Non-extensive statistical analysis of seismicity in the area of Javakheti, Georgia

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A B S T R A C T

The distribution of earthquake magnitudes in the Javakheti highlands was analyzed using a non-extensive statistical approach. The earthquakes occurring from 1960 to 2008 in this seismically active area of Southern Caucasus were investigated. The seismic catalog was studied using different threshold magnitude values. Analyses of the whole time period of observations as well as of sub-catalogs of consecutive 10-year span time windows were performed. In every case non-extensive parameter q and value α, the physical quantity characterizing energy density, were calculated from the modified frequency–magnitude relationship. According to our analysis the magnitude sequence in the Javakheti area for the whole period of observation is characterized by a non-extensivity parameter q = 1.81, in the upper limit of values reported elsewhere. While calculated non-extensivity parameters for consecutive 10-year windows fall within the range 1.6–1.7 reported worldwide. A significant increase of parameter q was identified in those 10-year sub-catalogs that included the strongest earthquakes within the period of observation. We suppose that this increase may be related to a more correlated behavior within the system of ‘fault fragments’ when a strong earthquake strikes or immediately after; during aftershock activity. Concurrently, smaller values of non-extensivity parameters q, found during seismically relatively quiet times, could be associated to the decreased correlations within the system during the earthquake generation stage, under an essentially decreased tectonic stress. The behavior of the energy density characteristic α almost mirrors the variation of parameter q: increases for seismically quiet periods in the Javakheti area and decreases in periods when strong earthquakes occur. We suggest that decreases of energy density characteristic α may point to a prevalent contribution of large size fragments to fragment–asperity interaction under the influence of a rapidly released stress, as opposed to relatively quiet periods when accumulated stress energy is supposedly released through the relative movement of smaller fragments.

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1. Introduction

Seismicity and earthquake triggering are highly complex phenomena in the spatial, temporal or energy domains. Therefore, the dynamics underlying these phenomena is not fully understood because we still not have a complete comprehension of the physical processes taking place in the Earth's lithosphere. Nevertheless, the key statistical properties of earthquake size (energy) and temporal distribution can satisfactorily be represented by the empirical Gutenberg–Richter (GR) law, which shows the log-linear relationship between frequency and magnitude of earthquakes larger than a certain threshold magnitude value (Gutenberg and Richter, 1944), does not fulfill for small and large magnitudes (Sotolongo-Costa and Posadas, 2004; Silva et al., 2006; Vilar et al., 2007). As it has been recently understood these constraints should not be attributed to the restricted sensitivity of instrumental methods but rather it is profoundly rooted in the physics of earthquake generation.

The complex spatio-temporal phenomena involved in earthquake generation are attributed to convective circulation in the mantle, provoking a relative motion of the faults along tectonic plate edges. This slow process creates frictional stress and storage of a large amount of energy in the Earth's crust. The faults move according to a so-called stick–slip mechanism: when the accumulated tectonic stress exceeds the frictional stress on the fault, the two previously stuck plates start to slip against each other. Subsequently, sudden ruptures of the earth's crust may occur, which leads to a rapid emission of energy in the form of seismic
waves (Kanamori and Brodsky, 2001). According to this model the irregularities in the tectonic fault profile (barriers, and roughness), by hindering movement, are considered to have a decisive role.

Recently, Sotolongo-Costa and Posadas, (2004) developed a new fragment–asperity interaction (SCP) model for earthquake dynamics that also assigns an important role to the fragments produced during plate break. According to the SCP model, the relative motion of two irregular faults may be influenced not only by the roughness of their profiles but also by the relative position of the different fragments in the space between, filled with the residues of the breakage of the tectonic plates. Thus, the large amount of accumulated pressure between two fault plates becomes the main factor conditioning the complexity of the fragment–asperity interaction. Those fragments may act either as roller bearings or as restraints of the relative motion of the plates, until the growing stress prompts their breaking, initiating the subsequent earthquake and the release of the energy \( e \) (Telesca, 2010a,b). The energy \( e \) released in this process should be proportional to the volume of the fragment.

Thus, it was assumed that the energy distribution of the earthquakes generated by the fragment–asperity interaction might reflect the volumetric distribution of the breakage fragments in the space between plates (Silva et al., 2006; Vilar et al., 2007). It should be noted that this process was examined in the framework of non-extensive statistics.

