Nat Hazards (2012) 60:501–515
DOI 10.1007/s11069-011-0026-z

ORIGINAL PAPER

Assessment of uncertainties related to seismic hazard using fuzzy analysis

N. Jorjiashvili · T. Yokoi · Z. Javakhishvili

Received: 29 June 2009 / Accepted: 31 October 2011 / Published online: 12 November 2011

Abstract Seismic hazard analysis in the last few decades has become a very important issue. Recently, new technologies and available data have been improved that have helped many scientists to understand where and why earthquakes happen, the physics of earthquakes, etc. Scientists have begun to understand the role of uncertainty in seismic hazard analysis. However, how to handle existing uncertainty is still a significant problem. The same lack of information causes difficulties in quantifying uncertainty accurately. Usually, attenuation curves are obtained in a statistical manner: regression analysis. Statistical and probabilistic analyses show overlapping results for the site coefficients. This overlapping takes place not only at the border between two neighboring classes but also among more than three classes. Although the analysis starts from classifying sites using geological terms, these site coefficients are not classified at all. In the present study, this problem is solved using fuzzy set theory. Using membership functions, the ambiguities at the border between neighboring classes can be avoided. Fuzzy set theory is performed for southern California in the conventional way. In this study, standard deviations that show variations between each site class obtained by fuzzy set theory and the classical manner are compared. Results of this analysis show that when we have insufficient data for hazard assessment, site classification based on fuzzy set theory shows values of standard deviations less than those obtained using the classical way, which is direct proof of less uncertainty.

Keywords Seismic hazard · Uncertainty · Fuzzy analysis · Attenuation relation

N. Jorjiashvili (✉) · Z. Javakhishvili (✉)
Institute of Earth Sciences, Ilia State University, Tbilisi, Georgia
e-mail: nato_jorjiashvili@iliauni.edu.ge

Z. Javakhishvili
e-mail: zurab_javakhishvili@iliauni.edu.ge

T. Yokoi
International Institute of Seismology and Earthquake Engineering,
Building Research Institute, Tokyo, Japan
e-mail: tyokoi@kenken.go.jp

1 Introduction

An estimation of expected ground motion is fundamental for earthquake hazard assessment. Generally, ground motion and damage are influenced by the magnitude of the earthquake, the distance from the seismic source to the site, the local ground conditions, and the characteristics of buildings (Atkinson 2004). Empirical attenuation relation, a practical way to estimate ground motion parameters, gives information about how these parameters depend on the above-mentioned source, path, and site effects (Field 2000). This, namely ground condition, must be considered, because the same earthquake recorded at the same distance may cause different damage according to ground conditions (Irikura et al. 2004).

In conventional approaches, grouping or classification is done for strong motion observation sites based on a priori given information expressed by geological terms (Wills et al. 2000). Then, regression analyses are applied to the ground motion parameters (peak ground acceleration (PGA), response spectra (SA), etc.) observed at each of these classes (Field 2000). Although amplification factors are assigned to all stations used in analyses, the majority of the sites in that seismic hazard are assessed; but, any ground motion recorded, which is not available, does not have this assigned value. Therefore, the mean value of each class has to be used for them in order to make a seismic hazard assessment.

The usage of the mean value, however, brings an uncertainty for site amplification into the obtained seismic hazard. Its cause is a widely scattered amplification factor, even in the same site class. Molas and Yamazaki (1995), for example, show the list of site factors for PGA. Significant overlap of widely scattered distribution is observed among different classes. It can be interpreted that the classification in geological terms done prior to the regression analysis may be incorrect, that is, the data scattered far from the mean value should be reclassified into another, because the borders between classes are essentially ambiguous, for example, due to weathering, etc. This can also cause one to misjudge the site classification when attenuation relation is applied to sites that do not have any ground motion record. The reliability of judging varies site to site, depending on the available information, that is, fresh cut along a road in a site and ground covered by newly filled sand in another site. It is possible to imagine that there exists a level of uncertainty, depending on available information at each site.

In the present study, fuzzy set theory is applied in order to handle the uncertainty caused by the site classification based on geological terms and its level of uncertainty.

2 Method of analysis: fuzzy set theory

2.1 Basics of fuzzy set theory

Fuzzy set theory is an extension of conventional (crisp) set theory. It handles the concept of partial truth (truth values between 1 (completely true) and 0 (completely false)). It was introduced by Prof. Lotfi A. Zadeh of UC Berkeley in 1965 as a means to model vagueness and ambiguity in complex systems (Zadeh 1965).

2.1.1 The membership function

For the universal X and given the membership degree function $\mu \rightarrow [0, 1]$, the fuzzy set A is defined as $\tilde{A} = \{[x, \mu_A(x)] \mid x \in X\}$. The membership function $\mu_A(x)$ quantifies the grade of membership of the elements x to the fundamental set. The value 0 means that the member is not included in the given set; 1 describes a fully included member. The values between 0 and 1 characterize fuzzy members (De Campos and Bolaños 1989).

