \in R^m$  and

$$\max\left\{|f(\cdot, x_1, \dots, x_m)| : \sum_{i=1}^m |x_i| \leq \rho\right\} \in L_{\omega} \quad \text{for any } \rho \in ]0, +\infty[.$$

For any function  $p : R \to R$  the notation  $p(t) \neq 0$  means that p is different from zero on the set of positive measure.

Everywhere in the sequel when we consider systems (1.1) and (1.2), it will be assumed that

$$p_{ik} \in L_{\omega}, \quad q_i \in L_{\omega} \quad (i, k = 1, 2) \tag{1.3}$$

and

$$f_i \in K_{\omega}(R \times R^2) \quad (i = 1, 2), \tag{1.4}$$

respectively.