At present Tsallis’ non-extensivity concept is widely accepted as a consistent theoretical tool to investigate nonequilibrium stationary states of complex systems, including earthquake faults, characterized by long-range interactions and multifractal self-similar properties, among others. Therefore, once the use of non-extensive statistics was recognized a suitable tool to describe the volumetric distribution function of the fragments, a model for earthquake generation was proposed, which results in a \( q \)-exponential form for the released energy distribution function (Salinas and Tsallis, 1999; Gell-Mann and Tsallis, 2004; Silva et al., 2006). It is important to mention that this form of energy distribution includes the Gutenberg–Richter law as a particular case (Sotolongo-Costa and Posadas, 2004).

Later, Silva et al. (2006) have revisited the previous Sotolongo-Costa model whilst taking into consideration the more reliable definition of mean values in the context of Tsallis non-extensive statistics, and introduced the following relationship between the released seismic energy and the linear size of fragments:

\[
\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right) \log \left[1 - \left(\frac{1-q}{2-q}\right) \left(\frac{10^{m}}{q^{2/\tau}}\right)\right]
\]

above Equation describes the distribution of the number \( N \) of earthquakes whose magnitude \( m \) is larger than a certain pre-defined threshold, normalized to the total number of events. This relationship describes more appropriately the energy distribution in a wider detectable range of magnitudes as compared to GR law (Silva et al., 2006; Telesca, 2010a; Darooneh and Mehri, 2010). Quantities \( q \) and \( a \) in Eq. (1) are a phenomenological non-extensivity parameter and an energy density characteristic value, accordingly (Vilar et al., 2007).

In the framework of the fragment–asperity model, the non-extensivity parameter \( q \) informs us about the scale of interactions: if \( q \) is close to 1, short-ranged spatial correlations are present and physical states are close to equilibrium. As \( q \) increases, the physical state goes away from equilibrium. In the case of geological seismic faults, this implies that the fault planes and the interstitial fragments are not in equilibrium and more seismic activity can be expected (Telesca, 2010a,b). Therefore, parameter \( q \) should reflect the variable character of the relationships among the constituents of the system ‘fault-fragments’. Indeed, authors recently investigating the dynamics of heavy ion collisions have interpreted non-extensivity parameter \( q \) as a measure of particle (fragment in our case) correlations within the system (Chinellato et al., 2010).

According to Silva et al. (2006), energy density value \( a \), with a dimension of volumetric energy density, is the proportionality coefficient between released relative energy \( e \) and the size of fragments \( 10^{m}/r^{3} \).

Along the last years the non-extensive approach to seismic-related-processes has been used in different seismic regions in USA, Spain, Turkey, Japan, or Italy (e.g. Sotolongo-Costa and Posadas, 2004; Silva et al., 2006; Darooneh and Dadashinia, 2008; Darooneh and Mehri, 2010; Darooneh and Mehri, 2010); Eftaxias, 2010; Sarlis et al., 2010). Results of these analyses indicate that the values of the non-extensivity parameter \( q \) seem to be universal and, in general, lie close to the range 1.6–1.7 (Vilar et al., 2007; Sarlis et al., 2010; Telesca, 2010c), though much larger and smaller values have been also reported (Telesca, 2010a, b; Darooneh and Mehri, 2010). On the other hand, energy density parameter \( a \) measured at different seismically active parts of the globe, varied by several orders of magnitude (Vilar et al., 2007; Darooneh and Dadashinia, 2008).

These results indicate that further investigation is needed to gain information on the non-extensive dimensions of earthquake generation at different spatial and temporal scales worldwide. In this respect it should be pointed that earthquakes in the highly seismically active Caucasian region have not yet been investigated adopting a non-extensive approach. This is why in this research we aimed to fill this gap by performing a non-extensive analysis of earthquake magnitude distribution in the Javakheti uplands, one of the most seismically active areas in the Southern Caucasus.
larger than a given $m$ was fitted to the Eq. (1). In the present study, the Levenberg–Marquardt (LM) nonlinear fitting method was used. The LM method is known to be an efficient and accurate fitting procedure that is often used for different purposes (Gallant, 1975; Bates and Watts, 1988). In our analysis fitting quality to real data was satisfactory, with a fitting accuracy of 0.94–0.98 and a standard error in the range of 0.04–0.06.

3. Results and discussion

As it was mentioned above all our datasets were analyzed adopting the non-extensive approach. Fig. 3 shows the results of the computed cumulative distribution in the Javakheti area for the whole period of analysis, a typical distribution of the number of events (black triangles) with magnitude $m$ greater than the threshold $M$ (here $M_{1.6}$) normalized to the total number of events. The dotted line represents the best-fit model function. In this case, and for all other catalogs and timeframes of analysis, we observed an agreement of Eq. (1) with field data.