2.1.2 Some operations and arithmetic on fuzzy sets and numbers

Let A and B be two fuzzy subsets of universal set X and $a(x)$ and $b(x)$ corresponding degree of membership of x in A and B, respectively. Then following operations on sets can be defined (Schmucker 1984).

$$\text{Intersection:}\quad A \cap B = \{\min[a(x), b(x)]\ /\ x \mid x \text{ is an element of } X\}, \tag{1}$$

$$\text{Union:}\quad A \cup B = \{\max[a(x), b(x)]\ /\ x \mid x \text{ is an element of } X\}. \tag{2}$$

The support of a fuzzy set A is the ordinary subset of X (Dubois and Prade 1988):

$$\text{supp}\, A = \{x \in X, \mu_A(x) > 0\}, \tag{3}$$

When we want to exhibit an element $x \in X$ that typically belongs to a fuzzy set A, we may demand its membership value to be a greater than some threshold $\alpha \in [0, 1]$. The ordinary set of such elements is the α-*cut* A_α of A

$$A_\alpha = \{x \in X, \mu_A(x) \geq \alpha\}, \tag{4}$$

The membership function of a fuzzy set A can be expressed in terms of the characteristic functions of its α-*cuts* according to the formula (Dubois and Prade 1988):

$$\mu_A(x) = \sup_{\alpha \in [0,1]} \min\left[\alpha, \mu_{A_\alpha}(x)\right], \tag{5}$$

where sup means the supremum, that is, the least upper bound.

$$\mu_{A_\alpha}(x) = \begin{cases} 1 & \text{if } x \in A_\alpha, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

A fuzzy subset A of a classical set X is called normal if there exists $x \in X$ such that $\mu_A(x) = 1$. Otherwise A is subnormal. An α-level set of a fuzzy set A of X is a non-fuzzy set denoted by $[A]^\alpha$ and is defined by

$$[A]^\alpha = \begin{cases} \{t \in X \mid A(t) \geq \alpha\} & \text{if } \alpha > 0, \\ cl(\text{supp}\, A) & \text{if } \alpha = 0, \end{cases} \tag{7}$$

where $cl\,(\text{supp}\, A)$ denotes the closure of the support of A (the smallest set which is closed with respect to some operations in set theory as it is known in classical set theory). A fuzzy set A of X is called convex if $[A]^\alpha$ is a convex subset of X, $\forall \alpha \in [0, 1]$ (Carlsson Ch and Fullér 2000).

A fuzzy number A is a convex, normalized fuzzy set whose membership function is at least segmentally continuous and has the functional value $\mu_A(x) = 1$ at precisely one element.

For the fuzzy number A, the interval of confidence for the level of presumption α $\alpha \in [0, 1]$ can be defined as follows:

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha, \alpha \in [0, 1]\}, \tag{8}$$

Let A and B be two fuzzy numbers and A_α and B_α their intervals of confidence for the level of presumption α $\alpha \in [0, 1]$. Then the addition can be defined as follows:

$$A_\alpha(+)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right](+)\left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right], \tag{9}$$

and subtraction can be written as follows:

$$A_\alpha(-)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right](-)\left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} - b_1^{(\alpha)}, a_2^{(\alpha)} - b_2^{(\alpha)}\right]. \tag{10}$$

Suppose we have n fuzzy numbers as $A_i \in U$, $i = 1, 2, 3, \ldots, n$, then

$$A_{i,\alpha} = \left[a_{1,i}^{(\alpha)}, a_{2,i}^{(\alpha)}\right], \quad \alpha \in [0, 1], \tag{11}$$

for the interval of confidence at the level α of A_i. Then, defining the mean as

$$a_1^{m\,(\alpha)} = (1/n)\sum_{i=1}^{n} a_{1,i}^{(\alpha)} \tag{12}$$

$$a_2^{m\,(\alpha)} = (1/n)\sum_{i=1}^{n} a_{2,i}^{(\alpha)} \tag{13}$$

the mean interval of confidence at the level of α of the mean fuzzy number A^m is described as

$$A_\alpha^m = \left[a_1^{m(\alpha)}, a_2^{m(\alpha)}\right]. \tag{14}$$

(14) and is called the fuzzy expected average (Kaufmann and Gupta 1985).

If R_i and W_i are a sequence of fuzzy sets, then the fuzzy weighted average of the R_i's using W_i's as weights is defined as follows:

$$R = \frac{\sum_{i=1}^{n}(R_i * W_i)}{\sum_{i=1}^{n} W_i}. \tag{15}$$

The fuzzy arithmetic operations, summation, multiplication, and division that are used in Eq. 15 are defined as follows (Schmucker 1984):

If

$$X = \{x(i)/i; 1 \le i \le n\}, \tag{16}$$

$$Y = \{y(i)/i; 1 \le i \le n\}, \tag{17}$$

where i, j, and n are integers; $x(i)$ and $y(i)$ are membership functions that characterize the fuzzy sets X and Y, respectively, then the fuzzy addition is defined as follows:

$$X + Y = \{\min(x(i), y(i))/(i + j); 1 \le i, j \le n\}, \tag{18}$$

The fuzzy summation is a repeated process of the fuzzy addition.