The main goal of the analysis was to investigate the variation of non-extensivity parameters $q_i$ and energy density characteristics $a_i$ of seismicity in the region of Javakheti for different threshold magnitudes and periods of observation. The analysis performed with different thresholds is very relevant for it because enables to assess the stability of measured non-extensivity and energy density attributes of the seismic process.

The variation of parameters $q_i$ and $a_i$ are presented in Figs. 4 and 5, respectively, for different timeframes and thresholds. In general, the observed variation of non-extensivity parameter $q_i$ is in the range of 1.6–1.7, found worldwide as mentioned. As it follows from the results, the first significant shift in the
variation of the non-extensivity parameter took place at the 10-year window that terminated in 1978, when the first M5.3 earthquake (one of three significant strong local events) struck the Javakheti area. Notably, the non-extensivity value rose to \(q_{10}=1.67\). This increase is observed at all threshold magnitudes and 10-year span windows. The next substantial shift in the variation of the non-extensivity parameter (\(q_{18}=1.7\)) took place at the 10-year window ending in 1986, when the strongest (M5.6) earthquake occurred in the area. At this and the following window, terminating in 1987, the non-extensivity parameter increases to the highest value (\(q=1.71\)) found in our analysis using 10-year windows.

During the two succeeding 10-year span windows \(q\) slightly decreased. It is interesting to note that in this period one of the strongest regional earthquakes (M6.9, Spitak, 07.12.1988), with its associated foreshocks and aftershocks, occurred 12 km from the southern edge of the area. Following this reduction, the six consecutive 10-year span windows show an increase of the non-extensivity parameter \(q\) up to the highest values (\(q=1.71\)). This rise starts in the window with the third strongest (M5.1) earthquake, occurred in 1990. It is worth to mention that this period, characterized by increased regional seismic activity, contains the second strongest event of the Southern Caucasus (M6.9, Racha earthquake, 29 April 1991), 95 km from the northern edge of the investigated area. In the next four 10-year windows, \(q\) values sharply fall relative to those observed at the initial windows (\(q_{10}=1.65\)). The lowest values of the non-extensivity parameter (\(q_{35}=1.6\)) were obtained for successive five 10-year span windows terminated in 2000 through 2004 consecutively. The gradual rise of the non-extensivity parameter was documented since 2002, when the minimal \(q_{35}=1.6\) value (for \(M=1.6\) threshold) was identified. This increase was relatively slow till 2004 but then accelerated and, in the window ending in 2008, we obtained \(q_{39-40}=1.6\) values close to those at the start of the analysis in the 1960s. It may be pointing to an escalation in the seismic activity in the Caucasus that was being documented by instrumental observations in the last decade. Two earthquakes of M5.1 and M6.0 occurred in 2006 and 2009, respectively, in the Greater Caucasus.

It is necessary to bear in mind that, in general, the variation of the non-extensivity parameter does not depend on the threshold magnitude adopted. Though some minor quantitative differences arise when the completeness of the catalog is neglected or when all registered events are included. This proves the stability of the estimation of \(q\) in our analysis, rooted in the scale-free nature of seismic processes. It seems that the behavior of the parameter \(q\) somehow recalls feature variation of \(b\) in the GR equation. In fact, a significant premonitory decrease of the \(b\) value has been observed before large earthquakes (Hainzl et al., 2003; Lei and Satoh, 2007).

The decrease of the non-extensivity parameter \(q\) observed during relatively quiet time windows (when small magnitude earthquakes occur), may point to the decreased order within the system of fault fragments in the course of strong earthquake generation, when the amount of accumulated stress is not yet enough to initiate a correlated behavior of the whole system (Chelidze and Matcharashvili, 2007). The situation changes when a strong earthquake occurs and much more correlated behavior of the system constituents is assumed to take place; the emergence of short and long range correlations is reflected in an increase of the non-extensivity parameter \(q\).

As opposed to the variation of parameter \(a\), the value \(a\) – a physical quantity characterizing the energy density (Silva et al., 2006) – rises when the seismic activity in the area decreases, and vice versa. As seen in Figs. 4 and 5, the variation of \(q\) mirrors \(q\) behavior. However, essential differences in the variation of these

![Fig. 3](image-url) Cumulative magnitude distribution of earthquakes vs. magnitude in the Javakheti area from 1960 to 2008 (crosses) and its fit to Eq. (1) (dotted line). Minimal threshold magnitude is M1.6.

![Fig. 4](image-url) Variation of the non-extensivity parameter \(q\) in the Javakheti area between 1960 and 2008 (all magnitudes represented with asterisks, \(M > 1.6\) with circles, and \(M > 2.2\) – with triangles), calculated for consecutive overlapping 10-year sliding windows with 1-year shift (symbols show terminal years of sliding windows).
characteristics have been found depending on the threshold magnitude. As expected, the energy density characteristic value increases at higher threshold values, because $a$ is the coefficient of proportionality between fragment size and released energy in the fragment–asperity model (Silva et al., 2006).

Since the amount of released energy rises with increased seismic activity, the observed decrease of $a$ values at all magnitude thresholds may point to a prevalent contribution of large size fragments in the fragment–asperity interaction. Thus, it can be concluded that decreases of $a$ are associated to activated movements of large size fragments, as opposed to quiet periods, when accumulated stress energy is mostly released through the relative movement of small fragments.

Next we study the non-extensive attributes for the whole time period between 1966 and 2008. As shown in Fig. 3, there is a good accordance between the modified GR law – Eq. (1) – and field data. It also follows that, for the Javakheti highlands, the non-extensive modification of the GR law differs from the standard pattern of the GR relationship at smaller magnitudes, similar to what has been found for other catalogs worldwide Darooneh and Mehri, 2010, and to the GR relationship at larger magnitudes starting about $M=2.2$.

The results of the analysis using 10-year span time windows ($q_i$) significantly differ from the value of the non-extensivity parameter ($q_w=1.81$) for the whole time set of observations. This value (distinctly large as compared to what has commonly been reported, in the 1.6–1.7 range) is close to the maximum ever detected, reported recently for California (Darooneh and Mehri, 2010). At the same time –as already explained above- the non-extensivity phenomenological parameters of $q_w$, calculated for shorter time periods, are in the range often found for different seismic catalogs worldwide (Vilar et al., 2007; Darooneh and Mehri, 2010; Telesca, 2010a). On the other hand, the energy density value $a_{w}=1.1 \times 10^6$, found in the Javakheti region for the whole time period of observation, is larger than those reported earlier for California and Iran Darooneh and Mehri, 2010, though it is very close to values recently found for the whole Italian seismic catalog (Telesca, 2010a).

According to our results (Fig. 5), the values of $a_w$ calculated using shorter windows notably change in the wide range. These changes in the energy density of seismic processes are observed for all thresholds. As said above, differences are observed in particular when the seismic activity in the Javakheti and adjacent area in Southern Caucasus markedly increased (1986–1992).

The difference between $q_w$ and $a_w$ values estimated for the whole time period in Javakheti, and the $q_i$ and $a_i$ values for consecutive 10-year windows may be explained by the integral nature of the first ones. Generally, a seismically active area with complex faulting and fault fragment interactions, may remain quiet or with a relatively low seismic activity for a considerably long period of time. This may lead to averaging calculated non-extensivity and energy density characteristic values for long time periods, observed both for $q_w$ and $a_w$.

4. Conclusions

In this research the seismically active area of Javakheti, in southern Caucasus, was studied from 1960 to 2008, adopting a non-extensive statistical approach. The analysis was performed using different magnitude threshold values, both for the whole period of observation and for consecutive 10-year span time windows. A value of the non-extensivity parameter $q=1.81$ was found for the whole time frame of observation, larger than what has been reported for most of the seismically active regions of the globe. At the same time non-extensivity parameters calculated for shorter 10-year sliding windows fall within the range (1.6–1.7) reported worldwide. A significant increase of the non-extensivity parameter was observed for those 10-year periods with strong earthquake events.

We suggest that variation of the non-extensivity parameter $q$ is related with changes in the long- and short-range correlation in the system ‘fault-fragments’ during the stages of strain accumulation and shock. The behavior of the energy density characteristic value $a$ almost mirrors the variation of parameter $q$. In the Javakheti area it is large during seismically quiet periods and decreases during periods of strong seismic activity.

We suppose that decreases in energy density value $a$ may evidence a prevalent role of large size fragments in fragment-fault interactions under the influence of a rapidly released stress during strong seismic events. Contrarily, in relatively quiet intervals accumulated stress energy seems to be released mostly through the relative movement of smaller fragments.
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