The fuzzy multiplication is defined as follows:

$$X * Y = \{\min[x(i), y(i)]/(i * j); 1 \le i, j \le n\}, \tag{19}$$

and the fuzzy division is defined as:

$$X/Y = \{\min[x(i), y(i)]/(i/j); 1 \le i, j \le n\}. \tag{20}$$

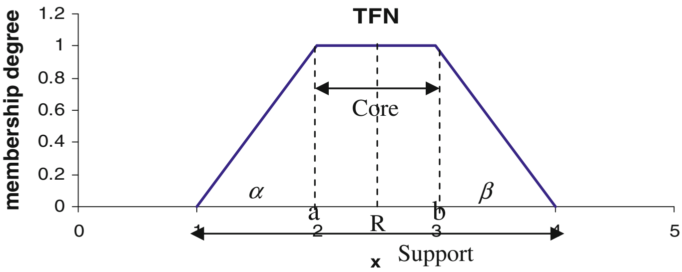

Fig. 1 Trapezoidal fuzzy number (TFN) and its centroid

2.1.3 Ranking method

The ranking method implemented in this study is based on a scoring function that measures a TFN's (trapezoidal fuzzy number) center of gravity (centroid) along the *x*-axis (Fig. 1).

For fuzzy numbers with a non-zero area, the score is calculated as follows:

$$R = R(TPZ(a, b, \alpha, \beta)) = a + \frac{b-a}{2} + \frac{\left(1 - \frac{1}{\sqrt{2}}\right)\left(\frac{\beta^2}{2} - \frac{\alpha^2}{2}\right)}{b - a + \frac{\alpha}{2} + \frac{\beta}{2}}, \tag{21}$$

where (a, b) is the core and α and β are the left and right bandwidth, respectively.

2.1.4 Expected value and standard deviation for trapezoidal fuzzy number

Let A be a fuzzy number with $[A]^\alpha = [a_1(\alpha), a_2(\alpha)]$, $\alpha \in [0, 1]$. The possibilistic (crisp) mean (or expected) value of A is defined as follows Eq. 22:

$$E(A) = \int_0^1 \alpha[a_1(\alpha) + a_2(\alpha)]\,\mathrm{d}\alpha = \frac{\int_0^1 \alpha \cdot \frac{a_1(\alpha)+a_2(\alpha)}{2}\,\mathrm{d}\alpha}{\int_0^1 \alpha\,\mathrm{d}\alpha}, \tag{22}$$

i. e., $E(A)$ is nothing but the level-weighted average of the arithmetic means of all α-level sets.

The variance (possibilistic) of A is defined as follows (23):

$$\sigma^2(A) = \int_0^1 \alpha\left(\left[\frac{a_1(\alpha)+a_2(\alpha)}{2} - a_1(\alpha)\right]^2 + \left[\frac{a_1(\alpha)+a_2(\alpha)}{2} - a_2(\alpha)\right]^2\right)\mathrm{d}\alpha$$
$$= \frac{1}{2}\int_0^1 \alpha[a_2(\alpha) - a_1(\alpha)]^2\mathrm{d}\alpha. \tag{23}$$

i. e., the possibilistic variance of A is defined as the expected value of the squared deviations between the arithmetic mean and the endpoints of its level sets.In case of $A = (a, b, \alpha, \beta)$ trapezoidal fuzzy number

$$E(A) = \int_0^1 \gamma[a - (1-\gamma)\alpha + b + (1-\gamma)\beta]\mathrm{d}\gamma = \frac{a+b}{2} + \frac{\beta - \alpha}{6} \tag{24}$$

and

$$\sigma^2(A) = \frac{(b-a)^2}{4} + \frac{(b-a)(\alpha+\beta)}{6} + \frac{(\alpha+\beta)^2}{24}. \tag{25}$$

where γ corresponds to α-level in Eqs. 22 and 23. Standard deviation is obtained as follows:

$$\sigma = \sqrt{\sigma^2(A)}. \tag{26}$$

2.1.5 Uncertainty scaling for reliability of classification

In the linguistic approach to decision analysis, the use of linguistic labels implies a level of uncertainty. The quantitative uncertainty is created by uncertainty in measurement or estimation of the source variable on which the label is placed, which also directly impacts the decision process (Debruin and Bregt 2001). In the case of environmental value, for example, the estimate provided is usually based on incomplete information, due to our limited understanding of the complex nature of natural systems (Christensen et al. 1996). The question arises how to include quantitative uncertainty into the analysis process .The operation can be described as follows:

$$U = \begin{pmatrix} \chi(a' - (d' - a')(u_n - u_b)), \chi(b' - (c' - b')(u_n - u_b)) \\ \chi(c' + (c' - b')(u_n - u_b)), \chi(d' + (d' - a')(u_n - u_b)) \end{pmatrix}, \tag{27}$$

where $a' = a - \alpha$; $b' = a - \alpha$; $c' = b$; $d' = b + \beta$, a, b, α, β are shown in Fig. 1. Schematically, uncertainty scaling operation is shown in Fig. 2.

U is the uncertainty scaling operation; u_n is the uncertainty term used in the scaling operation; u_b is the base uncertainty term (the term which represents the inherent quantitative uncertainty in the u_n-scaled term). In this study, we chose $u_b = 0.5$,

$$\chi(x) = \begin{cases} 1 & \text{if } x \geq 1 \\ 0 & \text{if } x \leq 0 \\ x & \text{otherwise} \end{cases}. \tag{28}$$

3 Data used in the present study

Data used here were downloaded via the Internet from the COSMOS VIRTUAL DATA CENTER (http://db.cosmos-eq.org/scripts/adv_search.plx, hereafter written CVDC) for

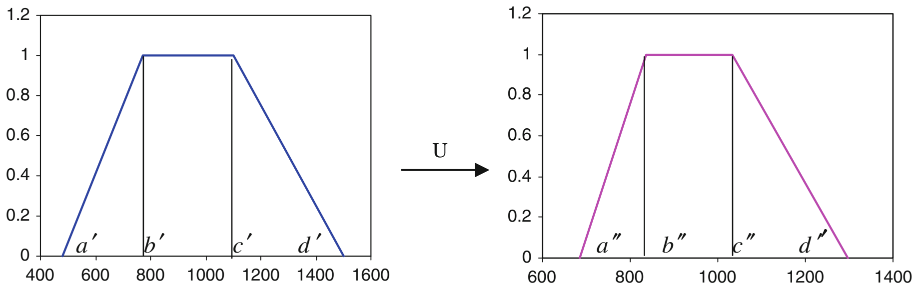

Fig. 2 Graphical representation of uncertainty scaling

California (latitude range between 32 N and 36 N), for 281 stations in southern California and 28 earthquakes with magnitudes larger than 5.0 determined between 32 N and 36 N. In this database, the 1994 Northridge earthquake is also included ($M = 6.7$) (Steidl and Lee 2000). There, stations are classified by their surface geology as summarized in Table 2. In the present study, BC class (Wills et al. 2000) is recognized in the possible range (left bandwidth) of B class here. At the same time, it is recognized in the possible range (right bandwidth) of C class. The same treatment is done for CD and DE classes.

Field (2000) obtained attenuation relations for PGA and SA at the period equal to 0.3, 1.0, and 3.0 s as follows.

$$\mu\left(M, r_{jb}, V_s\right) = b_1 + b_2(M-6) + b_3(M-6)^2 + b_5 \ln\left(\left(r_{jb}^2 + h^2\right)^{0.5}\right) + b_v \ln(V_s/V_a), \tag{29}$$

where μ is the predicted natural logarithm of the ground motion parameter (e.g., $\ln(SA)$), M the moment magnitude, r_{jb} the closest distance to the vertical projection of rupture (in km), and V_s the average 30-meter shear wave velocity at the site (m/s), the values of which are listed in Table 2. V_a is an arbitrary reference site velocity; here it is taken as 760 m/s in all cases, and h is a fictitious depth needed to prevent zero distance. The other parameters (b_1, b_2, b_3, b_5, h, b_v, and V_a) are solved by regression analysis from empirical data (Field 2000).

4 Application of fuzzy set theory

4.1 Designing of membership function

In the present study, non-symmetric linear trapezoidal membership functions are used for simplicity in order to capture the vagueness in geological classification. A standard linear trapezoidal fuzzy number (TFN) can be represented completely by a quadruplet $Tpz(a, b, \alpha, \beta)$ as shown in Fig. 1. It is also useful that crisp numbers ($a = b$, $\alpha = \beta$) can be represented in trapezoidal form (Bailey 2003).

In the CVDC database, average shear wave velocity to 30 m (V_S) is assigned to site classes as an indicator of site response (Table 1). The legend in geological terms of the site classes is found in the paper of Wills et al. (2000). Then, V_S is selected as the variable parameter for membership functions.

First, the distribution function against V_S is made for each site class of the present study, and the mean values μ and the standard deviations σ in statistical (classical) meaning and $f(x)$ the distribution function normalized by its value at μ are calculated.

Then, the membership function $q(V_s)$ is given as follows.

$$q(V_s) = 1.0 \quad \text{for} \ \frac{\mu - V_{s-}}{\mu - V_{s\,\min}} < 0.1, \quad \frac{V_{s+} - \mu}{V_{s\,\max} - \mu} < 0.1, \tag{30}$$

where V_{s-} denotes shear wave velocity values on the left side of the mean value, V_{s+} denotes shear wave velocity values on the right side of the mean value, $V_{s\,\min}$ means the minimum value of shear wave velocity obtained in the certain site class, and $V_{s\,\max}$ the maximum value for the same site class. From Eq. 30 , the uncertainty range can be determined (S_I^{c-}, S_I^{c+}).

Table 1 Summary of the data used were downloaded from COSMOS VIRTUAL DATA CENTER (http://db.cosmos-eq.org/scripts/adv_search.plx) with the parameters mentioned above

Site class CVDC	Number of stations	Number of stations with Vs	Mean value (m/s)	Standard deviation (m/s)
A	8	8	1,857.6	256
B	17	17	871.4	238
BC	47	47	728	209.2
C	132	132	470.8	139
CD	180	180	368	96
D	120	120	286	68
DE	45	45	226	36
E	4	4	157	45

The legend of site classes with geological terms (Wills et al. 2000)

Table 2 Site classes are used here in relation with others. The mean values and the standard deviations are calculated newly for the site classes of the present study

Present study			(Wills et al. 2000)				V_s (m/s) NEHRP	V_s (m/s) used by Field (2000)
Site class	Mean value (m/s)	Standard deviation (m/s)	Site class	*Vs* (m/s) Expected value	Mean value	Standard deviation		
A	1,857.6	256	A	N/A	N/A	N/A	$1{,}500 < V_s$	N/A
B	1,066	391	B	$760 < V_s$	686	275	$760 < V_s < 1{,}500$	1,000
			BC	$555 < V_s < 1{,}000$	724	368	N/A	760
C	471	219	C	$360 < V_s < 760$	464	147	$360 < V_s < 760$	560
			CD	$270 < V_s < 555$	372	98	N/A	360
D	285	76	D	$180 < V_s < 360$	301	104	$180 < V_s < 360$	270
			DE	$90 < V_s < 270$	298	225	N/A	180
E	157	45	E	Vs < 180	163	31	$V_s < 180$	N/A

$$
\begin{aligned}
q(V_s) &= \frac{0.9\left(x - S_I^{c-}\right)}{S_I^{c-} - f^{-1}(0.1)_-} + 1 \quad \text{for } f^{-1}(0.1)_- < x < S_I^{c-}, \\
q(V_s) &= \frac{-0.9\left(x - S_I^{c+}\right)}{f^{-1}(0.1)_+ - S_I^{c+}} + 1 \quad \text{for } S_I^{c+} < x < f^{-1}(0.1)_+, \\
q(V_s) &= 0.0 \ \text{ otherwise.}
\end{aligned}
\tag{31}
$$

where $f^{-1}(0.1)_-$ and $f^{-1}(0.1)_+$ denote the values of x at the left and right sides of μ, both of them that give 0.1 for the normalized distribution function $f(x)$. The data of which V_S is located between S_I^{c-} and S_I^{c+} are recognized "true", whereas those outside of the range limited by $f^{-1}(0.1)_-$ and $f^{-1}(0.1)_+$ are "false". Those in the ranges between $f^{-1}(0.1)_-$ and S_I^{c-}, and between S_I^{c+} and $f^{-1}(0.1)_+$ are "partially true–partially false." The linear trend line is added on the left side from the S_I^{c-} and on the right side from the S_I^{c+} in order to obtain trapezoidal membership function.

Table 3 Parameters of TFN for five site classes

Site class	S^c_{Imin}	S^{c-}_I	S^{c*}_I	S^{c+}_I	S^c_{Imax}
A	1,100	2,000	2,025	2,050	2,100
B	480	770	935	1,100	1,500
C	350	370	425	480	770
D	175	270	310	350	370
E	100	160	167.5	175	270

Table 4 Site class index and standard deviation for all site classes for classical and fuzzy case for $T = 1.0$ s

Site class	Values	Classical	By centroid of trapezoidal function
A class	Site class index (m/s)	1,857.6	1,799.8
	Stand. deviat. (m/s)	256	214.8
B class	Site class index (m/s)	1,066	966.5
	Stand. deviat. (m/s)	391	291
C class	Site class index (m/s)	471	490.1
	Stand. deviat. (m/s)	219	132.7
D class	Site class index (m/s)	285	273.4
	Stand. deviat. (m/s)	76	53
E class	Site class index (m/s)	157	159.9
	Stand. deviat. (m/s)	45	36

Giovinazzi and Lagomarsino (Giovinazzi and Lagomarsino 2004) express the four corners of TFN using S^c_{Imin}, S^{c-}_I, S^{c+}_I, S^c_{Imax} that correspond $a - \alpha$, a, b, $b + \beta$ in Fig. 1, respectively. S^c_{Imin} and S^c_{Imax} are called the lower and the upper bounds of the possible range. S^{c-}_I and S^{c+}_I are called the lower and the upper bounds of the uncertainty range. The most probable value S^{c*}_I is obtained as the mean value of the trapezoidal membership function as shown in Eq. 24. Tables 3 and 4 shows these five parameters for the five classes used in the present study.

The obtained membership function (Figs. 3, 4) for each site class looks as shown in Fig. 5.

First, for site class indexes, again centroids of trapezoidal membership functions are used shown in Eq. 21.

4.2 Uncertainty scaling

Once we obtain trapezoidal membership function for each site class, then we can use the uncertainty scaling operation for reliability of classification. Using such scaling operation, we can obtain different membership functions that correspond to each level of uncertainty as shown in Fig. 5. Then station coefficients can be obtained by several ways, depending on our purpose, which are shown later based on some examples.

Our purpose is to handle uncertainty. For this purpose, we use the uncertainty scaling. We choose a different level of uncertainty, such as “very uncertain,” “uncertain,”

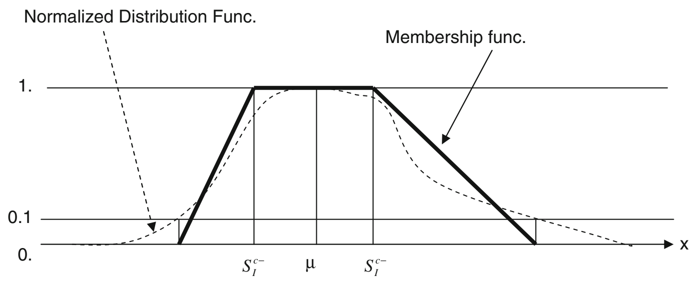

Fig. 3 Schematic figure for the process to obtain membership function based on the normalized distribution function

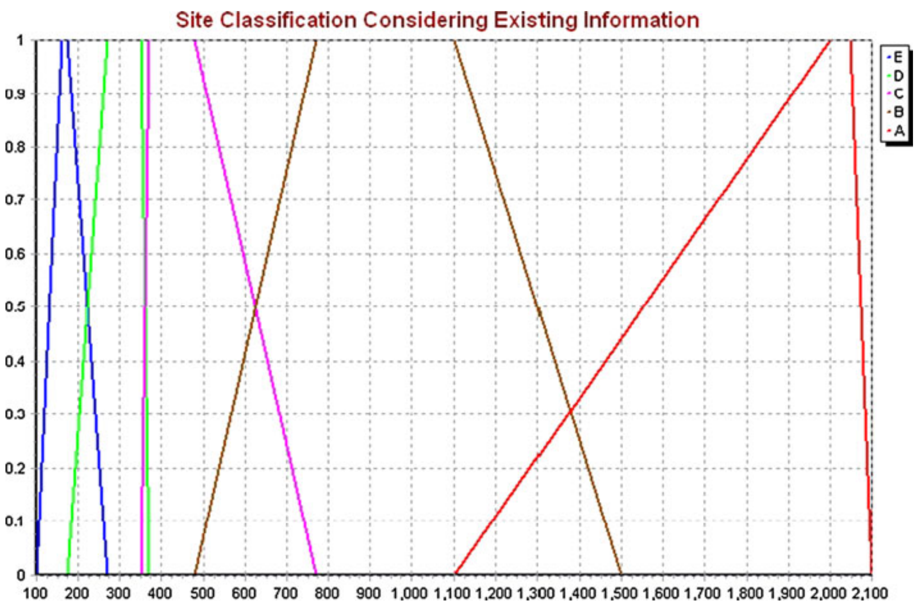

Fig. 4 Site classification considering shear wave velocity values

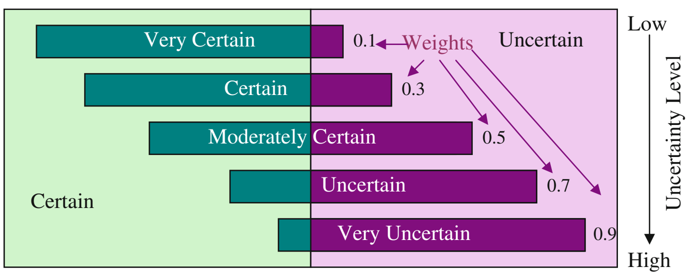

Fig. 5 Uncertainty scaling

"moderately certain," "certain," and "very certain," and give them some weights due to treatment of uncertainty. Then we can choose a specific level of uncertainty and perform fuzzy analysis for this level; or, we can consider some levels of uncertainty, neglect others, or otherwise leave only a certain level. We have many options depending on our purpose.

Then fuzzy analyses were applied for second type classification.

Then we define uncertainty semantically, as shown in Fig. 5. Then we use scaling operation shown in Eqs. 27 and 28. In those equations, instead of a', b', c', d', values of the membership function are used as an argument of χ function; for example, for B site class, we can get uncertainty scaling as follows:

For "very certain" case (weight: $u_n = 0.1$)

$$U(u_n) = (888,\ 902, 968,\ 1092), \tag{32}$$

and as an interval:

$$A_{0.1}(\alpha) = [14.001\alpha + 888.36; -123.46\alpha + 1087.22], \tag{33}$$

For "certain" case (weight: $u_n = 0.3$)

$$U(u_n) = (684, 836, 1034, 1296), \tag{34}$$

and as an interval:

$$A_{0.3}(\alpha) = [151.52\alpha + 681.82; -263.158\alpha + 1301.74], \tag{35}$$

For "moderately certain" case (weight: $u_n = 0.5$)

$$U(u_n) = (486, 770, 1100, 1500), \tag{36}$$

and as an interval:

$$A_{0.5}(\alpha) = [294.12\alpha + 486.82; -400\alpha + 1500], \tag{37}$$

For "uncertain" case (weight: $u_n = 0.7$)

$$U(u_n) = (276, 704, 1166, 1704), \tag{38}$$

and as an interval:

$$A_{0.7}(\alpha) = [434.78\alpha + 280.39; -526.32\alpha + 1667], \tag{39}$$

and for "very uncertain" case (weight: $u_n = 0.9$)

$$U(u_n) = (72, 638, 1232, 1908), \tag{40}$$

and as an interval:

$$A_{0.9}(\alpha) = [555.56\alpha + 70.67; -666.67\alpha + 1881.67], \tag{41}$$

where α is a level of confidence and $\chi(x) = \begin{cases} 1 & \text{if } x \geq 1 \\ 0 & \text{if } x \leq 0 \\ x & \text{otherwise} \end{cases}$.

Figure 6 shows uncertainty scaling for B site class. The blue line shows the membership function obtained by the second type of fuzzy classification. Again, as with other cases, "moderately certain" corresponds to this classification itself.

Site class indexes for the other two cases are performed in two different ways. One of them is a combined approach, considering only from "very certain" up to "moderately

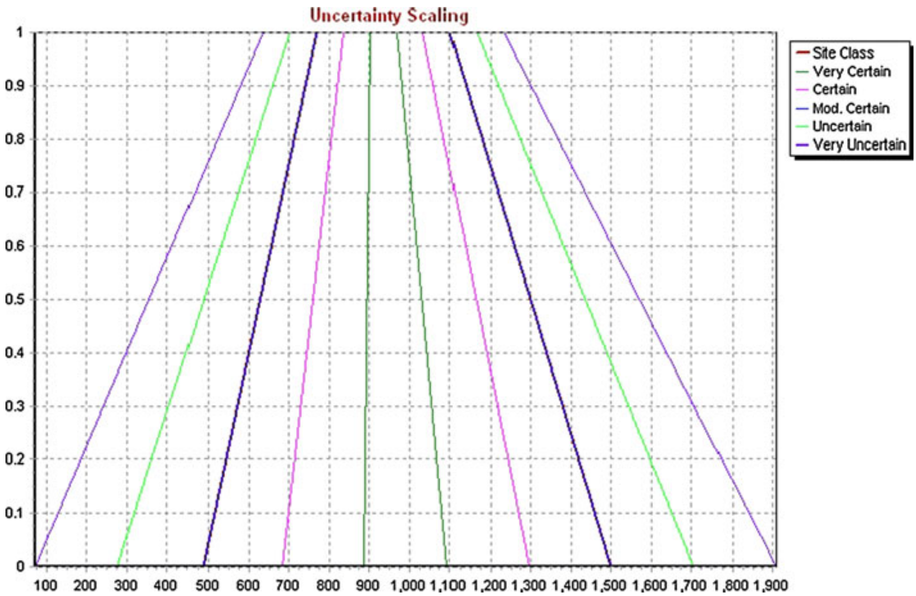

Fig. 6 Uncertainty scaling for B site class

Table 5 Expected values for each term and all site classes for SA for California

	Site classes	Membership	Very certain	Certain	Mod. certain
Expected values	A class	1,799.8	1,695.8	1,721	1,799
	B class	966.9	960.4	963.9	966.9
	C class	490.1	493.8	492	489.7
	D class	273.4	265.8	269.6	272.8
	E class	159.9	165.1	163.4	161

certain" levels of uncertainty. Fuzzy intervals of each level of confidence from 0 up to 1 are also used here. Then expected and maximum values are calculated (Table 5). We use both of these values for the calculation of the site class index (Kaufmann 1988).

Table 6 shows site class indexes and standard deviations obtained for B site class for all classical and fuzzy cases for period 1.0 s. These standard deviations are related to the variation of the station coefficients in site class (σ_s) for classical and fuzzy cases for B site class and for fuzzy cases obtained using Eqs. 25 and 26.

5 Results of analysis

Data from the paper (Field 2000) is used as an example for SA. The fuzzy analysis gives smaller values of the standard deviations than those of the classical case for SA.

First, the membership functions for SA are designed by considering existing information about shear wave velocity values. Field (2000) provides the information about site characteristics, that is, shear wave velocity at the surface. If there is no measurement or any

Table 6 Site class indexes and standard deviations for all site classes for classical and all fuzzy cases for $T = 1.0$ s

		Classical	Second type fuzzy classification						
			By centroid of trapezoidal function	Combined	Very certain	Certain	Moderately certain	Uncertain	Very uncertain
A class	Site class index (m/s)	1,857.6	1,799.8	1,798	1,695.8	1,721	1,799	1,802	1,813.5
	Stand. deviat. (m/s)	256	214.8	216	168	189	213	228	258
B class	Site class index (m/s)	1,066	966.5	966.8	963.7	965.4	966.9	967.9	968.7
	Stand.deviat. (m/s)	391	291	293	188.7	239	291	345	399
C class	Site class index (m/s)	471	490.1	491	493.8	492	489.7	486.6	482.2
	Stand. deviat. (m/s)	219	132.7	134	109	121	132	165	218
D class	Site class index (m/s)	285	273.4	274.3	265.8	269.6	272.8	276.7	283.5
	Stand. deviat. (m/s)	76	53	53.5	43	48	54	65	77
E class	Site class index (m/s)	157	159.9	160.2	165.1	163.4	161	158.3	156.7
	Stand. deviat. (m/s)	45	36	35	29	33	35.5	38	46

existing information, then membership function can be obtained as a linguistic term using a symmetric (quadruplet) membership function, but if there is a possibility to obtain some information, then such information can be used for the design of the membership function as it is done in the present work. Thus, membership functions are obtained more precisely; the site class indexes and the standard deviations are calculated quantitatively. In the fuzzy case, standard deviations are significantly reduced as compared to the classical case (Table 3 and 4).

As shown above, the membership functions for SA are obtained in an appropriate manner. Thus, the uncertainty scaling is applied to SA.

6 Discussion and conclusion

There are many causes of uncertainty throughout the seismic hazard analysis process. The attenuation relation of ground motion that plays a main role brings many uncertainties into the analysis. The present study focused on the uncertainty related with the site classification for the acceleration response spectra (SA).

In this study, standard deviations that show variations between each site class obtained by fuzzy set theory and the classical means are compared. Results on this analysis show that even when we have very insufficient data for hazard assessment, site classification based on fuzzy set theory shows values of standard deviations less than those obtained in the classical manner, which is direct proof of less uncertainty. The example performed in this work shows that variances based on station coefficients significantly reduce in the fuzzy case in comparison with the classical case. As shown here, designing membership function is very important. If we design appropriate membership function, then we can reduce uncertainty.

Given, membership functions allow the application of various methods of fuzzy set theory. In the present study, the uncertainty scaling is selected among them. This method can handle the difference of uncertainty level or reliability of judgment for site classification; for example, if we have to classify site conditions only based on geological information or some other non-numerical information, then we use membership function for the "very uncertain" level. If we have some geotechnical measurement or some more precise information, then we can select a lower uncertainty level (toward "very certain"). Also, not only the standard deviations but also the site class indexes can be changed by the uncertainty scaling. This shows a new aspect of the empirical attenuation relation.

All the discussions mentioned above prove the advantage and usefulness of fuzzy set theory, not only for attenuation relation of ground motion but also in many cases where some primary data are insufficient or ambiguous. Especially for seismic hazard assessment, it is necessary to handle uncertainty, and fuzzy set theory allows for this. Fuzzy set theory may be useful not only for seismic hazard but also for all kinds of natural hazards; however, there are many terms that are not understood very well quantitatively, which should be considered in hazard assessment.

References

Atkinson GM (2004) An overview of developments in seismic hazard analysis. 13th World conference on earthquake engineering. Vancouver, B.C., Canada, August 1–6, paper no 5001

Bailey D (2003) A decision support system for site selection of large-scale infrastructure facilities using natural language. In: proceedings of the conference: operations research into the 21st century. The Austrian Society for Operations Research (AOSM), pp 1–17

Carlsson Ch, Fullér R (2000) Real option evaluation in fuzzy environment. In: Proceedings of the international symposium of hungarian researchers on computational intelligence, Budapest, pp 69–77, [ISBN 963 00 4897 3]

Christensen NL, Bartuska AM, Brown JH, Carpenter S, D'Antonio C, Francis R, Franklin JF, MacMahon JA, Noss RF, Parsons DJ, Peterson CH, Turner MG, Woodmansee RG (1996) The report of the ecological society of America committee on the scientific basis for ecosystem management. Ecol Appl 6(3):665–691

De Campos LM, Bolaños MJ (1989) Representation of fuzzy measures through probabilities. Fuzzy Sets Syst 31:23–36

Debruin S, Bregt A (2001) Assessing fitness for use: the expected value of spatial data sets. Int J Geogr Inf Sci 15(5):457–471

Dubois D, Prade H (1988) Theory of possibility an approach to computerized processing of uncertainty. Plenm Press, New York

Field EH (2000) Modified ground-motion attenuation relationship for southern California that accounts for detailed site classification and a basin-depth effect. Bull Seismol Soc Am 90:S209–S221

Giovinazzi S, Lagomarsino S (2004) A macroseismic method for the vulnerability assessment of buildings. 13th World conference on earthquake engineering. Vancouver, B.C., Canada, paper no 896

Irikura K, Miyake H, Iwata T, Kamae K, Kawabe H (2004) Recipe for predicting strong ground motions from future large earthquakes and Luis Angel Dalguer. 13th World conference on earthquake engineering. Vancouver, B.C., Canada, paper no 1371

Kaufmann A (1988) Theory of expertons and fuzzy logic. Fuzzy Sets Syst 28(3):295–305

Kaufmann A, Gupta MM (1985) Introduction to Fuzzy arithmetic. Van Nostrand Reinhold Company Inc, New York

Molas GL, Yamazaki F (1995) Attenuation of earthquake ground motion in Japan including deep focus events. Bull Seismol Soc Am 85(5):1343–1358

Schmucker Kurt J (1984) Fuzzy sets, natural language computations, and risk analysis. Computer Science Press, Rockville

Steidl H, Lee Y (2000) The SCEC phase III strong-motion database. Bull Seismol Soc Am 90(6B):S113–S135

Wills CJ, Petersen M, Bryant WA, Reichle M, Saucedo GJ, Tan S, Tayler G, Treiman J (2000) A site-conditions map for Calofornia Based on Geology and Shear-Wave Velocity. Bull Seismol Soc Am 90(6B):S187–S208

Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353